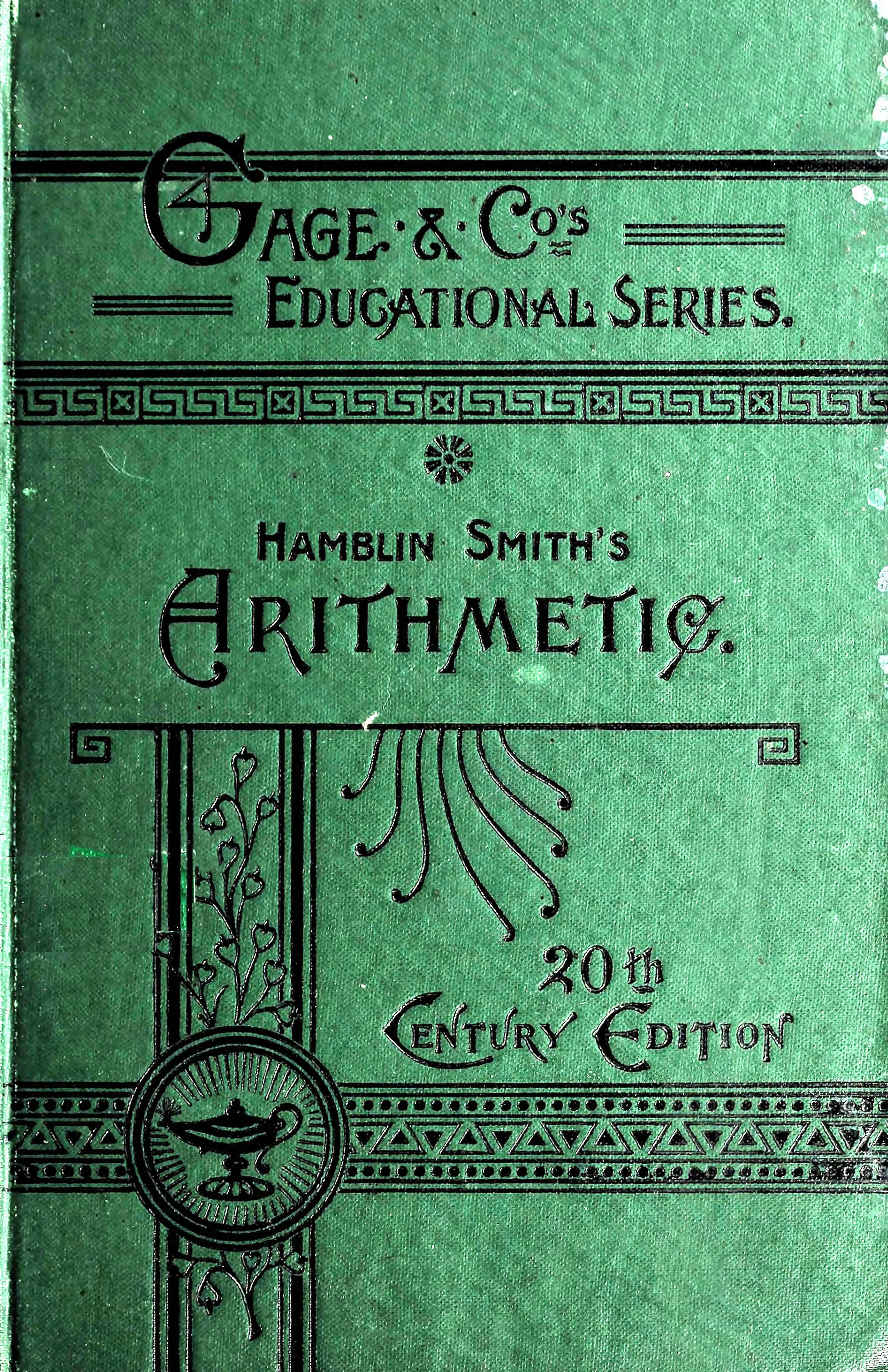

GAGE & CO'S
EDUCATIONAL SERIES.

HAMBLIN SMITH'S
ARITHMETIC.

20th
CENTURY EDITION

W. J. Gage & Co.'s Mathematical Series.

A TREATISE
ON
ARITHMETIC.

BY

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ADAPTED TO CANADIAN SCHOOLS

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PREFACE.

THIS edition of HAMBLIN SMITH'S ARITHMETIC differs much from the former one which has been used for a number of years in Canadian schools. The chapter on Decimal Notation, the elementary parts of the Simple Rules, Measures and Multiples, and the Compound Rules, have been omitted, the student having mastered these in the elementary stage. This has left space for a much more extended treatment of subjects than was given in the former edition, and has allowed the introduction of much new matter. The subjects of Percentage, Discount, Stocks, Exchange, Proportion, the Metric System, Mensuration, etc., have been treated at greater length in this than in the former edition. The chapter on Mensuration has been greatly enlarged, and the measurement of all the common surfaces and solids discussed. A chapter on Scales of Notation has been introduced to enable the student to obtain a greater mastery of the decimal system. The Metric System has been treated at considerable length, the tables have been given, and many problems, to illustrate the various measures, introduced. Many of the more simple miscellaneous problems, at the end of the former book, have been omitted and their places supplied with others more in keeping with the stage of advancement of the pupil. These have been classified, and their number increased from 350 to 528.

With the view of illustrating the requirements in this subject, in each Province of the Dominion, sets of Examination Papers, of recent issue, have been added. The answers to these have not been printed, the author believing that many teachers would prefer to test their pupils without this aid. These will be solved in the Key.

Thus, while the author has preserved all the characteristic features which made the former edition so helpful to students, he believes that the numerous changes and additions will add greatly to the value of the present work.

TORONTO, Oct. 23rd, 1900.

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ARITHMETIC

CHAPTER I.

Scales of Notation.

1. If two counters are taken and arranged into a group of two, there is one group and none over. These may be represented by the figures 10, where the 0 shows there are no single counters over, and the 1 represents one group of two counters.

If three counters are taken and arranged into a group of two and one, the group and single counter could be represented by 11, where the right-hand 1 represents one counter and the left-hand 1, one group of two counters.

Again if four counters are taken and arranged into groups of two, there would be two groups of two counters each and no counters over. If the two groups are arranged into a larger group or set with *two times two* counters in it, there would be one set of two times two counters and no groups of two counters over. Hence four would be represented by 100, where the right-hand 0 represents no single counters, the second 0, no groups of two counters, and 1, one group or set of two times two counters.

In a similar way

Five	would	be	represented	by	101
Six	“	“	“	“	110
Seven	“	“	“	“	111
Eight	“	“	“	“	1000
Nine	“	“	“	“	1001
Ten	“	“	“	“	1010
Eleven	“	“	“	“	1011
Twelve	“	“	“	“	1100

2. If instead of arranging counters into groups of two, they are arranged into groups of **Three :**

Three	would	be	represented	by	10
Four	“	“	“	“	11
Five	“	“	“	“	12
Six	“	“	“	“	20
Seven	“	“	“	“	21
Eight	“	“	“	“	22
Nine	“	“	“	“	100
Ten	“	“	“	“	101
Eleven	“	“	“	“	102
Twelve	“	“	“	“	110

3. If instead of arranging counters into groups of two or three, they are arranged into groups of **Ten :**

Ten	would	be	represented	by	10
Eleven	“	“	“	“	11
Fifteen	“	“	“	“	15
Eighty-five	“	“	“	“	85
Ninety-nine	“	“	“	“	99
One hundred	“	“	“	“	100

Notice that, in section 3, 10 represents ten and is to be read *ten ;* in section 1, 10 represents two and is to be read *one nought ;* in section 2, 10 represents three and is to be read *one nought.*

4. Instead of arranging counters as above, let us arrange them into groups of five counters each, until the remaining number of counters is less than five.

Let all the groups of five be arranged into *sets* or larger groups, each *set* to contain five groups, *i.e.*, five times five counters, until the remaining number of groups is less than five.

Let all the sets be arranged in *heaps*, or still larger groups, each *heap* to contain five times five times five counters, until the remaining number of *sets* is less than five, etc.

In such a grouping of numbers

Five = 10; Six = 11; Seven = 12;
Eight = 13; Nine = 14; Ten = 20;
Eleven = 21; Twelve = 22; Thirteen = 23;
Fourteen = 24; Fifteen = 30; Sixteen = 31;
Twenty = 40; Twenty-four = 44; Twenty-five = 100;
Thirty = 110; Thirty-four = 114; Forty-two = 132;

(Note: For the words, *would be represented by* = is used.)

In 132, the 2 denotes two counters; 3 denotes three groups of five counters each; and 1 denotes one set of five times five counters.

132 is not to be read one hundred and thirty-two but as *one three two.*

5. Such a system of notation as that adopted in section 1, is a *binary* scale of notation, and two is said to be its *radix;* that in section 2, is a *ternary* scale of notation and three is the radix; that in section 3, is a *denary* or *decimal* scale and ten is the radix; and that in section 4, is a *quinary* scale and five is the radix.

6. A Scale of a system of notation is the law of relation between its successive orders of units.

7. The Radix of the scale is the number which expresses the relation of the successive orders.

8. A scale whose radix is *two* is called *Binary;* three, *Ternary;* four, *Quaternary;* five, *Quinary;* six, *Senary;* seven, *Septenary;* eight, *Octenary;* nine, *Nonary;* ten, *Denary* or *Decimal;* eleven, *Undenary;* twelve, *Duodenary* or *Duodecimal*, etc.

9. From sections 1, 2, 3 and 4 it will be evident why the figure of greatest value used in any scale is one less than the radix of that scale.

10. To Change a Number from any Scale to the Decimal Scale.

Ex. 1. Change 1234 from the senary scale into the decimal one.

$$\begin{array}{rcl} & 4 = & 4 \\ 3 \times & 6 = & 18 \\ 2 \times & 6^2 = & 72 \\ 1 \times & 6^3 = & 216 \\ \hline & & 310 \end{array}$$

$\therefore$ 1234 senary = 310 decimal scale.

Or thus, 1234 One of the 4th order = 6 of the 3rd order
6 6 + 2 = 8 " " " "
8 Eight of the 3rd order = 48 " " 2nd order
6 48 + 3 = 51 " " " "
51 Fifty-one of the 2nd order = 306 " " 1st order
6 306 + 4 = 310 " " " "
310

Hence, *Multiply the number expressed by the left hand figure by the given radix and to the product add the number expressed by the next figure.*

Then multiply this sum by the radix and add to the product the number expressed by the next figure. Continue thus until all the figures have been used. The last sum will be the number in the decimal scale.

Examples i.

Express each of the following numbers in the decimal scale:—

1. 101011 binary.
2. 3024 quinary.
3. 34550 senary.
4. 67745 octenary.
5. 70808 nonary.
6. 4567 duodenary.

7. How many units are expressed by 10101 in the scale of radix 4?

8. A number is expressed in the scale of 8 by the figures 7070. Write the number in words in the scale of ten.

11. To Change a Number from the Decimal Scale into any other.

Ex. 2. Express 246 in the quaternary scale.

4 | 246
4 | 61 groups of 4 and 2 units over.
4 | 15 groups of 4^2 " 1 group of 4 over.
3 groups of 4^3 " 3 groups of 4^2 over.

∴ 246 decimal = 3312 quaternary.

Hence, *divide the number in the decimal scale continually by the radix of the proposed scale till the quotient is less than the radix.*

Write the last quotient and the successive remainders in order from left to right, placing a 0 *wherever there is no remainder.*

The result will be the number required.

Examples ii.

1. Change 4765 from the decimal scale to the quinary.

2. Change 5678 from the decimal to the octenary scale.

3. Transform 12345 from the decimal to the senary scale.

4. Express 7777 decimal scale in the septenary scale.

5. Express 846 decimal scale in the scale in which the figure of greatest value is 5.

12. The ordinary operations of arithmetic may be performed in any scale; but, remembering that the successive powers of the radix are not powers of ten, we must not divide by ten, but by the radix of the scale in question, to determine what must be carried.

Ex. 3. Add together 343, 123, 455, and 544 in the senary scale.

$$\begin{array}{r} 343 \\ 123 \\ 455 \\ 544 \\ \hline 2353 \end{array}$$

The sum of the first column is fifteen, which divided by six gives 2 to carry and 3 to set down; the sum of the second column is seventeen, which divided by six gives 2 to carry and 5 to set down; the sum of the third column is fifteen, which divided by six gives 2 and 3 to set down.

Ex. 4. From 347 nonary take 178 nonary.

$$\begin{array}{r} 347 \\ 178 \\ \hline 158 \end{array}$$

As 8 cannot be taken from 7, one is borrowed from 4; one of the second order = nine of the first; nine and seven = sixteen; eight from sixteen leaves 8, etc.

Ex. 5. Multiply 452 octenary scale by 6.

$$\begin{array}{r} 452 \\ 6 \\ \hline 3374 \end{array}$$

6 times 2 = 12; 12 ÷ 8 gives 1 to carry and 4 to set down.

6 times 5 = 30; 30 + 1 = 31; 31 ÷ 8 gives 3 to carry and 7 to set down, etc.

Ex. 6. Divide 5634 septenary scale by nine.

$$\begin{array}{r|r} 9 & 5634 \\ \hline & 442 \end{array}$$

Nine is not contained in 5.

$5 \times 7 + 6 = 41$; $41 \div 9$ gives 4 and 5 over.

$5 \times 7 + 3 = 38$; $38 \div 9$ gives 4 and 2 over, etc.

Examples iii.

1. Add in the quaternary scale, 333, 23, 1032, 222, 123.
2. Add in the nonary scale, 4567, 344, 7064, 8888, 3401, 7007.
3. Take 1010 binary scale from 10000 binary.
4. Take 3214 senary scale from 5011 senary.
5. Multiply 64325 septenary scale by 7.
6. Multiply 435 by 567 in the nonary scale.
7. Divide 1624 by 5 in the octenary scale.
8. Divide 337740 by 6 in the nonary scale, and express the quotient in the denary scale.

13. To Change a Number from one Scale to another Scale.

Ex. 7. Transform 2342 quinary scale into the quaternary one.

$$\begin{array}{rl}
4 & \underline{|\,2342} \\
4 & \underline{|\,321}-3 \\
4 & \underline{|\,41}-2 \\
4 & \underline{|\,5}-1 \\
 & \;\;1-1
\end{array}$$

The division by 4 is performed as illustrated in Ex. 6.

2342 quinary = 11123 quaternary.

Hence, *divide the number in the given scale continually by the radix of the proposed scale till the quotient is less than the radix.*

Write the last quotient and the successive remainders in order from left to right, placing a 0 *wherever there is no remainder.*

The result will be the number required.

Examples iv.

1. Convert 73421 from the octenary to the ternary scale.
2. Convert 30030 from the quaternary to the septenary scale.
3. By division convert 3456 septenary scale into the decimal one.
4. Express 760 decimal scale in the nonary one.
5. Convert 4*eet*3 duodecimal scale to the decimal one. (e = eleven and t = ten).
6. Express 9*tt* undenary scale in the denary one.
7. Reduce 46700 from the octenary scale to the decimal scale in three different ways.

14. Fractions may be transformed from one scale to another by expressing the numerator and denominator, respectively, in the scale required.

Thus, $\frac{35}{54}$ in denary = $\frac{55}{130}$ in senary = $\frac{43}{66}$ in octeneary.
$\frac{23}{40}$ in quinary = $\frac{14}{22}$ in nonary = $\frac{13}{20}$ in denary.

15. Fractions corresponding to decimal fractions are treated just as in decimals. The point is called the

Radix Point, and the fraction is known as the **Radix Fraction.**

Thus, $.2304 \text{ quinary} = \frac{2}{5} + \frac{3}{5^2} + \frac{0}{5^3} + \frac{4}{5^4}.$

16. A vulgar fraction expressed in any scale may be reduced to a radix fraction of that scale in a similar manner to its reduction to a decimal.

Ex. 1. Express $\frac{3}{4}$ of a unit in the senary scale.

$$\begin{array}{r} 4\ )\ 3.00 \\ \hline .43 \end{array}$$

We say, 3 units are 18 sixths of a unit. The fourth of 18 sixths is 4 sixths and remainder, 2 sixths. 2 sixths are 12 thirty-sixths; one fourth of 12 thirty-sixths is 3 thirty-sixths.

Ex. 2. Express $\frac{17}{30}$ nonary as a radix fraction.

$$\begin{array}{r} 30\ )\ 17.00\ (\ .53 \\ 160 \\ \hline 100 \\ 100 \\ \hline \end{array}$$

17 units nonary = 170 ninths of a unit. 170 ninths ÷ 30 gives 5 ninths for quotient, and remainder 10 ninths. 10 ninths $= 100\ (\frac{1}{9} \times \frac{1}{9})$; $100\ (\frac{1}{9} \times \frac{1}{9}) \div 30 = 3\ (\frac{1}{9} \times \frac{1}{9})$. Hence, $\frac{17}{30}$ nonary = .53 nonary.

Ex. 3. Reduce .4513 senary to the denary scale.

$$\begin{aligned} .4513 \text{ senary} &= \frac{4}{6} + \frac{5}{6^2} + \frac{1}{6^3} + \frac{3}{6^4} \\ &= \frac{4 \times 6^3 + 5 \times 6^2 + 1 \times 6 + 3}{6^4} \\ &= \frac{1053}{1296} = .8125 \end{aligned}$$

$$\begin{array}{r} \text{Or } .4513 \\ 10 \\ \hline 8.0430 \\ 10 \\ \hline 1.1300 \\ 10 \\ \hline 2.3000 \\ 10 \\ \hline 5.0000 \end{array}$$

.4513 senary = .8125 denary.

Ex. 4. Reduce $47.291\dot{6}$ from the scale of ten to the scale of twelve.

The whole number and the decimal must be worked separately.

$$12\,\underline{)\,47}\qquad\qquad\qquad \begin{array}{r} .291\frac{2}{3} \\ 12 \\ \hline 3.500 \\ 12 \\ \hline 6.000 \end{array}$$
$$\;\;3e$$

Hence, 47.2916 denary $= 3e.36$ duodenary.

Ex. 5. Reduce $.1\dot{5}$ septenary to the decimal scale.

$.1\dot{5}$ septenary $= \frac{14}{60}$ septenary $= \frac{11}{42}$ denary $= .\dot{2}61904\dot{7}$.

Examples v.

Reduce the following:

1. $\frac{17}{48}$ denary to duodenary. 2. $\frac{5}{16}$ octenary to denary.

3. $\frac{3}{25}$ senary to quaternary. 4. $\frac{47}{70}$ octenary to septenary.

Express as radix-fractions in the scale (i) of 3, (ii) of 6, (iii) of 10, (iv) of 12, (v) of 8.

5. $\frac{2}{3}$. 6. $\frac{3}{5}$. 7. $\frac{5}{6}$. 8. $\frac{7}{8}$.

Express as radix-fractions in the senary scale.

9. .25. 10. .375. 11. .125. 12. .0025.

Express as decimals.

13. .101 binary. 14. .203 quaternary. 15. .*te*3 duodenary.

EXAMINATION PAPERS.

I.

1. Name the scale of notation in which the digit of greatest value is 7; is 4; is *e* (eleven).

2. Explain what each figure of 40302 senary scale represents.

3. Change the largest number that can be expressed by four figures in the septenary scale into the denary one.

4. Change the smallest number that can be expressed by four figures in the quinary scale into the binary one.

5. Divide the difference between 1130305 and 235143 senary scale by 4.

II.

1. In every scale, what do the figures 10 always express? The figures 100? The figures 1000?

2. Multiply 41625 septenary scale by 6 and express the result in the denary scale.

3. Divide 1133133 by 111 in the scale of 4 and express the quotient in the scale of twelve.

4. From 40623 septenary take 44154 senary and express the result in the decimal scale.

5. Add together 345, 456, 471, 564, and 701 octenary scale and show wherein the process differs from the addition of £ s. d.

III.

1. Write the multiplication table for 5 times in the scale of radix 6.

2. Square in the octenary scale the largest number that can be expressed by three figures in the octenary scale.

3. In what scales from two to twelve can the following fractions be expressed (i) in terminating, (ii) in recurring form?

$$\tfrac{1}{3},\ \tfrac{5}{6},\ \tfrac{3}{4},\ \text{and}\ \tfrac{7}{36}.$$

4. Transform 4005·265 from the scale of 8 to that of 10.

5. Multiply the sum of the sum, difference, product and quotient of 5233 senary and 123 senary by 305 senary and express the result in the duodenary scale.

IV.

1. Transform 205 and 345 from the senary scale to the octenary and find the product of the answers in the undenary scale.

2. Multiply *tee* by *e*7 in the duodenary scale, and prove the result by division.

3. Divide 4336 by 23 in the nonary scale and multiply the result by *t*0*e* in the duodenary scale.

4. Any number in the scale of five, when divided by 4 gives the same remainder as the sum of its digits divided by 4.

5. Write down the first twenty numbers in the scale of 4, find their sum in this scale and express the result in the decimal scale.

V.

1. Show in what scale the sum of 4 and 4 is expressed by 13.

2. Find the scale in which the product of seven and eight is expressed by 62.

3. Find the G. C. M. and L. C. M. of 223 and 320 septenary scale.

4. By how many minutes was the month of February, 1896, longer than the same month in 1900? Express this number of minutes (1) in the denary scale; (2) in the octenary; and (3) in the undenary one.

5. The circumference of a bicycle wheel is $10\frac{1}{1}\frac{1}{1}0$ ft. binary scale. How often will it turn in going one mile?

VI.

1. Transform $\frac{37}{82}$ and $75\frac{50}{69}$ from radix ten to radix twelve.

2. Which of the weights 1 lb., 2 lb., 4 lb., 8 lb., 16 lb., 32 lb., etc., must be used to balance 45 lb.? 136 lb.?

3. Which of the weights 1 lb., 3 lb., 9 lb., 27 lb., 81 lb., etc., must be used to balance 433 lb., only one of each kind being used?

4. Divide 5633456 by 362 in the scale in which the digit of greatest value is 7.

5. Transfer *et.* 9 from the duodenary to the scale in which seventy-five is expressed by 83.

CHAPTER II.

Practical Methods of Shortening Labor and of Verifying Results.

17. Addition. The usual verification is to add both upwards and downwards, and see if the sums agree. This is generally sufficient.

Another method is to divide the addends into a number of groups and find the sum of each group, then find the sum of the answers, which must agree with the work it is to verify.

Examples vi.

Add together and verify the results:—

1. 7569475
5879847
3689765
7849765
7777777
9759756
6485869
4968497
6666666
7777888
5596877
9684769

2. 8647789
6948599
8796584
9797977
8989898
8476897
7648976
5869487
7777777
8888889
7485697
4785896

3. 9674789
3687896
5769467
8888888
7766885
4785689
3968796
6894785
5696879
7684967
7895948
4765839

4. 98765436
55667788
66666666
56756788
96789678
56945694
38756967
78948965
49678975
59483879
47676764
78964897

5. 78978966
66778899
88888888
45645677
99999999
78567865
49785678
58658984
78989786
89498768
47958965
68497689

6. 55667788
99999999
77777777
56785678
78907890
49678947
56978496
76898497
45687698
76896479
36854975
55566677

18. Subtraction. The correctness of the result in subtraction may be tested by adding the remainder, or difference to the subtrahend, when the result ought to be the same as the top line, or minuend.

Examples vii.

Subtract and verify the results:—

1. 720014539653
   488877664789

2. 700700700700
   123456789496

3. 584746340664
   428765082976

4. 304040607080
   125847283916

5. Subtract 5476587 ten consecutive times from 59487645 and add the ten successive remainders.

19. Multiplication. The proof of multiplication by *casting out the nines* depends on the following property of numbers:—

Any number divided by nine will leave the same remainder as the sum of its digits divided by nine.

This will be evident from the following example:—

$$\begin{aligned}\frac{6783}{9} &= \frac{6000}{9} + \frac{700}{9} + \frac{80}{9} + \frac{3}{9}\\ &= (666 + \tfrac{6}{9}) + (77 + \tfrac{7}{9}) + (8 + \tfrac{8}{9} + \tfrac{3}{9})\\ &= 666 + 77 + 8 + \tfrac{6}{9} + \tfrac{7}{9} + \tfrac{8}{9} + \tfrac{3}{9}\\ &= 751 + \frac{6+7+8+3}{9}.\end{aligned}$$

Hence it is clearly seen that the remainder, arising from the division of 6783 by 9 is the same as that arising from the division of the sum of the digits by 9.

This test may be given in the form of the following rule:—

Divide the sum of the digits in the Multiplicand by 9, and set down the remainder. Divide the sum of the digits in the Multiplier by 9, and set down the remainder. Mul-

tiply the two remainders together, divide the result by 9, *and set down the remainder. If the process be correct, this remainder will be the same as the remainder obtained by taking the sum of the digits in the Product and dividing it by* 9.

For example, if we multiply 76371 by 854 the product is 65220834.

Sum of digits in Multiplicand = 24,

and 24 ÷ 9 gives remainder 6.

Sum of digits in Multiplier = 17,

and 17 ÷ 9 gives remainder 8.

First remainder × second remainder = 48,

and 48 ÷ 9 gives remainder 3.

Sum of digits in the Product = 30,

and 30 ÷ 9 gives remainder 3.

This so-called proof is defective as a proof in the following, as its fails to detect errors in the product—

i. If the order of figures in the product is misplaced, as 37 for 73.

ii. If errors be made which counterbalance each other, as 35 written for 62, the sum of the digits in each case being the same.

iii. If 9 be written for 0, or 0 for 9, or either be omitted or inserted too often.

Examples viii.

Multiply the following and verify the results:—

1. 789658 by 976409
2. 9009067 by 4906708
3. 567008 by 980980
4. 6700097 by 8569067
5. 7080506 by 4890675
6. 4956742 by 8090067

20. Division. To prove division, multiply the divisor by the quotient, and add the remainder, if there is one, to the product. If the result is equal to the dividend, we have a verification of the first operation.

Division may also be proved by casting out the nines, but the proof is less direct than in multiplication. For instance, if we divided 417 by 29 the quotient is 14 with remainder 11. The most convenient form in which to apply the proof of nines is to write this in the form of 29 × 14 + 11 = 417. The remainder gives 2 × 5 + 2 or 12. This remainder and the dividend, 417, divided by 9, give a remainder 3, which therefore proves the work.

Examples ix.

Divide the following and prove the results correct:—

1. 978543964 by 8976
2. 239478596 by 4785
3. 678392067 by 54809
4. 796487006 by 19085
5. 789684700695 by 8 ten successive times.

21. Arithmetical Complement. The arithmetical complement of a number is defined to be the difference between any given number and the unit of the next superior order; thus 6 is the arithmetical complement of 4, 47 of 53, 8468 of 1532, and so on, being the differences respectively of 4, 53, 1532, and 10, 100, 10000, the next superior units of these numbers. Conversely, also, 4, 53, 1532 are the arithmetical complements of 6, 47, 8468 respectively.

The arithmetical complement of a number may be found by the following rule:—

Begin at the left hand and subtract every figure from 9 *until the last; subtract that from* 10.

The arithmetical complement may be used to find the difference between two numbers, thus: if 239 be subtracted from 576 the remainder is 337. But if 761, the arithmetical complement of 239, the less number, be added to 576, the greater, the sum will be 1337, one unit (1000 in this case) of the next superior order greater than the difference of the two numbers. By removing this unit, the number will be left equal to the difference of 239 and 576; so that the difference of the two numbers can be found by addition. The arithmetical complement may be written thus $\bar{1}761$, with the subtractive unit on the left, which when added to 576, the sum will be 337, the additive and subtractive units being together equal to zero.

This method is employed with great advantage to find the aggregate of several numbers when some of them are additive and some subtractive. Thus, if we have—

$$3795 - 1532 - 2019 + 8759 - 5104,$$

we arrange them as follows:—

	3795
A. C. of 1532 is	$\bar{1}8468$
" 2019 "	$\bar{1}7981$
	8759
" 5104 "	$\bar{1}4896$
	3899

the aggregate required.

Examples x.

Simplify the following:—

1. 7364 − 2685 − 3687 + 4617 − 2857 − 1856.
2. 968 − 245 − 268 − 456 + 989 − 168 − 246.
3. 54967 + 85364 − 24164 − 14867 − 23641 − 14567.
4. 76 + 89 + 76 − 41 − 52 − 63 − 78 − 29 + 86 + 98.
5. 364 − 187 + 569 − 287 + 367 − 354 + 567 − 413.

22. Contractions in Multiplication. The multiplication by any number from 12 to 19 inclusive, may be effected as follows:

Multiply by the figure of the Multiplier in the units' place, and to the number to be carried add the figure of the Multiplicand just multiplied.

Ex. 1. Multiply 2384 by 19.

$$\begin{array}{r} 2384 \\ 19 \\ \hline 45296 \end{array}$$

$4 \times 9 = 36$; set down 6 and carry 3.

$8 \times 9 + 3$ carried $+ 4$, the units' figure of the multiplicand $= 79$; set down 9 and carry 7.

$3 \times 9 + 7$ carried $+ 8$, the tens' figure of the multiplicand $= 42$; set down 2 and carry 4.

2 × 9 + 4 carried + 3, the hundreds' figure of the multiplicand = 25; set down 5 and carry 2.

2 carried + 2, the thousands' figure of the multiplicand = 4; set down 4.

The *back figure system*, as it is sometimes called, may be extended to numbers between 20 and 30, and between 30 and 40, by adding to the number to be carried the *double* or the *treble* of the figure of the multiplicand just multiplied.

Ex. 2. Multiply 34578 by 999.

Here 34578000 = 1000 times 34578.
and 34578 = 1 " "

34543422 = 999 times 34578.

Ex. 3. Find the product of 34578 by 699.

Here 699 = 700 − 1
and 24204600 = 700 times 34578.
34578 = 1 " "

24170022 = 699 times 34578.

Hence, any number can be multiplied by 99, 999, 9999, etc., by annexing 2, 3, 4, etc., ciphers to the multiplicand, and subtracting the multiplicand from this product. And in a similar way any number can be multiplied by another composed of a repetition of this figure with any other figure in the highest place.

Ex. 4. Multiply 9643287 by 378427.

	9643287
	(378)(42)(7)
7 times the multiplicand =	67503009
42 times the multiplicand = 6 times 7 times multiplicand = 6 times 67503009	= 405018054
378 times the multiplicand = 9 times 42 times the multiplicand = 9 times 405018054	= 3645162486
	3649280169549

23. To Square any Number.

$$78^2 = 80 \times 76 + 2^2 = 6084$$
$$34^2 = 30 \times 38 + 4^2 = 1156$$
$$75^2 = 80 \times 70 + 5^2 = 5625$$

Add to and subtract from, the given number such a number as is necessary to form an exact number of tens. Multiply the resulting number by the number of tens thus found and to the product add the square of the number added and subtracted. The sum is the square of the given number.

24. To Multiply Two Numbers Together When the Sum of the Units is Ten and the Number of Tens is the Same in the Multiplier and Multiplicand.

$$46 \times 44 = 50 \times 40 + 6 \times 4 = 2024.$$
$$78 \times 72 = 80 \times 70 + 8 \times 2 = 5616.$$
$$996 \times 994 = 1000 \times 990 + 6 \times 4 = 990024.$$

Hence, *multiply the number of units in the multiplicand and multiplier together and set down the product in the tens' and units' places of the product. Increase the number of tens in the multiplicand by one and multiply the sum by the number of tens in the multiplier and place the figures of the product to the left of the part already found. The resulting number is the product required.*

25. To Multiply Using Arithmetical Complements when there are the Same Number of Figures in the Multiplier and the Multiplicand.

Ex. Multiply 987 by 994.

A. C. of multiplicand = 13
A. C. of multiplier = 6
$6 \times 13 = 78$; 078
$987 - 6 = 981$; 981078 = product required.

Hence, *multiply the A. C. of the multiplicand by the A. C. of the multiplier. Set down the product, filling up as many places to the left with naughts as will make the number of places equal to the number of figures in the multiplicand. Subtract the A. C. of the multiplier from the* *multiplicand and set the remainder to the left of the figures already placed. The result will be the product required.*

Examples xi.

Multiply the following:—

1. 87656 by 14
2. 768478 by 18
3. 76895 by 99
4. 876546 by 9999
5. 87684 by 599
6. 948763 by 9999
7. 94768975 by 545135 in three lines of partial products.
8. 75647096 by 512864 in three lines of partial products.
9. 83 by 83
10. 996 by 996
11. 997 by 993
12. 9992 by 9998
13. 994 by 9997
14. 99725 by 99988

26. Special Contractions in Multiplication.

i. To multiply by 5, multiply by 10 and divide the product by 2. $5 = 10 \div 2$.

ii. To multiply by 25, multiply by 100 and divide the product by 4. $25 = 100 \div 4$.

iii. To multiply by 125, multiply by 1000 and divide the product by 8. $125 = 1000 \div 8$.

Examples xii.

1. Multiply 78968 by 5; by 25; by 125.
2. Multiply 96547 by 5; by 25; by 125.
3. Multiply 707077 by 5; by 25; by 125.
4. Multiply 9011091 by 5; by 25; by 125.

27. Abbreviations in Division.

Any number can be divided by 9, 99, 999, etc., by successively dividing the given number by 10, 100, 1000, etc., respectively, and taking the sum of the successive remainders for the true remainder; except when the sum of the latter exceeds the next higher unit; in that case both the quotient and remainder must be increased by unity.

Ex. Divide 65874 by 99.

$$\begin{array}{r} 100)658,74 \\ 6,58 \\ 6 \\ \hline 665,39 \end{array}$$

Here the sum of the partial remainder is 138, and both the quotient and remainder must be increased by unity. The reason of this we leave as an exercise for the student.

There is a method of dividing one number by another, termed the Italian method, which materially shortens the process. In this method all the partial subtrahends are omitted, and only the partial remainders retained in the working.

Ex. Divide 108419716121 by 5783.

$$\begin{array}{l} 5783)108419716121(18748005 \\ \hline \qquad\quad 50589 \\ \hline \qquad\qquad 43257 \\ \hline \qquad\qquad\quad 27761 \\ \hline \qquad\qquad\qquad 46296 \\ \hline \qquad\qquad\qquad\quad 32121 \\ \hline \qquad\qquad\qquad\qquad 3206 \text{ final remainder.} \end{array}$$

The first step is simply subtraction, giving 5058 for remainder. The work of the next step is as follows: 8 times 3 is 24; 4 from 9 gives 5 (which put down) and carry 2. 8 times 8 and 2 gives 66; 6 from 8 gives 2 (which put down) and carry 6. 8 times 7 and 6 gives 62; 2 from 5 gives 3 (put down) and carry 6. 8 times 5 and 6 gives 46; 46 from 50 gives 4 (put down).

It sometimes happens that *one* has also to be carried from the subtraction. For instance in this case—

$$\begin{array}{l} 5783\,)\,50581\,(\,8 \\ \hline \quad\;\; 4317 \end{array}$$

We say: 8 times 3 is 24; 4 from 11 gives 7 (put down) and carry 3 (instead of 2). Then 8 times 8 and 3 gives 67; 7 from 8 gives 1 (put down) and carry 6, etc.

28. Special Contractions in Division.

i. To divide by 5, multiply by 2 and divide by 10, $5 \times 2 = 10$.

ii. To divide by 25, multiply by 4 and divide by 100, $25 \times 4 = 100$.

iii. To divide by 125, multiply by 8 and divide by 1000, $125 \times 8 = 1000$.

iv. To divide by 75, 175, 225, or 275, multiply by 4 and divide the product by 300, 700, 900, or 1100 as the case may be.

Examples xiii.

1. Divide 786497 by 9; by 99; by 999, by means of addition.
2. Divide 786975 by 5; by 25; by 125.
3. Divide 348695 by 5; by 25; by 125.
4. Divide 6478900 by 75; by 175; by 225; by 275.

29. Tests of Exact Divisibility.

i. A number is exactly divisible by 2 if its right-hand figure is zero or an even digit.

ii. A number is exactly divisible by 4 if its two right hand figures are zeros or express a number exactly divisible by 4.

iii. A number is exactly divisible by 8 if its three right-hand figures are zeros or express a number exactly divisible by 8.

iv. A number is exactly divisible by 3 if the sum of its digits is exactly divisible by 3.

v. A number is exactly divisible by 9 if the sum of its digits is exactly divisible by 9.

vi. A number is exactly divisible by 5, 25, or 125 if the number ends in 1, 2, or 3 zeros or if the number expressed by the right-hand figure, or by the two, or by the three right-hand figures is exactly divisible by 5, 25, or 125 as the case may be.

vii. A number is exactly divisible by 11 when the difference between the sum of the digits in the odd places and the sum of the digits in the even places is either 0 or exactly divisible by 11. Thus, 24794 and 829191 are exactly divisible by 11.

Examples xiv.

Find whether the following numbers are exactly divisible by 2, 3, 4, 5, 8, 9, 10, or 11:—

1. 117
2. 288
3. 495
4. 1050
5. 23472
6. 42345
7. 27464
8. 32495
9. 84732
10. 6480
11. 619182718

12. Change one figure in each of the following numbers, so as to make the number divisible by 2. (State what change you make, and why.)

4379 6479 5243 7957 4343
5627 8123 2147 8971 5557
8291 4871 9281 3629 4441

13. Change one figure in each, so that the number shall be divisible by 5. Change the last figure in each, so that the number shall be divisible by 3. Change the first figure in each, so that the number shall be divisible by 9.

EXAMINATION PAPERS.

I.

1. Upon what principle is the method of subtracting by "equal additions" based?

2. What number must be added to the product of 783 and 907 to get the product of 787 and 907?

3. How many times can 101 be subtracted from one million, and what will remain?

4. One spring supplies 119 barrels of water in 7 hours; another, 390 barrels in 15 hours; and a third 324 barrels in 18 hours. In how many hours will the three springs together fill a cistern holding 1647 barrels?

5. The quotient of a division question is 17 times the divisor and the divisor is 59 times the remainder. Find the dividend when the remainder is 305.

II.

1. Explain the method for the multiplication of two numbers, each consisting of several figures, and multiply 30071 by 20590, explaining the reason for each step of the process.

2. Multiply 76894754 by 112756 in three lines of partial products.

3. By what number must the product of the sum and difference of 8376 and 5684 be increased so that the result may be exactly divisible by 7859?

4. A drover bought 527 sheep at $2 per head; twice as many calves at thrice as much per head, 19 cows at $29 per head, and thrice as many horses as cows at four times as much apiece. How much did the whole drove cost him?

5. One-half the sum of two numbers is 4331, and one-half their difference is 3353. Find the numbers.

III.

1. Eight head of cattle at $23 each, and 7 horses at $89 each, were given for 3 acres of land. What was the land worth per acre?

2. If 18 men can reap a field in 76 days, how long will it take 19 men to reap the same field?

3. A man bought an equal number of sheep and cows for $6300. Each sheep cost $3.50, and each cow $21.50. How many of each did he buy?

4. It was found that after 789 had been subtracted 375 times from a certain number that the remainder was 362. Find the number.

5. The ages of three brothers are 19, 17, and 15 years, and their father wills them his property worth $35,700 according to their ages. What does each get?

IV.

1. There is a number which, when divided by 4, and the quotient diminished by 35^2 and the result multiplied by 10, and the product decreased by the difference between the arithmetical complements of 7846 and 3479 gives 883. Find the number.

2. If 5 lb. of tea are worth 15 lb. of coffee, and 4 lb. of coffee are worth 8 lb. of sugar, how many pounds of sugar are worth 75 lb. of tea? 2

3. Find the number from which if 13675 be taken the remainder will be 45209 less 27645.

4. A horse is worth 8 times as much as a saddle, and both together are worth $261. Find the value of the horse.

5. A dealer in cattle gave $6400 for a certain number, and sold a part of them for $3600 at $18 each, and by so doing lost $2 per head. For how much a head must he sell the remainder to gain $800 on the whole?

V.

1. Any number may be multiplied by 5, 25, 125, etc., by annexing 1, 2, 3, etc., ciphers respectively to the number, and then dividing it by 2, 4, 8, etc. Explain the reason of this rule.

2. Of what number is 99995 both divisor and quotient?

3. A person bequeathed his property to his 3 sons. To the youngest he gave $1789; to the second 5 times as much as to the youngest; and to the eldest 3 times as much as to the second; find the value of the property.

4. In walking a certain distance John takes 17694 steps; how many steps will James take in walking half the distance, John taking 3 steps for every 4 of James'?

5. A merchant failed and his goods were worth $7770. Out of this he can pay his creditors 37 cents on the dollar. One of his creditors got $1998 as his share. Find the merchant's indebtedness, and what he owed the one creditor.

VI.

1. In the multiplication of numbers, how do you prove the correctness of the operation by casting out the nines? Explain and give reasons for the rule, and show the errors to which it is liable.

2. Multiply together 172814412 and 987654321 in three lines of partial products.

3. Simplify 1 − 2 + 4 − 8 + 16 − 32 + 64 − 128 + 256 − 512 + 1024 − 2048 + 4096 − 8192 + 16384 − 32768 + 65536 − 131072 + 262144 − 524288 + 1048576 − 2097152 + 4194304.

4. Divide 7864643457 by 9999.

5. The quotient is equal to 6 times the divisor, and the divisor to 6 times the remainder, and the three together amount to 516; find the dividend.

VII.

1. Divide the sum of the products of (64 × 39) and (36 × 39) by 100, and tell why the digits of the answer should be the same as those of the multiplicand.

2. From 1000 subtract 482. Multiply 689 by the subtrahend, and also by the remainder. Add the two products. How could the product be found without performing the work in full?

3. How often may 7897 be subtracted from 978648, and what is the last remainder?

4. The product of three numbers is 196790480, the smallest is 365, and the product of this and the largest is 396755. What are the other two numbers?

5. Using the arithmetical complement square the following numbers:
99, 999, 9999, 99999. 98, 998, 9998, 99998, 97, 997, 9997, 99997.

VIII.

1. The divisor is 789, the quotient 789, and the remainder the largest possible. Find the dividend.

2. What is the nearest number to 37401 that can be divided by 784 without a remainder?

3. What is the nearest number to 25000 that can be divided by 575 without a remainder?

4. Find the least number which added to 65343214 will make it exactly divisible by 999.

5. The divisor and quotient are equal, and the remainder, 752, is the greatest possible. Find the dividend.

For additional examples, see page 294.

CHAPTER III.

MEASURES AND MULTIPLES.

30. The following rule for finding the H. C. F. will be found very convenient in practice:

i. *Divide all the given numbers by the least of them, and bring down the remainders.*

ii. *Divide the first divisor and all of the first remainders by the least of them, and bring down the remainders.*

iii. *Proceed in this manner until a remainder is found that will divide all the other remainders, and the divisor last used, and this will be the highest common factor required.*

Ex. 1. Find the H. C. F. of 365, 511 and 803

365,	511,	803.
365,	146,	73

We divide by 365, writing down the remainders 146 and 73. 73 will divide the first divisor, 365, and the other remainders, and is therefore the H. C. F.

Ex. 2. Find H. C. F. of 232, 290 and 493.

232,	290,	493.	
232,	58,	29.	H. C. F. is 29.

Ex. 3. Find H. C. F. of 492, 1476, and 1763.

492,	1476,	1763.	
492,	0,	287.	
205,		287.	
205,		82.	
41,		82.	H. C. F. is 41.

Ex. 4. Find H. C. F. of 148, 444, 592, 703.

148,	444,	592,	703.	
148,	0,	0,	111.	
37,			111.	H. C. F. is 37.

31. To find the Highest Common Factor by inspection.

The H. C. F. of two numbers is a factor of their difference, and of three or more numbers it is a factor of their *smallest* difference.

Ex. 1. Find the H. C. F. of 323 and 357.

The difference between 323 and 357 is 34,

$$34 = 2 \times 17.$$

It is seen that 2 is not a common factor; hence if these numbers have a common factor it must be 17. By trial 17 is found to be contained 21 times in 357, and must be contained in 323, (21 − 2) times, or 19 times.

Ex. 2. Find the H. C. F. of 1829 and 2419.

$$2419 - 1829 = 590$$
$$590 = 10 \times 59.$$

10 is not a common factor. Hence, if these numbers have a common factor it must be 59.

$$1829 \div 59 = 31$$
$$\therefore 2419 \div 59 = (31 + 10) = 41.$$

Ex. 3. Find the H. C. F. of 84, 105, 140, and 154.

The smallest difference is 14.

$$14 = 2 \times 7.$$

2 is not a common factor. By trial 7 is found to be a common factor, and is hence their H. C. F.

Examples xv.

Find the H. C. F. of the following:—

1. 529 and 667.
2. 296 and 407.
3. 506 and 308.
4. 1825 and 2555.
5. 110, 140, and 350.
6. 444, 592, and 703.

32. The following method of finding the L. C. M. should be followed by advanced pupils:

Set down the numbers in a line, then strike out any that are contained in any of the others. Divide those not struck out by any number that will exactly divide one of them; under any that it exactly divides, place the quotient; under any which contain some factor common to it, set down the *quotient, after striking out this factor; and bring down all the other numbers.*

Proceed in this way with the new line; and so on, until all the numbers left in any line have no common measure, but unity. Then the continued product of the numbers in this line and all the divisors is the L. C. M. of the given numbers.

Ex. 1. Find the L. C. M. of 4, 8, 10, 12, 16, 20, 24, 25, 30.

$$\begin{array}{r|l} 24 & \cancel{4}, \cancel{8}, \cancel{10}, \cancel{12}, 16, 20, 24, 25, 30. \\ \hline & \qquad\qquad\quad 2, \ \cancel{5}, \ 1, 25, \ \cancel{5}. \end{array}$$

$$\therefore \text{L. C. M.} = 24 \times 2 \times 25 = 1200.$$

Examples xvi.

Find the L. C. M. of

1. 18, 24, 40, 50, 60, 90.
2. 18, 35, 50, 60, 144.
3. 16, 39, 40, 65, 88, 120.
4. 27, 33, 54, 69, 132.
5. 63, 84, 99, 156.
6. 15, 26, 39, 65, 180.

EXAMINATION PAPERS.

I.

1. Find the least number which, divided by 13, 15, and 17, gives remainder 12 in each case.

2. If *A*, *B*, and *C* walk 103950 inches together, how often will they step at the same moment, *A* taking 33 inches at a step, *B* 27, and *C* 30?

3. How many rails will enclose a field 23023 feet long by 17765 feet wide, the fence being straight, and 6 rails high, the rails of equal length, and the longest that can be used?

4. Two cog-wheels containing 210 and 330 cogs respectively are working together. After how many revolutions of the larger wheel will two cogs which once touch, touch again?

5. Three numbers between 30 and 140 have 12 for their H. C. F., and 2772 for their L. C. M. Find the numbers.

II.

1. Explain how to find (1) the H. C. F. and (2) the L. C. M. of a series of numbers by resolving them into their prime factors.

2. A farmer has 600 bushels of wheat. What are the three smallest sized bags, and the three largest bins, holding an exact number of bushels, that will each measure the same without a remainder?

3. What is the smallest sum of money with which I can buy sheep at $5 each, cows at $22 each, or horses at $75 each?

4. Three horses are running round a race-course of 5280 yards; the first horse runs 440 yards a minute, the second 352 yards, and the third 264 yards. Find the time between their once coming all together, and their coming all together again.

5. Find the least number which divided by 675, 1050 and 4368, will leave the same remainder, 32.

III.

1. Explain how you would find all the divisors which a number has. Find those of 8100.

2. The L. C. M. of 2, 3, 4, 5, 6, 8, 9, and another number prime to them is 10440. What is this number?

3. How do you determine whether a number is prime or composite?
Which of the following numbers are prime and which composite: 3391, 2699, 14787 and 1477?

4. Three men, *A*, *B* and *C*, start together from the same place to walk round an island 60 miles in circumference. They walk in the same direction, *A* at the rate of 5 miles per hour, *B* at 4, and *C* at 3. In what time will all be together for the first time after starting, and how many miles will each have gone?

5. Find the greatest weight, in grains, that will measure both pounds Avoirdupois and pounds Troy, there being 5760 grains in one pound Troy, and 144 lbs. Avoirdupois contain as many grains as 175 lbs. Troy.

IV.

1. Define Factor, Measure, Multiple, and explain when a number is Prime and when Composite. In what digits must prime numbers end?

2. The product of two numbers is 1270374, and half of one of them is 3129. What is the other?

3. The fore-wheel of a carriage was 11 feet in circumference, and the hind one 13 feet. There being 5280 feet in a mile, how many miles had a carriage gone when the same spots which were

on the ground at the time of starting, had been on the ground 360 times at the same instant?

4. *A* can dig 36 post holes in a day; *B* can dig 32, and *C* 30 in the same time. What is the smallest number which will furnish exact days' labor either for each working alone or for all working together?

5. How many firkins of butter, each containing 56 lb. at 23 cents per lb., must be given for 14 bbl. of sugar, each containing 276 lb. at 8 cents per lb.?

V.

1. Explain the use of zero in decimal notation.

2. Find the greatest number which will divide 10974 and 15336, leaving as remainders respectively 54 and 36.

3. The digits in the units' and millions' places of a number are 2 and 7 respectively. What will be the digits in the same places when 999999 is taken from the number?

4. An avenue 3 miles long is planted with 5 rows of trees. The trees are placed in the different rows at the distances of 6, 8, 9, 10, and 12 feet respectively. If the rows start from the same straight line, (1) how often will 5 trees be in a line, there being 5280 feet in a mile? and (2) how many trees will there be in the avenue?

5. A number is composed of the following factors: 2^4, 3^2, 5^3, 11, and 17. Find the number.

VI.

1. What numbers between 400 and 500 will divide 211850 without a remainder?

2. The product of five consecutive numbers is 254251200. Find the numbers.

3. Prove that the product of any four consecutive numbers is exactly divisible by $1 \times 2 \times 3 \times 4$.

4. From a heap of cannon balls weighing 13092 lb., a number weighing 9852 lb. was taken. Find the greatest possible weight of each ball, supposing they were of equal weight.

5. On counting out the marbles in a bag, 5 at a time, or 6 at a time, or 7 at a time, there are always 4 over. But on counting them 11 at a time, there are none over. What is the least number of marbles in the bag?

For additional examples, see page 295.

CHAPTER IV.

FRACTIONS.

33. Numbers are the measures of quantities.

A *Quantity* is anything which may be regarded as being made up of parts, like the whole.

Thus a sum of money is a quantity, because we may regard it as made up of parts like the whole.

To measure any quantity we fix upon some known quantity of the same kind for our standard or UNIT, and the NUMBER, which expresses how many times this Unit is contained in the quantity, is called the MEASURE of the quantity.

To put this in a more *practical* shape, we give the following illustration: We measure large sums of money by the *Unit* which we call a *Dollar*, and when we say that a man's income is *two thousand a year*, we mean that he receives yearly a sum of money which contains the unity two thousand times, and we call the number Two Thousand the *measure* of his income.

34. Now we can conceive that a unit of measurement may be divided into a number of parts of *equal* magnitude.

For instance, if we take a dollar as the *Unit* by which we measure sums of money, we suppose this Unit to be divided into one hundred equal parts, and we call each of these parts *one-hundredth* of a Dollar; *two* such parts will be two-hundredths, *three* will be three-hundredths of a Dollar. Such parts are called Fractions of a Dollar, or other Unit, and we give the following definition:—

DEF.—A **Fraction** is an expression representing one or more of the equal parts of a **Unit.**

The number of equal parts into which the **Unit** is divided is called the **Denominator** of the Fraction, and the number expressing how many of these parts are taken to form the Fraction is called the **Numerator** of the Fraction.

These operations are denoted by the following symbols: we represent a fraction by writing the numerator above the denominator, and separating them by a horizontal line.

Thus $\frac{3}{4}$ represents the Fraction of which the Numerator is 3 and the Denominator 4.

Such Symbols are called Fraction-Symbols, or, for brevity, Fractions.

35. The symbol $\frac{1}{2}$ is read one-half.
The symbol $\frac{1}{3}$ is read one-third.
The symbol $\frac{3}{4}$ is read three-fourths.
The symbol $\frac{6}{7}$ is read six-sevenths,
and so on.

36. The Numerator and Denominator of a Fraction are called the **Terms** of the Fraction.

A **Proper** Fraction is one in which the Numerator is less than the Denominator, as $\frac{3}{5}$.

An **Improper** Fraction is one whose Numerator is not less than its Denominator, as $\frac{7}{4}$, $\frac{7}{7}$.

37. A whole number, or integer, can be written as a fraction, by putting 1 beneath the number as a denominator: thus 5 may be written as a fraction, thus $\frac{5}{1}$.

Also, since $\frac{5}{1}=\frac{10}{2}=\frac{15}{3}=\frac{20}{4}$, and so on, it is clear that we can represent a whole number by a fraction whose denominator is any whole number we please to select.

38. A *Mixed Number* is a number made up of an integer and a fraction, as $4\frac{2}{7}$. This may be read thus, *four and two-sevenths*, and must be regarded as the *sum* of 4 and $\frac{2}{7}$.

A mixed number can be brought into the form of an improper fraction, by multiplying the integer by the denominator of the fraction, adding to the product the numerator of the fraction, and making the sum the numerator of a fraction of which the denominator is the denominator of the original fraction.

Thus, $4\frac{2}{7} = \frac{30}{7}$,
for $4\frac{2}{7} = 4 + \frac{2}{7} = \frac{28}{7} + \frac{2}{7} = \frac{30}{7}$.

Conversely, an improper fraction can be reduced to a mixed number, by dividing the numerator by the denominator, setting down the quotient as the integral part, and making the remainder the numerator of the fractional part of the mixed number, the denominator being the denominator of the original fraction.

Thus, $\frac{25}{7} = 3\frac{4}{7}$,
for $\frac{25}{7} = \frac{21+4}{7} = \frac{21}{7} + \frac{4}{7} = 3 + \frac{4}{7} = 3\frac{4}{7}$.

Examples xvii.

Convert into improper fractions:—

1. $7\frac{4}{9}$.
2. $23\frac{15}{47}$.
3. $216\frac{13}{29}$.
4. $173\frac{19}{1000}$.
5. $43\frac{7}{9}$.
6. $54\frac{17}{19}$.
7. $90\frac{8}{15}$.
8. $901\frac{41}{71}$.
9. $506\frac{11}{13}$.
10. $700\frac{6}{101}$.
11. $705\frac{300}{707}$.
12. $609\frac{7}{409}$.

Examples xviii.

Convert into whole or mixed numbers:—

1. $\frac{427}{10}$.
2. $\frac{3477}{1000}$.
3. $\frac{4293}{137}$.
4. $\frac{65943}{71}$
5. $\frac{600}{23}$.
6. $\frac{888}{25}$.
7. $\frac{1000}{73}$.
8. $\frac{6784}{1941}$.
9. $\frac{6001}{607}$.
10. $\frac{7694}{83}$.
11. $\frac{76856}{2237}$.
12. $\frac{12345}{709}$.

39. To show that $\frac{2}{3} = \frac{8}{12}$.

Suppose a **Unit** to be divided into 3 equal parts.

Then $\frac{2}{3}$ will represent 2 of these parts . . . . . . (1).

Next, let each of the 3 parts be subdivided into 4 equal parts.

Thus the **Unit** has been divided into 12 equal parts, and $\frac{8}{12}$ will represent 8 of these subdivisions . . . . . . (2).

Now, one of the parts in (1) is equal to 4 of the subdivisions in (2).

$\therefore$ 2 parts are equal to 8 subdivisions,

and $\therefore \frac{2}{3} = \frac{8}{12}$.

We draw from this proof two inferences:

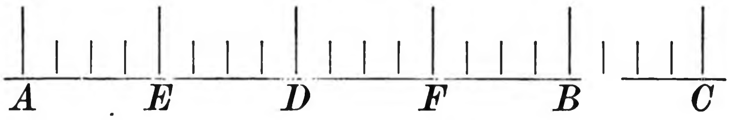
i. If the numerator and denominator of a fraction be *multiplied* by the same number, the value of the fraction is not altered.

Thus $\frac{3}{7} = \frac{12}{28}$ and $\frac{4}{15} = \frac{40}{150}$.

ii. If the numerator and denominator of a fraction be *divided* by the same number, the value of the fraction is not altered.

Thus $\frac{14}{20} = \frac{7}{10}$ and $\frac{90}{100} = \frac{9}{10}$.

40. To make the important theorem established in Article 39 clearer, we shall give a *practical* proof that $\frac{4}{5} = \frac{16}{20}$, by taking a straight line as the unit of length.



Let the line AC be divided into 5 equal parts.

Then, if B be the point of division nearest to C,

AB is $\frac{4}{5}$ of AC . . . . (1).

Next, let each of the parts be subdivideded into 4 equal parts.

Then AC contains 20 of these subdivisions,

and AB contains 16 of these subdivisions;

$\therefore AB$ is $\frac{16}{20}$ of AC . . . . (2).

Comparing (1) and (2) we conclude that

$$\frac{4}{5} = \frac{16}{20}$$

41. A fraction is in its *lowest terms* when the numerator and denominator have no common factor except unity:

Thus $\frac{2}{3}$, $\frac{5}{7}$, $\frac{17}{19}$, represent fractions in their lowest terms.

To reduce a fraction to its lowest terms we have the following Rule:

Divide the Numerator and Denominator by their H. C. F.

Thus, if we have to reduce $\frac{18}{81}$ to its lowest terms, we know that 9 is the H. C. F. of 18, and 81, and dividing the numerator and denominator by 9, we have the resulting fraction $\frac{2}{9}$.

Again, to reduce $\frac{25}{500}$ to its lowest terms, we find 25 to be the H. C. F. of 25 and 500, and therefore $\frac{1}{20}$ will be the reduced fraction.

When we see, by inspection or by an application of the tests of divisibility given in Art. 29, that a factor is common to both Numerator and Denominator, we may divide both by this factor and reduce the fraction to *lower* terms, without going through the process of finding the H. C. F.

Thus, to reduce the fraction $\frac{270}{9360}$ we see that both terms are divisible by 10, and $\therefore \frac{270}{9360} = \frac{27}{936}$.

Now 27 and 936 are both divisible by 9 (Art. 29),

$$\text{and } \therefore \frac{27}{936} = \frac{3}{104}.$$

Examples xix.

Reduce the following fractions to their lowest terms:—

1. $\frac{24}{80}$.
2. $\frac{72}{280}$.
3. $\frac{42}{210}$.
4. $\frac{192}{576}$.
5. $\frac{5184}{6912}$.
6. $\frac{1680}{1920}$.
7. $\frac{6409}{7395}$.
8. $\frac{319}{5687}$.
9. $\frac{1989}{2873}$.
10. $\frac{9495}{15615}$.
11. $\frac{3178}{5221}$.
12. $\frac{2813}{3783}$.

42. Two fractions may be replaced by two equivalent fractions with a Common Denominator by the following rule:—

Find the L. C. M. of the denominators of the given fractions.

Divide the L. C. M. by the denominator of each fraction.

Multiply the first Numerator by the first Quotient.

Multiply the second Numerator by the second Quotient.

The two Products will be the Numerators of the equivalent fractions whose common denominator is the L. C. M. of the original denominators.

The same rule holds for three, four or more fractions.

Ex. 1. Reduce to equivalent fractions with the lowest common denominator, $\frac{3}{8}$ and $\frac{4}{7}$.

Denominators 8, 7.
L. C. M. 56.
Quotients 7, 8.
New numerators 21, 32.
Equivalent fractions $\frac{21}{56}$, $\frac{32}{56}$.

Ex. 2. Reduce to equivalent fractions with the lowest common denominator, $\frac{2}{3}$, $\frac{4}{9}$, $\frac{13}{72}$.

Denominators 3, 9, 72.
L. C. M. 72.
Quotients 24, 8, 1.
New numerators 48, 32, 13.
Equivalent fractions $\frac{48}{72}$, $\frac{32}{72}$, $\frac{13}{72}$.

Examples xx.

Reduce to equivalent fractions with the lowest common denominator

1. $\frac{3}{4}$, $\frac{5}{7}$.
2. $\frac{4}{9}$, $\frac{5}{18}$, $\frac{7}{27}$.
3. $\frac{3}{5}$, $\frac{4}{7}$, $\frac{6}{11}$.
4. $\frac{5}{12}$, $\frac{13}{20}$, $\frac{17}{80}$, $\frac{19}{120}$.
5. $\frac{4}{7}$, $\frac{15}{17}$, $\frac{26}{51}$, $\frac{65}{102}$.
6. $\frac{1}{3}$, $\frac{3}{5}$, $\frac{1}{6}$, $\frac{5}{18}$.
7. $\frac{3}{10}$, $\frac{5}{27}$, $\frac{7}{90}$, $\frac{11}{360}$.
8. $\frac{3}{7}$, $\frac{17}{21}$, $\frac{29}{35}$, $\frac{23}{60}$, $\frac{47}{70}$.

43. To compare the values of two or more fractions, we convert them into equivalent fractions with a common denominator. Then the comparison of the values of the original fractions can be made by comparing the numerators of the new fractions.

For example, to compare the value of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{7}$.

The equivalent fractions are $\frac{56}{84}$, $\frac{63}{84}$, $\frac{60}{84}$.

The descending order of value of the numerators is 63, 60, 56;

∴ the descending order of value of the given fractions is $\frac{3}{4}$, $\frac{5}{7}$, $\frac{2}{3}$.

44. We may also compare fractions by reducing them to fractions with a common *Numerator*, and assigning the greatest value to that one of the resulting fractions which has the *least* denominator.

Thus, to compare the values of

$$\frac{3}{5},\ \frac{27}{31},\ \text{and } \frac{81}{95}.$$

The equivalent fractions are

$$\frac{81}{135},\ \frac{81}{93},\ \text{and } \frac{81}{95}.$$

∴ the descending order of the given fractions is

$$\frac{27}{31},\ \frac{81}{95},\ \frac{3}{5}.$$

Examples xxi.

Compare the values of

1. $\frac{3}{7}, \frac{4}{5}, \frac{9}{13}$.
2. $\frac{5}{6}, \frac{7}{9}, \frac{12}{17}$.
3. $\frac{9}{11}, \frac{13}{15}, \frac{17}{21}$.
4. $\frac{3}{20}, \frac{7}{40}, \frac{11}{70}$.
5. $\frac{7}{33}, \frac{9}{43}, \frac{11}{53}$.
6. $\frac{2}{17}, \frac{5}{34}, \frac{7}{51}$.

Addition of Fractions.

45. The rule for adding two or more fractions together is this :—

Reduce the fractions to equivalent fractions having the Lowest Common Denominator.

Then add the numerators of the eqivalent fractions, and place the result as the Numerator of a fraction whose Denominator is the common denominator of the equivalent fractions.

The fraction will be equal to the sum of the original fractions.

For example, to find the sum of $\frac{1}{3}$ and $\frac{1}{4}$.

$$\frac{1}{3}=\frac{4}{12} \text{ and } \frac{1}{4}=\frac{3}{12}.$$

$$\therefore \frac{1}{3}+\frac{1}{4}=\frac{4}{12}+\frac{3}{12}=\frac{7}{12}.$$

46. To add mixed numbers.

Find the sum of the whole numbers; find the sum of the fractions. To the sum of the whole numbers add the sum of the fractions; the result will be the sum of the mixed numbers.

Ex. 1. **Find** the sum of $4\frac{2}{3}$ and $3\frac{4}{7}$.

$$\begin{aligned}4\tfrac{2}{3}+3\tfrac{4}{7}&=4+\tfrac{2}{3}+3+\tfrac{4}{7}\\&=4+3+\tfrac{2}{3}+\tfrac{4}{7}\\&=7+\tfrac{14}{21}+\tfrac{12}{21}\\&=7+\tfrac{26}{21}\\&=7+1\tfrac{5}{21}\\&=8\tfrac{5}{21}.\end{aligned}$$

and, similarly, when three or more mixed numbers are to be added, we may separate the fractions from the integers, and make a distinct operation for each class.

Examples xxii.

Find the sum of the following fractions :—

1. $\frac{1}{7}$ and $\frac{2}{5}$.
2. $\frac{2}{3}$ and $\frac{3}{14}$.
3. $\frac{2}{7}$, $\frac{3}{11}$ and $\frac{2}{33}$.
4. $\frac{4}{9}$, $\frac{5}{14}$ and $\frac{5}{42}$.
5. $\frac{5}{13}$, $\frac{2}{39}$, $\frac{25}{78}$ and $\frac{7}{156}$.
6. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ and $\frac{1}{32}$.
7. $\frac{5}{11}$, $\frac{2}{27}$, $\frac{7}{55}$ and $\frac{4}{135}$.
8. $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{9}$ and $\frac{1}{11}$.
9. $\frac{2}{3}$, $\frac{3}{40}$, $\frac{7}{240}$ and $\frac{11}{2880}$.
10. $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{15}$ and $\frac{1}{10}$.
11. $\frac{1}{6}$, $\frac{1}{20}$, $\frac{3}{40}$ and $\frac{7}{10}$.
12. $\frac{5}{7}$, $\frac{3}{14}$, $\frac{23}{28}$ and $\frac{8}{49}$.
13. $2\frac{1}{4}$, $3\frac{1}{8}$, $5\frac{7}{16}$, $7\frac{5}{24}$, $3\frac{7}{36}$ and $11\frac{5}{18}$.
14. $3\frac{1}{5}$, $7\frac{2}{9}$, $4\frac{5}{18}$, $\frac{7}{24}$, $5\frac{7}{27}$ and $2\frac{13}{45}$.
15. $4\frac{3}{8}$, $5\frac{11}{12}$, $3\frac{5}{16}$, $7\frac{1}{24}$, $8\frac{7}{15}$ and $\frac{23}{25}$.

Subtraction of Fractions.

47. The rule for subtracting a fraction from a greater fraction is this :—

Reduce the fractions to equivalent fractions having the Lowest Common Denominator. Then subtract the numerator of the smaller of the equivalent fractions from the numerator of the greater, and place the result as the numerator of a fraction, whose denominator is the common denominator of the equivalent fractions. This fraction will be equal to the difference of the original fractions.

For example, to find the difference between $\frac{2}{3}$ and $\frac{5}{7}$.

$$\tfrac{2}{3}=\tfrac{14}{21} \text{ and } \tfrac{5}{7}=\tfrac{15}{21},$$
$$\therefore \tfrac{5}{7}-\tfrac{2}{3}=\tfrac{15}{21}-\tfrac{14}{21}=\tfrac{1}{21}.$$

Examples xxiii.

Find the difference of the following fractions:—

1. $\frac{4}{5}$ and $\frac{5}{6}$.
2. $\frac{3}{7}$ and $\frac{15}{19}$.
3. $\frac{11}{12}$ and $\frac{12}{13}$.
4. $\frac{13}{51}$ and $\frac{235}{357}$.
5. $\frac{17}{63}$ and $\frac{29}{108}$.
6. $\frac{9}{38}$ and $\frac{43}{209}$.
7. $\frac{146}{273}$ and $\frac{268}{637}$.
8. $\frac{199}{200}$ and $\frac{359}{360}$.
9. $\frac{347}{1242}$ and $\frac{835}{1998}$.
10. $\frac{73}{100}$ and $\frac{37}{700}$.

48. In subtracting one mixed number from another we can employ the same method as was used in adding mixed numbers, but a little care is necessary. Suppose we have to take

$$3\tfrac{4}{7} \text{ from } 4\tfrac{2}{3},$$

Reducing the *fractional* parts of the numbers to equivalent fractions with a common denominator, we have

$$3\tfrac{12}{21} \text{ and } 4\tfrac{14}{21}.$$

We can now take the integral part of the first number from the integral part of the second, and the fractional part of the first from the fractional part of the second, and we have

$$4\tfrac{14}{21} - 3\tfrac{12}{21} = 1\tfrac{2}{21}.$$

But suppose we have to take $3\frac{5}{7}$ from $10\frac{2}{5}$,

Since $\frac{5}{7} = \frac{25}{35}$ and $\frac{2}{5} = \frac{14}{35}$

$\frac{5}{7}$ is *greater* than $\frac{2}{5}$,

and we cannot take away the fractional part of $3\frac{25}{35}$ from the fractional part of $10\frac{14}{35}$. We escape from the difficulty by the device of adding *unity* to each expression, to $3\frac{25}{35}$ in the form of 1, and to $10\frac{14}{35}$ in the form of $\frac{35}{35}$.

$$\text{Thus } 10\tfrac{14}{35} - 3\tfrac{25}{35} = 10\tfrac{49}{35} - 4\tfrac{25}{35} = 6\tfrac{24}{35}.$$

Take another illustration of a *practical* nature.

From $5\frac{1}{4}$*d.* take away $3\frac{3}{4}$*d.*

We add four farthings, *i.e.*, $\frac{4}{4}$ of a penny, to the former sum, and 1 penny to the latter, and reason thus:—

$$5\tfrac{1}{4}d. - 3\tfrac{3}{4}d. = 5\tfrac{5}{4}d. - 4\tfrac{3}{4}d. = 1\tfrac{2}{4}d. = 1\tfrac{1}{2}d.$$

Examples xxiv.

Find the difference of the following:—

1. $1\frac{5}{8}$ and $4\frac{3}{16}$.
2. $21\frac{7}{9}$ and $24\frac{1}{3}$.
3. $5\frac{11}{21}$ and $20\frac{11}{28}$.
4. $11\frac{7}{33}$ and $\frac{17}{22}$.
5. $27\frac{4}{5}$ and $21\frac{24}{25}$.
6. $7\frac{5}{7}$ and 10.
7. 21 and $15\frac{17}{25}$.
8. 100 and $91\frac{17}{83}$.
9. $101\frac{1}{2}$ and $47\frac{11}{16}$.
10. $50\frac{5}{9}$ and $40\frac{25}{27}$.

Multiplication of Fractions.

49. A fraction is multiplied by a whole number by multiplying the numerator by that number and leaving the denominator unchanged.

Thus $\frac{2}{7}$ multiplied by 3 becomes $\frac{6}{7}$,
for each of the symbols $\frac{2}{7}$ and $\frac{6}{7}$ implies that a unit has been divided into 7 equal parts, and three times as many of those parts are taken to form the fraction represented by the latter as are taken to form the fraction represented by the former.

50. *To prove that* $\frac{2}{3}$ of $\frac{4}{5}=\frac{8}{15}$.

$\frac{2}{3}$ of $\frac{4}{5}=\frac{2}{3}$ of $\frac{12}{15}$. Art. 39.

Now, suppose a unit to be divided into 15 equal parts,
then $\frac{2}{3}$ of $\frac{4}{5}=\frac{2}{3}$ of 12 of such parts,
$=\frac{8}{12}$ of 12 of such parts, Art. 39.
$=8$ of such parts;
but $\frac{8}{15}=8$ of such parts;
$\therefore \frac{2}{3}$ of $\frac{4}{5}=\frac{8}{15}$.

Hence we derive the Rule for what is called **Multiplication of Fractions.**

We extend the meaning of the sign ×, and define $\frac{2}{3}\times\frac{4}{5}$ to mean $\frac{2}{3}$ of $\frac{4}{5}$, and we conclude that $\frac{2}{3}\times\frac{4}{5}=\frac{2\times4}{3\times5}$, which in words gives us this rule:—

"*Take the product of the numerators to form the numerator of the resulting fraction, and the product of the denominators to form the denominator.*"

The same rule holds good for the multiplication of three or more fractions.

Before effecting the multiplication, common factors should be removed from the numerator and denominator. It will be well for the learner to be familiar with the principles laid down in Art. 39.

For example, to find the value of $\frac{14}{25}$ of $\frac{35}{51}$ of $\frac{17}{49}$ we proceed thus:—

$$\frac{14}{25} \text{ of } \frac{35}{51} \text{ of } \frac{17}{49} = \frac{14 \times 35 \times 17}{25 \times 51 \times 49}$$
$$= \frac{2 \times 7 \times 5 \times 7 \times 17}{5 \times 5 \times 3 \times 17 \times 7 \times 7}$$

and, removing common factors from numerator and denominator,

$$= \frac{2}{5 \times 3}$$
$$= \frac{2}{15}.$$

Examples xxv.

Reduce to their simplest form

1. $\frac{3}{7}$ of $\frac{5}{9}$.
2. $\frac{3}{4} \times \frac{5}{7} \times \frac{9}{11}$.
3. $\frac{42}{65} \times \frac{39}{56} \times \frac{24}{27}$.
4. $\frac{84}{85} \times \frac{102}{117} \times \frac{91}{132}$.
5. $\frac{17}{82} \times \frac{27}{38} \times \frac{123}{153}$.
6. $\frac{100}{161} \times \frac{35}{72} \times \frac{33}{40}$.
7. $\frac{9}{10}$ of $\frac{11}{12}$ of $\frac{13}{14}$ of $\frac{21}{29}$.
8. $\frac{292}{465}$ of $\frac{573}{657}$ of $\frac{248}{1719}$.
9. $\frac{1205}{2763}$ of $\frac{712}{2169}$ of $\frac{1535}{1068}$.
10. $\frac{28}{125}$ of $\frac{46}{84}$ of $\frac{325}{690}$.

Division of Fractions.

51. A fraction is divided by a whole number by multiplying the denominator by that number, and leaving the numerator unchanged.

Thus $\frac{2}{7}$ divided by 3 becomes $\frac{2}{21}$,

for $\frac{2}{7}$ implies that a unit has been divided into 7 equal parts. $\frac{2}{21}$ implies that a unit has been divided into 21 equal parts, and hence each part in the former is three times as great as each part in the latter, and since the same number of parts is taken in both cases, the latter fraction is one-third of the former.

52. *To show that* $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4}$.

The quotient resulting from the division of $\frac{2}{3}$ by $\frac{4}{5}$ is such a number that, when it is multiplied by the divisor $\frac{4}{5}$, the product must be equal to the dividend $\frac{2}{3}$, that is

$$\tfrac{4}{5} \text{ of the Quotient} = \tfrac{2}{3},$$
$$\therefore \tfrac{5}{4} \text{ of } \tfrac{4}{5} \text{ of the Quotient} = \tfrac{5}{4} \text{ of } \tfrac{2}{3},$$
$$\therefore \tfrac{20}{20} \text{ of the Quotient} = \tfrac{5}{4} \text{ of } \tfrac{2}{3},$$
$$\therefore \text{ the Quotient} = \tfrac{5}{4} \text{ of } \tfrac{2}{3},$$
$$\text{that is, } \tfrac{2}{3} \div \tfrac{4}{5} = \tfrac{5}{4} \text{ of } \tfrac{2}{3},$$
$$\text{or, } \tfrac{2}{3} \div \tfrac{4}{5} = \tfrac{2}{3} \times \tfrac{5}{4}.$$

Hence we obtain the following rule for what is called **Division of Fractions** :—

Invert the divisor, and proceed as in Multiplication.

$$\text{Thus, } \tfrac{12}{49} \div \tfrac{16}{35} = \tfrac{12}{49} \times \tfrac{35}{16} = \tfrac{15}{28}.$$

Examples xxvi.

Divide

1. $\frac{12}{49}$ by $\frac{3}{7}$.
2. $\frac{25}{39}$ by $\frac{10}{13}$.
3. $\frac{85}{256}$ by $\frac{51}{64}$.
4. $\frac{16}{261}$ by $\frac{4}{27}$.
5. $\frac{38}{275}$ by $\frac{133}{385}$.
6. $\frac{91}{369}$ by $\frac{78}{287}$.
7. $\frac{49}{550}$ by $\frac{313}{1450}$.
8. $\frac{695}{1544}$ by $\frac{1251}{2316}$.
9. $\frac{1535}{2421}$ by $\frac{921}{1076}$.
10. $\frac{616}{3315}$ by $\frac{2200}{4199}$.

53. In the application of the rules of Multiplication and Division of Fractions to *Mixed Numbers*, we *may* in all cases change the Mixed Numbers into Improper Fractions, and proceed as in the foregoing examples. In Division we *must* proceed thus :—

For example,

$$4\tfrac{5}{9} \div 12\tfrac{3}{10} = \tfrac{41}{9} \div \tfrac{123}{10} = \tfrac{41}{9} \times \tfrac{10}{123} = \tfrac{10}{27}$$
$$16 \div 12\tfrac{4}{5} = \tfrac{16}{1} \div \tfrac{64}{5} = \tfrac{16}{1} \times \tfrac{5}{64} = \tfrac{5}{4} = 1\tfrac{1}{4}.$$

In Multiplication it is usually the best course thus :—

$$7\tfrac{2}{3} \times 5\tfrac{4}{7} = \tfrac{23}{3} \times \tfrac{39}{7} = \frac{23 \times 13}{7} = \tfrac{299}{7} = 42\tfrac{5}{7}.$$

Examples xxvii.

Simplify the following fractions :—

1. $6\frac{2}{3} \times 9\frac{5}{6}$.
2. $9\frac{4}{15} \times 19\frac{2}{7}$.
3. $14 \times 3\frac{2}{21}$.
4. $12\frac{3}{5} \times 17\frac{6}{7}$.
5. $13\frac{1}{11} \times 11\frac{11}{12}$.
6. $16\frac{1}{13} \times 15\frac{4}{11}$.
7. $4\frac{2}{7} \div 3\frac{7}{9}$.
8. $8\frac{3}{5} \div 6\frac{1}{7}$.
9. $103\frac{1}{9} \div 53\frac{7}{13}$.
10. $20\frac{1}{18} \div 2\frac{1}{9}$.
11. $17\frac{6}{7} \div 12\frac{1}{2}$.
12. $31\frac{5}{8} \div 17\frac{1}{4}$.

54. The following examples should be carefully noticed:—

i. From 17 take $4\frac{5}{21}$.

$$17 - 4\frac{5}{21} = 16 + 1 - 4\frac{5}{21} = 16 - 4 + 1 - \frac{5}{21}.$$
$$= 12 + \frac{16}{21} = 12\frac{16}{21}.$$

Or
$$\begin{aligned} & 17 - 4\frac{5}{21} \\ &= 17\frac{16}{21} - \left(4\frac{5}{21} + \frac{16}{21}\right) \\ &= 17\frac{16}{21} - 5 \\ &= 12\frac{16}{21}. \end{aligned}$$

ii. From 317 take $\frac{5}{49}$.

$$317 - \frac{5}{49} = 316 + 1 - \frac{5}{49} = 316 + \frac{44}{49} = 316\frac{44}{49}.$$

iii. Multiply $\frac{999}{1000}$ by 397.

Since $\frac{999}{1000} = 1 - \frac{1}{1000}$

$$397 \times \frac{999}{1000} = 397 - \frac{397}{1000} = 396 + 1 - \frac{397}{1000}$$
$$= 396 + \frac{603}{1000} = 396\frac{603}{1000}.$$

55. A Compound Fraction is defined to be the fraction of a fraction.

Thus $\frac{2}{3}$ of $\frac{5}{7}$ and $\frac{3}{4}$ of $2\frac{1}{4}$ of $5\frac{2}{7}$ are compound fractions.

They are reduced to simple fractions by the process of Multiplication.

Thus $\frac{3}{4}$ of $2\frac{1}{4}$ of $5\frac{2}{7} = \frac{3}{4} \times \frac{9}{4} \times \frac{37}{7} = \frac{3 \times 9 \times 37}{4 \times 4 \times 7} = \frac{999}{112} = 8\frac{103}{112}$.

56. A Complex Fraction is one of which the Numerator or Denominator is itself a fraction or a mixed number.

Thus $\dfrac{\frac{3}{4}}{7}$ $\dfrac{2}{\frac{5}{9}}$ and $\dfrac{4\frac{2}{7}}{5\frac{3}{8}}$ are complex fractions.

They are reduced to simple fractions by the process of Division.

Thus $$\frac{\frac{3}{4}}{7} = \frac{3}{4} \div 7 = \frac{3}{4} \div \frac{7}{1} = \frac{3}{4} \times \frac{1}{7} = \frac{3}{28}$$

and $$\frac{2}{\frac{5}{9}} = 2 \div \frac{5}{9} = \frac{2}{1} \times \frac{9}{5} = \frac{18}{5} = 3\frac{3}{5}.$$

Examples xxviii.

Simplify the following fractions:—

1. $\frac{3}{4}$ of $5\frac{1}{9}$ of $7\frac{1}{5}$.
2. $4\frac{3}{7}$ of $11\frac{1}{5}$ of 15.
3. $\frac{7}{8}$ of $2\frac{5}{9}$ of $3\frac{4}{7}$ of 90.
4. $\dfrac{\frac{2}{5}}{\frac{7}{8}}$.
5. $\dfrac{6\frac{2}{9}}{4\frac{2}{3}}$.
6. $\dfrac{14}{3\frac{1}{2}}$.
7. $\dfrac{30\frac{1}{4}}{11}$.
8. $\dfrac{16\frac{2}{3}}{\frac{25}{39}}$.

The Highest Common Factor and the Least Common Multiple of Fractions.

57. The H. C. F. or L. C. M. of fractions can be readily found by considering that the denominator is simply the name of so many units represented by the numerator. No difficulty is ever experienced in finding the H. C. F. or L. C. M. of \$12 and \$16, or of 12 apples and 16 apples. In fractions the name is written *under* the number representing the collection of units of that name.

Thus to find the H. C. F. of $\frac{12}{36}$ and $\frac{16}{36}$, proceed as in whole numbers; find the H. C. F. of 12 and 16, which is 4, and call it by its name, which in this case is thirty-sixths. Hence the H. C. F. is $\frac{4}{36}$.

Similarly to find the L. C. M. of $\frac{12}{36}$ and $\frac{16}{36}$, find the L. C. M. of 12 and 16, which is 48, and call it by its proper name. Hence the L. C. M. is $\frac{48}{36}$.

Find the H. C. F. and the L. C. M. of $\frac{2}{3}$, $\frac{4}{9}$ and $\frac{8}{15}$.

Reduced to common denominators these become

$$\frac{30}{45},\ \frac{20}{45} \text{ and } \frac{24}{45}.$$

The H. C. F. of 30, 20 and 24 is 2.

Hence the H. C. F. of $\frac{2}{3}$, $\frac{4}{9}$ and $\frac{8}{15}$ is $\frac{2}{45}$

$$= \frac{\text{H. C. F. of Numerators.}}{\text{L. C. M. of Denominators.}}$$

Again the L. C. M. of 30, 20 and 24 is 120.

Hence the L. C. M. of $\frac{2}{3}$, $\frac{4}{9}$ and $\frac{8}{15}$ is $\frac{120}{45} = \frac{8}{3}$

$$= \frac{\text{L. C. M. of Numerators.}}{\text{H. C. F. of Denominators.}}$$

NOTE.—Each fraction should be in its lowest terms.

Hence, to find the H. C. F. of fractions we have the following rules:—

Reduce each fraction to its lowest terms.

Change them to others having the same name or denominator, and find the H. C. F. of their numerators. This placed over the common denominator will be the H. C. F. of the fractions.

Or, *Find the H. C. F. of the numerators, and under this place the L. C. M. of the denominators. The resulting fraction will be the H. C. F. required.*

To find the L. C. M. of fractions: *Change them to others having a common denominator, and find the L. C. M. of the numerators. Place this over the common denominator and reduce the fraction to its lowest terms. The resulting fraction will be the L. C. M. of the fractions.*

Or, *Find the L. C. M. of the numerators, and under this place the H. C. F. of the denominators of the fractions. The resulting fraction will be the L. C. M. required.*

Examples xxix.

Find the H. C. F. of the following fractions:—

1. $\frac{1}{7}$ and $\frac{2}{5}$.
2. $\frac{17}{63}$ and $\frac{29}{108}$.
3. $\frac{1}{2}$, $3\frac{1}{4}$, $4\frac{1}{8}$, and $5\frac{2}{3}$.
4. $\frac{2}{3}$, $\frac{3}{7}$, $\frac{12}{13}$, $4\frac{1}{5}$, and $5\frac{1}{4}$.

Find the L. C. M. of the following fractions:—

5. $\frac{2}{3}$ and $\frac{5}{6}$.
6. $2\frac{1}{2}$ and $7\frac{1}{8}$.
7. $4\frac{1}{3}$, $5\frac{2}{9}$, and $3\frac{5}{12}$.
8. $\frac{1}{2}$ of $2\frac{2}{3}$ of $\dfrac{7}{3\frac{1}{2}}$ and $\frac{3}{7}$ of $\dfrac{8}{1\frac{1}{3}}$ of $2\frac{1}{3}$.

On the Use of Brackets.

58. When an expression is inclosed in a bracket, ( ), it is intended to show that the whole of the expression is affected by some symbol which precedes or follows the bracket.

Thus $24 \times (3\frac{1}{2} + 7\frac{1}{4})$ means, that 24 times the sum of the numbers $3\frac{1}{2}$ and $7\frac{1}{4}$ is to be taken, which we may effect by combining $3\frac{1}{2}$ and $7\frac{1}{4}$ by addition, and multiplying the result by 24.

Again, $2\frac{5}{7} \div (4\frac{3}{4} - 2\frac{1}{2})$ signifies that $2\frac{5}{7}$ is to be divided by the difference between $4\frac{3}{4}$ and $2\frac{1}{2}$; and therefore the result will be

$$2\tfrac{5}{7} \div 2\tfrac{1}{4}, \text{ or } \tfrac{19}{7} \div \tfrac{9}{4}, \text{ or } \tfrac{19}{7} \times \tfrac{4}{9}, \text{ or } \tfrac{76}{63}.$$

And, generally, we may say, that when numbers are included in a bracket, the expression within the bracket must be brought into the simplest form before combining it with the expressions not in the bracket.

59. The methods of denoting a bracket are various; thus, the marks [ ] and { } are often employed. Brackets are made to enclose one another, as in the expression,

$$3 \div [2 + 3 \div \{4 + 5 \div (2 + \tfrac{1}{3})\}].$$

In removing such brackets it is best to commence with the *innermost*, and to remove the brackets one by one, thus,

$$\begin{aligned} & 3 \div [2 + 3 \div \{4 + 5 \div (2 + \tfrac{1}{3})\}] \\ &= 3 \div [2 + 3 \div \{4 + 5 \div \tfrac{7}{3}\}] \\ &= 3 \div [2 + 3 \div \{4 + \tfrac{15}{7}\}] \\ &= 3 \div [2 + 3 \div \tfrac{43}{7}] \\ &= 3 \div [2 + \tfrac{21}{43}] \\ &= 3 \div \tfrac{107}{43} = \tfrac{129}{107}. \end{aligned}$$

We have worked out this example at length because it will teach the learner how to simplify with neatness a peculiar class of fractions called *Continued* Fractions, which appear in a form like the following:—

$$\cfrac{1}{4 + \cfrac{1}{1 - \cfrac{1}{2 - \frac{9}{16}}}}$$

This fraction, by the aid of brackets, may be represented thus,

$$1 \div [4 + 1 \div \{1 - 1 \div (2 - \tfrac{9}{16})\}],$$

and then we can simplify it by the gradual removal of the brackets, the final result being $\frac{7}{51}$.

60. There is another method of simplifying Complex and Continued Fractions, which we may explain by the following examples:—

Ex. 1. To simplify $\dfrac{5}{2+\frac{3}{7}}$.

Multiply all the terms of the fraction by 7, and it becomes

$$\frac{35}{14+3} \text{ or } \frac{35}{17}.$$

Ex. 2. To simplify $\dfrac{\frac{2}{3}}{5+\frac{3}{10}}$.

Multiply the terms by 30, and we get

$$\frac{20}{150+9} \text{ or } \frac{20}{159}.$$

Ex. 3. To simplify $\dfrac{\frac{2}{3}-\frac{3}{7}}{\frac{5}{6}-\frac{5}{14}}$.

Multiply all the terms by 42, and we get

$$\frac{28-18}{35-15} \text{ or } \frac{10}{20} \text{ or } \frac{1}{2}.$$

Ex. 4. To simplify $\dfrac{3}{3+\dfrac{4}{9+\frac{2}{7}}}$

$$\frac{3}{3+\dfrac{4}{9+\frac{2}{7}}}=\frac{3}{3+\frac{28}{65}}=\frac{195}{195+28}=\frac{195}{223}.$$

Ex. 5. To simplify $\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{2}}}}$

$$\frac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{2}}}}=\frac{1}{1+\dfrac{1}{1+\frac{2}{3}}}=\frac{1}{1+\frac{3}{5}}=\frac{5}{8}.$$

Examples xxx.

Simplify the following fractions :—

1. $\dfrac{6}{5+\frac{3}{4}}$

2. $\dfrac{7}{19-\frac{3}{11}}$

3. $\dfrac{\frac{4}{5}}{7-\frac{2}{3}}$

4. $\dfrac{6\frac{7}{8}}{11-\frac{5}{12}}$

5. $\dfrac{\frac{4}{5}-\frac{3}{20}}{\frac{7}{10}-\frac{9}{40}}$

6. $\dfrac{\frac{11}{16}-\frac{5}{24}}{\frac{7}{12}+\frac{3}{4}}$

7. $\dfrac{2}{5+\dfrac{6}{9+\frac{3}{4}}}$

8. $\dfrac{3}{2+\dfrac{1}{3+\frac{1}{3}}}$

9. $\dfrac{5}{2-\dfrac{1}{4-\frac{2}{5}}}$

10. $\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\frac{2}{5}}}}$

61. If two brackets stand side by side, *with no sign between them,* as $(\frac{2}{3}+\frac{3}{4})\ (\frac{5}{6}-\frac{2}{7})$, it is implied that the contents of one bracket are to be *multiplied* by the contents of the other.

The following cases will illustrate the generally received usage in Arithmetic respecting these signs :—

1. *The operations indicated by "of," ×, and ÷ should be performed before adding or subtracting.*

Ex. 1.

$$\begin{aligned}&\tfrac{3}{4} + \tfrac{2}{3} \text{ of } \tfrac{9}{11} - \tfrac{1}{4} \div \tfrac{1}{3} + \tfrac{2}{3} \times \tfrac{6}{11}\\ &= \tfrac{3}{4} + (\tfrac{2}{3} \text{ of } \tfrac{9}{11}) - (\tfrac{1}{4} \div \tfrac{1}{3}) + (\tfrac{2}{3} \times \tfrac{6}{11})\\ &= \tfrac{3}{4} + \tfrac{6}{11} - \tfrac{3}{4} + \tfrac{4}{11}\\ &= \tfrac{10}{11}.\end{aligned}$$

2. *The operations indicated by × and ÷ should be performed in the order in which they occur.*

Ex. 2.

$$\begin{aligned}&\tfrac{2}{3} \times \tfrac{6}{11} \div \tfrac{3}{4}\\ &= \tfrac{2}{3} \times \tfrac{6}{11} \times \tfrac{4}{3}\\ &= \tfrac{16}{33}.\end{aligned}$$

Ex. 3.

$$\begin{aligned} &\tfrac{2}{3} \div \tfrac{6}{11} \times \tfrac{3}{4} \\ &= \tfrac{2}{3} \times \tfrac{11}{6} \times \tfrac{3}{4} \\ &= \tfrac{11}{12}. \end{aligned}$$

Ex. 4.

$$\begin{aligned} &\tfrac{3}{4} \times \tfrac{5}{6} \div \tfrac{2}{3} \times \tfrac{1}{4} \\ &= \tfrac{3}{4} \times \tfrac{5}{6} \times \tfrac{3}{2} \times \tfrac{1}{4} \\ &= \tfrac{15}{64}. \end{aligned}$$

3. *The operation indicated by "of" should be performed before that indicated by* ÷ ; this is the only case in which custom makes a distinction between × and "of."

Ex. 5.

$$\begin{aligned} &\tfrac{3}{8} \text{ of } 2\tfrac{5}{7} \div 1\tfrac{6}{7} \text{ of } \tfrac{3}{4} \\ &= (\tfrac{3}{8} \times \tfrac{19}{7}) \div (\tfrac{13}{7} \times \tfrac{3}{4}) \\ &= \tfrac{3}{8} \times \tfrac{19}{7} \times \tfrac{7}{13} \times \tfrac{4}{3} \\ &= \tfrac{19}{26}. \end{aligned}$$

Examples xxxi.

Simplify the following expressions:—

1. $3\frac{2}{5} \div (2\frac{1}{3} + 1\frac{5}{7})$.
2. $(4\frac{3}{11} + 2\frac{1}{5}) \div 35\frac{3}{5}$.
3. $\dfrac{2}{1 + \dfrac{5}{7 + \frac{2}{3}}}$.
4. $\dfrac{1}{1 + \dfrac{1}{1 + \frac{1}{2}}}$.
5. $5\dfrac{3}{4 - \dfrac{5}{7 + \frac{2}{5}}}$.
6. $\frac{3}{7} + \frac{5}{9}$ of $\frac{3}{10} - \frac{2}{21}$.
7. $\frac{2}{3}$ of $\frac{5}{9} + \frac{3}{7} \div \frac{4}{5}$.
8. $(\frac{11}{13} \div \frac{2}{7})$ of $7\frac{7}{12} - 1\frac{3}{5}$.
9. $(\frac{4}{9} - \frac{3}{11})(2\frac{3}{4} + 3\frac{2}{3})$.
10. $(\frac{3}{13} - \frac{2}{39}) \div (\frac{5}{78} + \frac{7}{156})$.
11. $\dfrac{(2 + \frac{1}{5}) \div (3 + \frac{1}{7})}{(\frac{1}{2} - \frac{1}{3}) \times (4 - 3\frac{3}{7})}$.
12. $\dfrac{(3\frac{1}{3} - 2\frac{1}{2}) \div \frac{5}{6} \text{ of } \frac{3}{8}}{2\frac{2}{3} \div (\frac{1}{2} + \frac{1}{4})}$.

62. We shall conclude this Chapter with a set of Miscellaneous Examples on Fractions.

Examples xxxii.

1. Add together $\frac{17}{33}, \frac{5}{12}, \frac{8}{44}, \frac{3}{28}, \frac{15}{56}$.
2. Add $\frac{2}{5}$ of $\frac{3}{7}$ to $\frac{3}{7}$ of $2\frac{1}{3}$,
   and multiply the result by $(\frac{2}{3}$ of $\frac{5}{6}) \div (\frac{5}{4} + \frac{4}{5})$.

3. Subtract $\frac{2}{3}$ of $\frac{5}{6}$ from $1\frac{1}{2}$ of $\frac{4}{9}$,
and divide the result by $(\frac{2}{3} - \frac{4}{7}) \times (\frac{4}{5} - \frac{5}{8})$.

4. Simplify the fractions $\frac{221}{629}$, $\frac{72816}{85288}$, and find their product.

5. Divide the product of $3\frac{2}{5}$ and $3\frac{3}{7}$ by the product of $1\frac{5}{7}$ and $1\frac{13}{21}$.

6. Multiply together the fractions $4\frac{1}{3}$, $2\frac{3}{4}$, and add the result to $4\frac{3}{4} + 3\frac{1}{3}$.

7. Multiply the difference between $\frac{21}{71}$ and $\frac{201}{701}$ by the sum of $4\frac{7}{15}$ and $1\frac{3}{8}$, and multiply the result by the difference between $10\frac{2}{5}$ and $5\frac{2}{3}$.

8. Simplify $(\frac{1}{3} + \frac{4}{7}) \dfrac{20\frac{1}{4}}{3\frac{6}{7} + 2\frac{1}{4}}$.

9. Simplify $(3\frac{4}{5} + 5\frac{1}{9} - \frac{1}{45})(4\frac{1}{5} - 3\frac{1}{4})$
divided by $1\frac{5}{11} + 2\frac{1}{8} - (2\frac{9}{16} - \frac{1}{8} - \frac{1}{22})$.

10. Simplify $(1\frac{1}{3} + 2\frac{2}{7})\left(\dfrac{5\frac{1}{16}}{4\frac{6}{7} + 1\frac{1}{4}}\right)$.

11. Simplify $(7\frac{1}{9} + 1\frac{4}{5} - \frac{1}{45})(2\frac{1}{4} - \frac{4}{5})$
divided by $(4\frac{1}{8} - \frac{6}{11}) - (2\frac{7}{8} - \frac{7}{16} - \frac{1}{22})$.

12. Simplify $\dfrac{6\frac{3}{4} - 1\frac{5}{14}}{2\frac{1}{6} + 1\frac{3}{7}}$ and $(\frac{5}{7} \text{ of } 1\frac{6}{13}) \div \dfrac{2\frac{5}{7}}{3\frac{1}{4}}$.

13. Simplify $\dfrac{1}{4 - \dfrac{1}{2 - \dfrac{1}{1 - \frac{5}{13}}}}$ and $\dfrac{1}{4 + \dfrac{1}{1 - \dfrac{1}{2 - \frac{9}{16}}}}$

14. Simplify $\dfrac{10\frac{2}{5} - 1\frac{5}{7}}{7\frac{1}{8} + 3\frac{3}{40}}$ and $(\frac{3}{7} \text{ of } 2\frac{1}{17}) \div \dfrac{1\frac{2}{3}}{2\frac{3}{7}}$.

15. Simplify $\dfrac{8\frac{7}{8} - 7\frac{6}{7} + 5\frac{5}{6} - 4\frac{4}{5}}{9\frac{9}{10} - 8\frac{13}{15} + 7\frac{7}{8} - 6\frac{6}{7}}$ and $1\frac{21}{43} \times \frac{37797}{75008}$.

16. Simplify

$$\frac{5-\dfrac{1}{5-\frac{1}{5}}}{3-\dfrac{1}{3-\frac{1}{3}}}\times\frac{9}{23}\text{ of }7\quad\text{and}\quad\frac{6+\dfrac{1}{6-\frac{1}{6}}}{4-\dfrac{1}{4-\frac{1}{4}}}\times 10\tfrac{8}{9}.$$

17. Simplify

$$\frac{8\frac{3}{5}-7\frac{3}{4}+5\frac{2}{3}-4\frac{1}{2}}{13-11\frac{9}{10}+10\frac{7}{9}-9\frac{17}{20}}\times\frac{2}{11}\text{ of }365.$$

18. Simplify

$$\frac{\frac{1}{21}\times5\frac{17}{23}\times6\frac{3}{11}+6\frac{19}{51}\times1\frac{23}{49}\div2\frac{5}{17}+1\frac{10}{49}}{9\frac{16}{57}\times1\frac{22}{23}\div5\frac{17}{38}+3\frac{11}{78}\times6\frac{17}{21}\div7\frac{21}{32}}\times12\tfrac{4}{9}.$$

19. Simplify

$$\frac{\frac{1}{23}\text{ of }6\frac{13}{17}\text{ of }24\frac{11}{13}-4\frac{13}{18}\times3\frac{33}{34}\div3\frac{37}{96}}{8\frac{17}{19}\times5\frac{14}{39}\div4\frac{15}{32}-7\frac{19}{20}\times5\frac{11}{65}\div14\frac{21}{25}}\times4\tfrac{8}{23}.$$

20. Simplify

$$\frac{19}{7\times\dfrac{2}{3-1\frac{2}{3}}}\times\frac{7735}{67184}\div\left(1\tfrac{3}{16}-\tfrac{47}{48}\right).$$

21. Simplify

$$\frac{1}{2+\dfrac{3}{4+\frac{5}{6}}}\times\frac{4862}{4147}\div\left(1\tfrac{1}{2}-\tfrac{23}{38}\right).$$

22. Simplify

$$\frac{\dfrac{7}{4-\frac{5}{6}}-\dfrac{5}{6-\frac{3}{8}}}{\dfrac{4}{7-\frac{4}{7}}+\dfrac{2}{4-\frac{2}{5}}}\times\frac{\dfrac{1}{\frac{1}{2}-\frac{27}{59}}-13}{19-\dfrac{1}{\frac{1}{2}-\frac{6}{31}}}.$$

23. Simplify

$$\frac{\dfrac{2}{3-\frac{4}{5}}+\dfrac{3}{4-\frac{5}{6}}}{\dfrac{3}{2-\frac{1}{4}}-\dfrac{1}{3-\frac{4}{5}}}\times\frac{\dfrac{1}{\frac{2}{3}-\frac{2}{9}}-\dfrac{1}{1\frac{1}{2}-\frac{19}{22}}}{\dfrac{1}{1\frac{1}{7}-\frac{1}{8}}-\dfrac{2}{6\frac{3}{5}-2\frac{5}{11}}}.$$

EXAMINATION PAPERS.

I.

1. Explain how to reduce a mixed number to an improper fraction, and show the reason for each step.

2. Bought $18\frac{3}{4}$ yards of silk at \$$2\frac{2}{5}$ a yard, and $27\frac{1}{2}$ lb. of cheese at \$$\frac{3}{20}$ per lb. How much money did I spend?

3. How many times does the sum of $12\frac{4}{5}$ and $8\frac{7}{8}$ contain their difference?

4. *B*, who owns $\frac{5}{11}$ of a ship, sells $\frac{4}{7}$ of his share for \$3,600. What is the ship worth?

5. There are two numbers whose sum is $4\frac{1}{5}$ and whose difference is $2\frac{4}{7}$. Find the numbers.

II.

1. What is meant by expressing one number as the fraction of another? Explain how to express $3\frac{1}{2}$ as the fraction of $9\frac{1}{5}$.

2. How may the relative magnitude of two or more fractions be compared? Arrange the fractions $\frac{7}{15}$, $\frac{8}{35}$, $\frac{23}{44}$, $\frac{29}{56}$ in the order of descending magnitude.

3. Add together $\frac{12}{91}$, $\frac{6}{77}$, and $\frac{9}{143}$, and find what is the least fraction with denominator 1000, which must be added in order that the sum may be greater than unity.

4. Show that the value of $\frac{2+5}{3+7}$ lies between $\frac{2}{3}$ and $\frac{5}{7}$.

5. A ship and her cargo are valued at \$60,000, and $\frac{3}{8}$ of the value of the ship is equal to $\frac{1}{4}$ of the value of the cargo. Find the value of each.

III.

1. Define Numerator and Denominator, and explain why they are appropriately applied to the terms of a fraction.

2. If $\frac{4}{7}$ of $\frac{3}{5}$ of $2\frac{1}{2}$ bbl. of flour is worth \$$7\frac{1}{3}$, what is the value of $2\frac{2}{11}$ bbl.?

3. If any number of fractions be equal, then any of them is equal to the fraction whose numerator is equal to the sum of all the numerators, and whose denominator is equal to the sum of all the denominators. Exemplify this in the case of six equal fractions.

4. Add together $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, and subtract the sum from 2; multiply the difference by $\frac{3}{5}$ of $\frac{27}{40}$ of 88, and find what fraction the product is of 999.

5. *A*'s age is $\frac{5}{12}$ of *B*'s, and *B*'s is $\frac{4}{7}$ of *C*'s, and *C* 12 years ago was 72. What are their respective ages?

IV.

1. Before adding fractions together, why is it necessary to change them to others having the same denominator?

2. What is the least number which must be taken from $17\frac{1}{5}$ so that it may contain $3\frac{2}{7}$ an exact number of times?

3. There is a number which divided by $8\frac{4}{17}$, and the quotient increased by $2\frac{3}{4}$, and the sum multiplied by $\frac{2\frac{1}{2}}{3}$, and the result diminished by $\frac{1}{2}$ of $\frac{3}{7}$ of $14\frac{4}{9}$, gives $2\frac{2}{3}$. Find the number.

4. *A* bought a horse and carriage for \$225, and paid for the harness $\frac{2\frac{1}{2}}{12}$ of what he paid for the horse. The carriage cost $\frac{7}{8}$ of the value of the horse. What was the price of each?

5. Divide \$8888 among *A*, *B*, and *C*, so that *A* may receive \$88 less than 3 times *B*'s share, and *C* \$176 more than one-half of *A* and *B*'s shares.

V.

1. Explain each step in the process of reducing a complex fraction to a simple one.

2. Simplify $3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} - 1$ divided by $3\frac{1}{3} \times 3\frac{1}{3} - 1$.

3. What is the smallest sum of money with which *A* can purchase sheep at \$$4\frac{1}{3}$ each, calves at \$$5\frac{1}{6}$ each, or pigs at \$$2\frac{1}{2}$ each. How many of each can be bought with this sum?

4. John spent \$80 less than $\frac{2}{3}$ of his money at one time, and at another \$40 more than $\frac{3}{7}$ of the remainder, and now has \$40 left. How much had he at first?

5. One-fourth of $\frac{2\frac{1}{3}}{7}$ of the length of a pole is in the mud; two-thirds of the remainder is in the water, and there are $5\frac{1}{2}$ feet in the air. What is the length of the pole?

VI.

1. Show that $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5}$.

2. Find three fractions whose numerators shall be 3, 5, and 7 respectively, and their sum equal to unity.

3. From the sum of $3\frac{3}{4}$ and $4\frac{5}{6}$ subtract $6\frac{6}{7}$, multiply the difference by $2\frac{2}{5}$, and divide the product by $4\frac{1}{7}$.

4. *A* sold a watch for $\frac{1}{5}$ more than it cost him to *B*, who sold it to *C* for $36, which was $\frac{1}{4}$ less than it cost him. What did the watch cost *A* ?

5. There are three rooms $21\frac{2}{3}$, $18\frac{3}{4}$, and $17\frac{7}{9}$ feet long respectively. Find the longest plain ruler with which the three rooms can be measured.

VII.

1. Give a definition of multiplication that will apply to fractions.

2. A person dies worth $40000, and leaves $\frac{1}{3}$ of his property to his wife, $\frac{1}{2}$ to his son, and the rest to his daughter. The wife at her death leaves $\frac{3}{5}$ of her legacy to the son and the rest to the daughter ; but the son adds his fortune to his sister's, and gives her $\frac{1}{3}$ of the whole. How much will the sister gain by this ? and what fraction will her gain be of the whole ?

3. One-half of a population can read ; $\frac{21}{30}$ of the remainder can read and write ; $\frac{4}{25}$ of the remainder can read, write, and cipher, while the rest, 243600, can neither read, write, nor cipher. What is the population ?

4. Three men, *A*, *B*, *C*, run round a circle in 5, 6, and $7\frac{1}{2}$ minutes respectively. If they start from the same point at the same time and run in the same direction, how long will they run before they are all together again ? and how often will each have gone round it ?

5. *A* owned $\frac{5}{9}$ of a ship, and sold $\frac{3}{4}$ of his share to *B*, who sold $\frac{4}{7}$ of what he bought to *C*, who sold $\frac{9}{11}$ of what he bought to *D*. What part of the whole ship did each now own ?

VIII.

1. What are the advantages in arithmetical operations of employing fractions expressed by the smallest number possible ? State how fractions expressed by large numbers may be reduced to equivalent fractions expressed by smaller numbers. Is this always possible ?

2. Is $\frac{4}{21}$ more nearly equal to $\frac{1}{6}$ or to $3\frac{1}{2} - 2\frac{20}{21} + \dfrac{1\frac{1}{4}}{3\frac{1}{4}}$ of $2\frac{1}{3} - 1\frac{1}{7}$, and by how much?

3. Of the sovereigns who have reigned in England since the Norman conquest, there are $\frac{1}{6}$ of one name, $\frac{2}{9}$ of another, $\frac{1}{12}$ of another, $\frac{1}{9}$ of each of two others, and $\frac{1}{18}$ of each of three others, and there are 5 besides. Find how many sovereigns have reigned in England since the conquest.

4. Three horses start from the same point, and at the same time, upon a race course 300 rods in circuit; the first horse passing over $\frac{1}{2}$ the circuit, the second $\frac{2}{5}$, the third $\frac{1}{3}$, in a minute. In how many minutes will they all be together again, and how far will each have travelled?

5. Divide the difference of $13\frac{1}{3} \div \{(2\frac{6}{7} - 2\frac{8}{11}) \times 1\frac{4}{7}\}$ and $13\frac{1}{3} \div (2\frac{6}{7} - 2\frac{8}{11}) \times 1\frac{4}{7}$ by $13\frac{1}{3} \div 2\frac{6}{7} - 2\frac{8}{11} \times 1\frac{4}{7}$.

IX.

1. A tree, whose length was 136 ft., was broken into two pieces by falling; $\frac{2}{3}$ of the length of the longer piece equaled $\frac{3}{4}$ of the length of the shorter. What was the length of each piece?

2. If $17\frac{4}{7}$ bushels of wheat sow $9\frac{4}{7}$ acres, how many bushels will be required to sow $5\frac{7}{12}$ acres?

3. Suppose that $\frac{2}{25}$ is represented by unity, what number will represent $\frac{4}{35}$?

4. In a regiment consisting of English, Irish, and Scotch, $\frac{2}{5}$ of the regiment was Irish, $\frac{3}{10}$ Scotch; but after 200 Irish and 250 Scotch were added to the regiment, $\frac{9}{40}$ were English. Find the original strength of the regiment, and the number of English, Irish, and Scotch, respectively.

5. Five brothers join in paying a sum of money. The eldest pays $\frac{2}{7}$ of it, and the others pay the remainder in equal shares. It is found that the eldest brother pays $270 more than a younger brother's share. Find the sum of money.

For additional questions involving fractions see page 297.

CHAPTER V.

DECIMAL FRACTIONS.

63. The multiples of 10 are 10, 20, 30, 40, 50, and so on.

The Powers of 10 are 10, 100, 1000, 10000, and so on, and these are called the first, second, third, fourth...... Powers of 10.

64. A Fraction, which has for its denominator one of the Powers of 10, is called a **Decimal Fraction,** or for shortness' sake, a **Decimal.** All other fractions are, by way of distinction, called **Vulgar Fractions.**

65. To save the trouble of writing the denominators of decimal fractions, a method of notation is used, by which we can express the value of the denominator in every case. This method will be best explained by the following examples:—

.3 stands for $\frac{3}{10}$, and is read thus, *three-tenths.*

.25 stands for $\frac{25}{100}$, and is read thus, *twenty-five hundredths.*

.347 stands for $\frac{347}{1000}$, and is read thus, *three hundred and forty-seven thousandths.*

The figures which follow the **Point .** are those which form the *Numerator* of the fraction in each case.

The *number* of the figures which follow the Point corresponds to the number denoting the particular Power of 10, which forms the *Denominator* of the fraction in each case.

Now, as the first power of 10 is 1 followed by one zero, and the second power of 10 is 1 followed by two zeros, and the third power of 10 is 1 followed by three zeros, and so on, we can in every case write the denominator by affixing to 1 a number of zeros equal to the number of figures that follow the Point.

Thus, .426789 stands for $\frac{426789}{1000000}$, *six* zeros being affixed to the 1, because the number of figures that follow the Point is in this case *six*.

Again, .07 stands for $\frac{7}{100}$,
.005 stands for $\frac{5}{1000}$,
.00025 stands for $\frac{25}{100000}$,

the zeros which come between the Point and the figures 7, 5, and 25, not being set down in the numerators of the fraction, as having no effect on the value of the numerators, seeing that 07 and 7 stand for the same number, and that 005 and 5 stand for the same number.

But these zeros affect the value of the denominators, as for instance,

$$.7=\tfrac{7}{10}, \text{ while } .07=\tfrac{7}{100}, \text{ and } .007=\tfrac{7}{1000}.$$

66. Zeros *affixed* to a decimal have no effect on its value: that is

.7, .70, .700 are all equal:

$$\text{for } .7=\tfrac{7}{10},$$
$$.70=\tfrac{70}{100}=\tfrac{7}{10},$$
$$.700=\tfrac{700}{1000}=\tfrac{70}{100}=\tfrac{7}{10}.$$

67. The method of representing Decimal Fractions is merely an extension of the method by which Integers are represented, as will be seen from the following considerations.

As the *local* value of each digit increases tenfold as we advance from right to left, so does the local value of each decrease in the same proportion as we advance from left to right.

If, then, we affix a line of digits to the right of the units' place, each one of these having from its position a value, one-tenth part of the value which it would have if it were one place farther to the left, we shall have on the right hand of the units' place a series of fractions of which the denominators are successively 10, 100, 1000, ... while the numerators may be any numbers between 9 and zero.

Thus 246.4789

$$=2\times100+4\times10+6+\tfrac{4}{10}+\tfrac{7}{100}+\tfrac{8}{1000}+\tfrac{9}{10000}.$$

68. A number made up of an integer and a decimal, as 4.5, may be expressed in a fractional form by writing as the Numerator all the figures in the number, and as the Denominator 1 followed by as many zeros as there are figures *after the point*.

$$\text{Thus,}\quad 4.5 = \frac{45}{10},$$
$$\text{for}\quad 4.5 = 4 + \frac{5}{10} = \frac{40}{10} + \frac{5}{10} = \frac{45}{10}.$$
$$\text{Again,}\quad 14.075 = \frac{14075}{1000},$$
$$\text{for}\quad 14.075 = 14 + \frac{75}{1000} = \frac{14000}{1000} + \frac{75}{1000} = \frac{14075}{1000}.$$

Examples xxxiii.

Express, by means of fraction-symbols in their lowest terms,

1. .5.
2. .25.
3. .75.
4. .375.
5. .00243.
6. .0000725.
7. 14.8.
8. 104.235.
9. 50.0004.
10. 100.001.

Express in the abbreviated form

11. $\frac{9}{10}$.
12. $\frac{37}{100}$.
13. $\frac{4579}{10000}$.
14. $\frac{3}{1000}$.
15. $\frac{17295}{100}$.
16. $\frac{59}{100000000}$.
17. $\frac{25679}{10000000}$.
18. $\frac{325793}{100000}$.
19. $\frac{19}{10000}$.

69. We call

.5, 3.7, 15.9 decimal expressions of the *first* order ;
.25, 4.39, 143.73 decimal expressions of the *second* order ;
.043, 5.006, 27.009 decimal expressions of the *third* order;

the number of the order depending on the number of figures that *follow the point*.

The number denoting the order we call the **Index** of the order: thus 1 is the index of the *first* order, 2 of the *second* order, and so on.

70. From what is stated in Art. 66 we learn that a decimal of any order may be made into an equivalent decimal of a *higher* order by affixing one, two, three zeros, according as the index of the higher exceeds the index of the lower by 1, 2, 3.

Thus .43 may be made into an equivalent decimal of the *fifth* order by affixing *three* zeros, thus, .43000, and .047 may be made into an equivalent decimal of the *seventh* order, by affixing *four* zeros, thus, .0470000.

Addition of Decimal Fractions.

71. To add .27 to .45 we proceed thus: we set down the decimals one under another, point under point, add the figures as if they stood for whole numbers, and place the point in the result under the other points, thus,

$$\begin{array}{r} .27 \\ .45 \\ \hline .72 \\ \hline \end{array}$$

72. If the decimals to be added be not of the same order, as for instance .37 and .049, we reason thus,

.049 is a decimal of the third order,

.37 is a decimal of the second order, but it can be made into an equivalent decimal of the third order by affixing a cipher, thus, .370.

Then we proceed to add the decimals, thus,

$$\begin{array}{r} .370 \\ .049 \\ \hline .419 \\ \hline \end{array}$$

Now, suppose we have to add more than two decimal expressions, as .0074, .72, .05, and .123456.

Of these four expressions the last is of the *sixth* order, and we may make the other three into equivalent decimals of the sixth order, and set them down thus,

$$\begin{array}{r} .007400 \\ .720000 \\ .050000 \\ .123456 \\ \hline .900856 \end{array}$$

When the learner is thoroughly acquainted with the principle on which this process of addition depends, he may omit the affixed zeros, since they have no effect on the result, and may write the sum just worked out in the following way:—

.0074
.72
.05
.123456
———
.900856

If the numbers to be added be made up of integers combined with decimals, we keep the points in a vertical line, and proceed as in addition of integers.

Thus, to add 4.27, 15.004, .9007, and 23, we proceed thus,

4.2700	or thus,	4.27
15.0040		15.004
.9007		.9007
23.0000		23.
43.1747		43.1747

Examples xxxiv.

Find the sum of

1. .275 and .425.
2. .007 and .2394.
3. .001 and .0002.
4. 13.279, 3.00046, 742.000372.
5. .000493, 3.24, 15, 42.6, 324.42037.
6. 49.327, .458, 8317.05, 341.875, 32.4962.
7. 700.372, 894.0009, .347, .00082, 5370.006.
8. 560.379, .45687, 350.0036, 7.074, 52.257.

Subtraction of Decimal Fractions.

73. If we have to find the difference between .47 and .35, where both decimals are of the same order, and .47 is the larger of the two, we proceed thus,

From .47
Take .35

Result .12

performing an operation like that of Subtraction of Integers, and keeping the points in a vertical line.

That this method gives the correct result is evident, for

$$.47 - .35 = \frac{47}{100} - \frac{35}{100} = \frac{12}{100} = .12.$$

74. If we have to find the difference between .888 and .9, we may make the latter into a decimal of the third order, thus, .900, and since this is larger than .888, we proceed thus,

From .900
Take .888

Result .012

If we have to find the difference between .998 and 1, we observe that 1, being an integer, must be greater than .998, which is a Proper Fraction, *i.e.*, $\frac{998}{1000}$, and we proceed thus,

From 1.000
Take .998

Result .002

Examples xxxv.

Find the difference between

1. 56.429 and 5.218.
2. 9.005 and 7.462.
3. 53.316 and 5.0867.
4. .799 and .8.
5. 6.047 and 5.9863.
6. 850.007 and 270.8796.
7. .0000086 and .00001.
8. .00537 and .000985.
9. 10 and .0002.
10. .09999 and .101.

Multiplication of Decimals.

75. In finding the product of .12 and .11, we might proceed thus,

$$.12 \times .11 = \frac{12}{100} \times \frac{11}{100} = \frac{12 \times 11}{100 \times 100} = \frac{132}{10000} = .0132,$$

the result being a decimal of the *fourth* order.

Again, if we have to find the product of 4.32 and .00012,

$$4.32 \times .00012 = \tfrac{432}{100} \times \tfrac{12}{100000} = \tfrac{5184}{10000000} = .0005184,$$

the result being a decimal of the *seventh* order.

And, generally, the product of any two decimal expressions is a decimal expression of an order whose index is the sum of the indices of the orders of the two expressions.

Hence, we deduce the following rule for Multiplication of Decimals:—

Multiply as in the case of integers, and mark off in the product a number of decimal places equal to the sum of the number of decimal places in the two factors.

For example, to multiply 2.4327 by 4.23.

$$\begin{array}{r} 2.4327 \\ 4.23 \\ \hline 72981 \\ 48654 \\ 97308 \\ \hline 10.290321 \end{array}$$

Again, to multiply 43.672 by .00000047.

$$\begin{array}{r} 43.672 \\ .00000047 \\ \hline 305704 \\ 174688 \\ \hline 2052584 \end{array}$$

We have now to mark off eleven decimal places from this product, and as the product contains only seven figures, we must prefix four zeros, and put the point on the left of these, thus, .00002052584, and this will be the required product.

One more case must be considered.

Suppose we have to multiply .235 by .48.

$$\begin{array}{r} .235 \\ .48 \\ \hline 1880 \\ 940 \\ \hline .11280 \end{array}$$

This decimal of the *fifth* order is equivalent to a decimal of the *fourth* order, .1128 (Art. 66), and this is the simplest form of the result.

Examples xxxvi.

Multiply

1. 7.5 by 4.7.
2. 3.62 by 5.23.
3. .427 by .235.
4. .562 by .00074.
5. 3.00704 by 4.0205.
6. .0009 by 1000.
7. 623.4075 by 24.0259.
8. .00746 by .006235.
9. 1432.6749 by .00004030705.
10. 50704.042 by .004007090061.

Find the value of the following:—

11. $.407 \times 4.03 \times .006$.
12. $1.01 \times 1000 \times .001$.
13. $.52 \times .007 \times 4.3 \times .02$.

Find the continued product of

14. .07, 4.6, .009, and 52.47.
15. 42.6, .795, 4.03, and .00074.
16. What is the cube of 2.74?
17. Raise 3.5 to the fourth power.

Division of Decimals.

76. If we have to divide .27 by 3, we might proceed thus,

$$.27 \div 3 = \tfrac{27}{100} \div 3 = \tfrac{9}{100} = .09.$$

Again, if we have to divide .00625 by 25, we might proceed thus,

$$.00625 \div 25 = \tfrac{625}{100000} \div 25 = \tfrac{25}{100000} = .00025.$$

In both cases *the Quotient is a decimal of the same order as the dividend.*

Hence we derive the following Rule:—

If the Divisor be an integer, perform the operation of Division as if the Dividend were also an integer, and mark off in the Quotient as many decimal places as there are decimal places in the Dividend.

For example, suppose we have to divide .0086751 by 243.

$$
\begin{array}{r}
243\,)\,.0086751\,(\,357 \\
729 \\
\hline
1385 \\
1215 \\
\hline
1701 \\
1701 \\
\hline
\end{array}
$$

The Quotient is to be a decimal of the *seventh* order,

∴ the result is .0000357.

77. Next observe that, if the divisor be a *decimal* expression, *we can in every case change it into an Integer* by a process which we shall now explain.

If we multiply a decimal expression

by 10, the effect is to move the point one place to the right;

by 100, the effect is to move the point two places to the right;

by 1000, the effect is to move the point three places to the right;

and so on.

For instance, $123.456 \times 10 = 1234.56$,

and $123.456 \times 100 = 12345.6$.

The reason is obvious,

$$
\begin{aligned}
\text{for } 123.456 \times 10 &= \tfrac{123456}{1000} \times 10 = \tfrac{123456}{100} = 1234.56, \\
\text{and } 123.456 \times 100 &= \tfrac{123456}{1000} \times 100 = \tfrac{123456}{10} = 12345.6.
\end{aligned}
$$

Hence we can transform any Divisor into an Integer by multiplying it by 10, 100, 1000, ... according as the Divisor is a decimal of the first, second, third . . . . order.

For example, if the Divisor be .000492, and we multiply it by 1000000, we transform it into the Integer 492.

Now, we may multiply a Divisor by any number, if we multiply the Dividend by the same number.

For instance, if the Divisor be 8 and the Dividend 32, we may multiply each by 10,

so that the Divisor becomes 80, and the Dividend 320;

and whether we divide 32 by 8, or 320 by 80, the Quotient will be the same number, that is, 4.

78. We can now lay down a general rule for Division of Decimals.

If the Divisor be a decimal, change it into an Integer by removing the point a sufficient number of places to the right, and also remove the point in the Dividend the same number of places to the right. Divide as in the case of integers. Then, if the Dividend be an integer, the Quotient will be an integer, and if the Dividend be a decimal, the Quotient will be a decimal of the same order.

The process will be better understood from the following examples:—

Ex. 1. Divide .625 by .025.

$$.625 \div .025 = \frac{.625}{.025} = \frac{625}{025} = \frac{625}{25}$$
$$= 25.$$

Here the Quotient is an *Integer*, because the Dividend is an Integer;

∴ the Quotient required is 25.

Ex. 2. Divide 108.997 by 2.3.

$$108.997 \div 2.3 = \frac{108.997}{2.3} = \frac{1089.97}{23.} = \frac{1089.97}{23}$$
$$= 47.39.$$

Here the Quotient, 4739, is a decimal of the *second* order, because the Dividend is a decimal of the second order;

∴ the Quotient required is 47.39.

Ex. 3. Divide .625 by .00025.

$$.625 \div .00025 = \frac{.625}{.00025} = \frac{62500.}{00025.} = \frac{62500}{25}$$
$$= 2500.$$

Here the Quotient is an *Integer*, because the Dividend is an Integer;

∴ the Quotient required is 2500.

Ex. 4. Divide .00169 by 1.3.

$$.00169 \div 1.3 = \frac{.00169}{1.3} = \frac{0.0169}{.13} = \frac{.0169}{13}.$$
$$= .0013.$$

Here the Quotient, 13, is a decimal of the *fourth* order, because the Dividend is a decimal of the fourth order;

∴ the Quotient is .0013.

Ex. 5. Divide 625 by .25.

$$625 \div .25 = \frac{625}{.25} = \frac{62500}{25}$$
$$= 2500.$$

Here the Quotient is an *Integer*, because the Dividend is an Integer;

∴ 625 ÷ .25 = 2500.

These are cases of *exact* division, that is, when, on the process of division being carried out, *there is no remainder*.

Examples xxxvii.

Divide

1. 1.296 by .108.
2. 17.28 by .0012.
3. .00169 by 1.3.
4. 2921 by .23.
5. 15633.0062 by 362.9.
6. 1 by .0001.
7. .03096 by .000072.
8. .7644 by .0052.
9. .0000615228 by 307.
10. 746.44808 by 7.58.
11. .24294591 by 36.9.
12. 63987.42 by .000073.
13. .26986365 by 3500.
14. 26986.14 by .00009.
15. .00131053 by .0065.
16. 617325 by .00025.
17. .830676 by .000231.
18. .00019517 by 673.
19. 1.0191 by .00079.
20. 2078.61 by 579.
21. 241.16047 by .527.
22. .65220834 by .00854.
23. 4700460.66583 by .00518963.

79. We next take the following example:—

Divide 347 by .64.

$$\text{Here } 347 \div .64 = \frac{347}{.64} = \frac{34700}{64.} = \frac{34700}{64}.$$

and we proceed thus,

```
64 ) 34700.0000 ( 542.1875
     320
     ———
      270
      256
      ———
       140
       128
       ———
        120
         64
        ———
         560
         512
         ———
          480
          448
          ———
           320
           320
           ———
```

When the units' figure has been brought down and there is a remainder, we may carry on the division further by placing a decimal point at the end of the Dividend, and affixing as many zeros as we please, observing that all the figures which will come after those already in the Quotient will be decimals.

Examples xxxviii.

Divide

1. 7.45 by .32.
2. 14.327 by 12.8.
3. 43.26 by 12.5.
4. 7432.976 by .225.
5. 1.2 by 625.
6. .217 by 1250.

80. The student is now to observe that, by employing Short Division, the example just worked out may be put in a very concise form. Thus, taking up the work at the point where we have to divide 34700 by 64, we proceed thus,

$$\begin{array}{r|l} 8 & 34700.0 \\ \hline 8 & 4337.5000 \\ \hline & 542.1875 \text{ Quotient.} \end{array}$$

So, also, if we have to divide 43672.509 by 36, we proceed thus,

$$\begin{array}{r|l} 4 & 43672.50900 \\ \hline 9 & 10918.12725 \\ \hline & 1213.12525 \text{ Quotient.} \end{array}$$

Again, to divide .0000013932 by 32, we proceed thus,

$$\begin{array}{r|l} 4 & .0000013932 \\ \hline 8 & .0000003483000 \\ \hline & .0000000435375 \text{ Quotient.} \end{array}$$

NOTE.—Division by 10, 100, 1000 . . . is effected by moving the decimal place in the Dividend one, two, three . . . places to the left.

$$\text{Thus } 24.6 \div 10 = 2.46$$
$$.47 \div 100 = .0047.$$

Examples xxxix.

Employ Short Division in finding the Quotient when we divide

1. 426.478 by 16.
2. .07849782 by 72.
3. 362.47 by .025.
4. .00007263 by 4.5.
5. 42.007437 by .24.
6. .00463 by 50.
7. 2.4715 by .000016.
8. 9000 by .00036.
9. .001 by 100.
10. .001001001 by 2000.

N.B.—The process of Division may often be shortened by Multiplying the Dividend and Divisor by a number which will transform the Divisor into a power or a multiple of 10; thus, if we have to divide 24.46927151 by 12.5, we multiply both by 8.

$$\text{Then } \frac{24{\cdot}46927151}{12{\cdot}5} = \frac{195.75417208}{100} = 1.9575417208.$$

81. In the examples hitherto given the cases are all those of *exact* division.

In all cases we may proceed with the division till there is no remainder, or till certain figures in the Quotient recur again and again in the same order.

We shall have an example of this recurrence of figures in Art. 82, but first we must observe that we often require to find the Quotient *up to a certain place of decimals.*

For example, suppose we have to find the Quotient arising from the division of 2.47 by .37, to four places of decimals.

$$2.47 \div .37 = \frac{2.47}{.37} = \frac{247}{37} = 6.6756\ldots.$$

Hence, the Quotient, correct to four places of decimals, is 6.6756.

Examples xl.

Find the Quotient to three places of decimals when we divide.

1. 42.5 by .0023.
2. .197 by .79.
3. 37.9 by 409.
4. 27100 by .00313.
5. .0269 by .281.
6. 229 by .007.

82. If we continue the division further in the example given in Art. 81, we find the figures 675 coming again and again in the same order in the Quotient, so that the Quotient is 6.675675675 . . . without any termination.

Let us now take this example,

Divide 90 by .0011.

$$\text{Here } 90 \div .0011 = \frac{90}{.0011} = \frac{900000}{11}$$

$$\begin{array}{r} 11\ \underline{)\ 900000} \\ 81818 \end{array}$$

Up to this point the Quotient is an Integer: but, if we proceed further with the division, we shall obtain a decimal expression: thus, if we affix two more zeros, preceded by a decimal point, to the dividend, we shall have

$$\begin{array}{r} 11\ \underline{)\ 900000.00} \\ 81818.18 \end{array}$$

If we carry on the division to any extent, we shall have the two figures 18 coming again and again in the same order. A decimal of this kind is called *Periodic, Circulating, or Recurring.*

83. The extent of the Period is denoted by placing a dot over the *first*, and another dot over the *last* of the figures in it.

Thus 18 denotes a decimal *of an order such that it can be represented by no finite index*, since it runs on .18181818 . . . to an infinite number of figures.

So also, $6.\dot{7}5\dot{6}$ stands for 6.756756756 . . . .
$.\dot{0}4\dot{7}$ stands for .047047047 . . . .
$.4\dot{3}7\dot{2}$ stands for .4372372372 . . . .
$26.04\dot{7}\dot{9}$ stands for 26,04797979 . . . .
$.0092\dot{6}$ stands for .009266666 . . .

84. A Vulgar Fraction may be converted into a Decimal Fraction by the following process:—

Reduce the fraction to its lowest terms, and then find the Quotient resulting from the division of the numerator by the denominator by the rule for division of decimals.

Thus, to reduce $\frac{3}{8}$ to a decimal, we proceed thus,

$$\begin{array}{r} 8\,)\,\underline{3.000} \\ .375 \end{array}$$

$$\therefore \tfrac{3}{8} = .375.$$

Again, to reduce $\frac{47}{32}$ to a decimal, we divide 47 by 32.

$$47 \div 32 = 1.46875.$$
$$\therefore \tfrac{47}{32} = 1.46875.$$

Or, we might work by Short Division, thus,

$$\begin{array}{r|l} 4 & \underline{47.00} \\ 8 & \underline{11.75} \\ & 1.46875. \end{array}$$

Again, to reduce $\frac{1}{7}$ to a decimal, we proceed thus,

$$\begin{array}{r|l} 7 & \underline{1.00000000} \\ & .14285714\ldots \end{array}$$

$$\therefore \tfrac{1}{7} = .\dot{1}4285\dot{7}.$$

85. To show that, when a Vulgar Fraction is reduced to a Decimal, either the operation must terminate or the figures of the Quotient must recur in the same order.

Consider the operation by which such a fraction as $\frac{1}{7}$ is reduced to a decimal. The only remainders that can occur are 0, 1, 2, 3, 4, 5, 6. If the remainder 0 should occur, the division terminates: if not, we can only have *six* different remainders, and when any of these occurs a second time, we must have a recurrence of the former remainders in the same order.

When a fraction in its lowest terms is reduced to a decimal and produces a recurring decimal, the *extreme* limit of the number of places in the *period* of the recurring decimal is one less than the denominator.

Thus $\frac{1}{7}$ produces a recurring decimal of 6 places.
$\frac{2}{19}$ produces a recurring decimal of 18 places.
$\frac{3}{29}$ produces a recurring decimal of 28 places.

86. When a Vulgar Fraction is in its lowest terms it can only be expressed as an Exact Decimal when the denominator is composed of factors, each of which is one of the numbers 2 or 5.

Thus $\frac{3}{8}$ can be expressed as an exact decimal because $8 = 2 \times 2 \times 2$.
$\frac{3}{20}$ can be expressed as an exact decimal because $20 = 2 \times 2 \times 5$.
$\frac{4}{125}$ can be expressed as an exact decimal because $125 = 5 \times 5 \times 5$.

The reason for this is, that no Vulgar Fraction can be expressed as an Exact Decimal unless it can be transformed to one which has ten, or some power of 10, for its denominator. Now, no number can by multiplication be made a power of 10 unless it is composed of factors each of which is 2 or 5.

Thus 8 can be made into a power of 10 by multiplying it by $5 \times 5 \times 5$.
125 can be made into a power of 10 by multiplying it by $2 \times 2 \times 2$.

40 can be made into a power of 10 by multiplying it by 5 × 5.

Hence $\frac{3}{8}=\frac{3}{2\times2\times2}=\frac{3\times5\times5\times5}{2\times2\times2\times5\times5\times5}=\frac{375}{1000}=.375.$

$\frac{7}{125}=\frac{7}{5\times5\times5}=\frac{7\times2\times2\times2}{5\times5\times5\times2\times2\times2}=\frac{56}{1000}=.056.$

$\frac{9}{40}=\frac{9}{2\times2\times10}=\frac{9\times5\times5}{2\times2\times10\times5\times5}=\frac{225}{1000}=.225.$

But such numbers as 7, 12, 30, cannot be made into powers of 10 by multiplication, and hence $\frac{3}{7}$, $\frac{5}{12}$, $\frac{11}{30}$, cannot be reduced to exact decimals.

It may also be remarked that, when a Vulgar Fraction in its lowest terms is reduced to an exact decimal, the order of that decimal is expressed by the greatest number of times that either of the factors 2 or 5 occurs in the denominator.

Examples xli.

Convert into decimals the following vulgar fractions:—

1. $\frac{7}{20}$.
2. $\frac{11}{25}$.
3. $\frac{6}{7}$.
4. $\frac{1}{99}$.
5. $\frac{1}{999}$.
6. $\frac{1}{41}$.
7. $\frac{11}{21}$.
8. $\frac{13}{60}$.
9. $\frac{17}{1375}$.
10. $\frac{129}{55}$.

Contractions in Multiplication and Division of Decimals.

87. When the number of decimal places is great, the figures obtained by the ordinary mode of multiplication are often unnecessarily numerous. Thus, in multiplying 62.37416 by 2.34169 by the ordinary method, there would be ten places of decimals in the product, while for all practical purposes three or four are quite enough.

Ex. Multiply 62.37416 by 2.34169 so as to retain only 4 places of decimals.

ORDINARY METHOD.		CONTRACTED METHOD.
62.37416		62.37416
2.34169		96143.2
56	136744	1247483 = 623741 × 2 + 1
374	24496	187122 = 62374 × 3
623	7416	24950 = 6237 × 4 + 2
24949	664	624 = 623 × 1 + 1
187122	48	374 = 62 × 6 + 2
1247483	2	56 = 6 × 9 + 2
146.0609	467304	146.0609

By comparing the contracted method with the ordinary method, the reason of the preceding operation will be readily understood.

Since the product of any order of units by units is of the same order as the figure multiplied, the units' figure of the multiplier is written under the place to be retained. For convenience, the other figures are written in an inverted order. Now (Art. 75) 4, a decimal of the *third* order, multiplied by 3, a decimal of the *first* order, will give a decimal of the *fourth* order; also, 7, a decimal of the *second* order, multiplied by 4, a decimal of the *second* order, will give a decimal of the *fourth* order, etc., etc.

Now, to the product of 2 and 1, 1 must be added: since, if 6 had not been rejected, there would have been 1 to carry; then the other figures are multiplied in the usual way. Next, multiply 4 by 3 and set down 2 under the 3, and multiply the other figures by 3 in the usual way.

Next, multiply 7 by 4, and to the product add 2: since, if 416 had not been rejected the product would have approximated to 2000, etc.

Hence we have the following Rule:—

Write the Multiplier with the order of its figures reversed under the Multiplicand, so that the units' figure may be under that figure of the Multiplicand which is the lowest decimal to be retained in the Product. Then multiply by each figure of the Multiplier, neglecting all the figures of the Multiplicand to the right of it, except to find what is to be carried, and carrying one more when the rejected part of any product is 5 *or greater than* 5. *Arrange the partial products so that their right-hand figures may stand in the same vertical column. Their sum will be the product required. From this product cut off the desired number of decimal places.*

88. When the divisor consists of several figures, the work will be much shortened by cutting off a figure from the divisor at each successive step of the division, instead of annexing a figure to the dividend. Care must be taken to increase each product by what would have been carried if the figure or figures had not been cut off.

Ex. 1. Divide 3.784169 by 2.716418 correct to three places of decimals.

```
2716418 ) 3784169 ( 1393
    ...   2716418
          -------
          1067751
           814925
          -------
           252826
           244478
           ------
             8348
             8149
             ----
              199
```

By comparing the units of the highest order in the divisor with the units of the same order in the dividend, it is evident that there must be one figure to the left of the point in the quotient; hence the answer is 1.393.

Ex. 2. Divide 763.14163 by 21.3642 correct to four places of decimals.

```
      ORDINARY METHOD.          |        CONTRACTED METHOD.
213642 ) 76314163 ( 357205      |  213642 ) 76314163 ( 357205
         640926|                |     ....  640926
         ------|-               |           ------
         122215|6               |           122215
         106821|0               |           106821
         ------|--              |           ------
          15394|63              |            15394
          14954|94              |            14955
          -----|---             |            -----
            439|690             |              439
            427|284             |              427
            ---|------          |              ---
             12|40600           |               12
             10|68210           |               11
             --|-----           |               --
              1|72390           |                1
```

Here the figures of the quotient are 357205, and by comparing the 2 tens of the divisor with the 76 tens of the dividend, it is plain there must be 2 places to the left of the point; hence the quotient is 35.7205.

From considering these cases we have the following Rule:—

Compare the number of figures in the divisor to the left of the point with the number of figures in the dividend to the left of the point, and thus determine the position of the point in the quotient. Then divide as in Ex. 1, *dropping a figure from the right of the divisor at each step of the division.*

NOTE.—Care should be taken to mark the figures dropped by placing a dot or other mark beneath them.

Examples xlii.

1. .863541 × .10983 to five places of decimals.
2. .053407 × .047126 to six places of decimals.
3. 3.141592 × 52.7438 to four places of decimals.
4. 325.701428 × .7218393 to three places of decimals.
5. 3.1729432 × 8.316259 to four places of decimals.
6. 2.3748 ÷ 1.4736 to three places of decimals.
7. 31.47 ÷ 839.27656 to four places of decimals.
8. 252070.520751 ÷ 591.57 to three places of decimals.
9. 73.64 ÷ .43232 to four places of decimals.
10. 6.5555 ÷ 7.06249 to three places of decimals.

Recurring Decimals.

89. *Pure* Recurring Decimal Fractions are those in which the period commences immediately after the decimal point.

Thus $.\dot{3}$, $.\dot{2}\dot{7}$, $.\dot{0}42\dot{9}$, are pure recurring decimals.

Mixed recurring Decimal Fractions are those in which one or more figures precede the period.

Thus $.2\dot{3}$, $.24\dot{2}\dot{7}$, $.35\dot{0}42\dot{9}$ are mixed recurring decimals.

90. To find the Vulgar Fraction which is equivalent to a given Pure Recurring Decimal.

Ex. 1. Find the Vulgar Fraction equivalent to $.\dot{3}$.

The decimal $= .333 \ldots.$

From 10 times the decimal, or $3.333 \ldots.$

take the decimal, or $.333 \ldots.$

Then 9 times the decimal $= 3.000 \ldots.$

$\therefore$ the decimal $= \frac{3}{9} = \frac{1}{3}$.

Ex. 2. Find the Vulgar Fraction equivalent to $.\dot{2}4\dot{7}$.

The decimal $= .247247 \ldots.$

From 1000 times the decimal, or $247.247 \ldots.$

take the decimal, or $.247 \ldots.$

Then 999 times the decimal $= 247.000 \ldots.$

$\therefore$ the decimal $= \frac{247}{999}$.

Ex. 3. Find the Vulgar Fraction equivalent to $.\dot{0}42\dot{3}$.

The decimal $= .04230423 \ldots.$

From 10000 times the decimal, or $423.0423 \ldots.$

take the decimal, or $.0423 \ldots.$

Then 9999 times the decimal $= 423.0000 \ldots.$

$\therefore$ the decimal $= \frac{423}{9999} = \frac{47}{1111}$

Examples xliii.

Convert into Vulgar Fractions in their lowest terms

1. $.\dot{6}$.
2. $.\dot{2}\dot{7}$.
3. $.\dot{0}4\dot{5}$.
4. $.\dot{3}12\dot{3}$.
5. $.\dot{0}07\dot{2}$.
6. $.\dot{4}02\dot{3}$.
7. $.\dot{0}005\dot{4}$.
8. $.\dot{0}000\dot{9}$.

91. Hence we deduce the following Rule for reducing a Pure Recurring Decimal to a Vulgar Fraction :—

Take one of the periods to form the numerator, and for the denominator the number formed by repeating 9 *as many times as there are figures in the period.*

Thus $.\dot{7} = \frac{7}{9}$.

$.\dot{0}\dot{5} = \frac{5}{99}$.

$.\dot{4}32\dot{7} = \frac{4327}{9999}$.

92. To find the Vulgar Fraction which is equivalent to a given Mixed Recurring Decimal.

Ex. 1. Find the Vulgar Fraction equivalent to $.2\dot{3}\dot{7}$.

The decimal $= .23737 \ldots$

From 1000 times the decimal, or $237.37 \ldots$

Take 10 times the decimal, or $2.37 \ldots$

Then 990 times the decimal $= 235.00 \ldots$

$\therefore$ the decimal $= \frac{235}{990} = \frac{47}{198}$.

Ex. 2. Find the Vulgar Fraction equivalent to $.04\dot{7}2\dot{6}$.

The decimal $= .04726726 \ldots$

From 100000 times the decimal, or $4726.726 \ldots$

take 100 times the decimal, or $4.726 \ldots$

Then 99900 times the decimal $= 4722.000 \ldots$

$\therefore$ the decimal $= \frac{4722}{99900} = \frac{787}{16650}$.

Ex. 3. Find the Vulgar Fraction equivalent to $3.1\dot{4}$.

The decimal $= 3.1444 \ldots$

From 100 times the decimal, or $314.44 \ldots$

Take 10 times the decimal, or $31.44 \ldots$

Then 90 times the decimal $= 283.000$

$\therefore$ the decimal $= \frac{283}{90}$.

Examples xliv.

Convert into Vulgar Fractions in their lowest terms

1. $.4\dot{2}\dot{5}$.
2. $.47\dot{5}\dot{9}$.
3. $4.25\dot{3}$.
4. $.004\dot{2}\dot{6}$.
5. $7.2\dot{0}1\dot{1}$.
6. $2.53\dot{0}\dot{6}$.
7. $53.00\dot{2}4\dot{3}$.
8. $18.3\dot{6}05\dot{6}$.
9. $267.356\dot{1}$.

93. Hence we deduce the following rule for reducing a Mixed Recurring Decimal to a Vulgar Fraction:—

Form the Numerator by taking from the figures up to the end of the first period the figures that precede the first period; and form the Denominator by setting down 9 *as many times as there are figures in the period, and affixing* 0 *as many times as there are figures between the decimal point and the first period.*

$$\begin{aligned}\text{Thus,}\quad .2\dot{4}\dot{5} &= \frac{245-2}{990} = \frac{243}{990}.\\ .004\dot{7}\dot{3} &= \frac{473-4}{99000} = \frac{469}{99000}.\\ 4.\dot{5} &= \frac{45-4}{9} = \frac{41}{9}.\\ 7.34\dot{5} &= \frac{7345-734}{900} = \frac{6611}{900}.\end{aligned}$$

94. The method of performing arithmetical operations with Recurring Decimals will be best explained by taking the operations separately.

i. **Addition.**

Find the sum of $3.4\dot{9}$, $4.\dot{0}4\dot{7}$, and $.14\dot{6}\dot{3}$.

If absolute accuracy is required, proceed as follows:—

First make the repetends **similar,** *i.e., make them all begin to repeat in the same place to the right of the point.* (See (i) below).

Next, make the repetends **coterminous,** *i.e., make them all end in the same place to the right of the point.*

This is done by finding the L. C. M. of 1.3.2, the number of figures in each repetend (ii.) below.

$$\begin{array}{rlrlr} & & \text{(i)} & & \text{(ii)}\\ 3.4\dot{9} &= & 3.49\dot{9} &= & 3.49\dot{9}9999\dot{9}\\ 4.\dot{0}4\dot{7} &= & 4.04\dot{7}0\dot{4} &= & 4.04\dot{7}0470\dot{4}\\ .14\dot{6}\dot{3} &= & .14\dot{6}\dot{3} &= & .14\dot{6}3636\dot{3}\\ \hline & & & & 7.69\dot{3}4106\dot{8}\end{array}$$

In ascertaining how many are to be carried add up the first column of repetends, thus, $6+7+9=22$, carry 2.

ii. **Subtraction.**

Here we proceed on the same principle as in Addition.

Thus to subtract $5.24\dot{7}$ from $8.\dot{0}5\dot{9}$,

$$\begin{array}{rlrlr} 8.\dot{0}5\dot{9} &= & 8.05\dot{9}0\dot{5} &= & 8.05\dot{9}0\dot{5}\\ 5.24\dot{7} &= & 5.24\dot{7} &= & 5.24\dot{7}7\dot{7}\\ \hline & & & & 2.81\dot{1}2\dot{8}\end{array}$$

iii. In **Multiplication** and **Division** the recurring decimals should be converted into vulgar fractions, and when the product or quotient of these fractions has been found, it may be converted into a decimal.

Thus, $4.\dot{5} \times 3.\dot{7} = \frac{45-4}{9} \times \frac{37-3}{9} = \frac{41}{9} \times \frac{34}{9} = \frac{1394}{81}$,

and $.0\dot{5} \div .04\dot{2} = \frac{5}{90} \div \frac{38}{900} = \frac{5}{90} \times \frac{900}{38} = \frac{5 \times 10}{38} = \frac{25}{19}$.

We may then, if it be required, convert $\frac{1394}{81}$ and $\frac{25}{19}$ into decimals by the process explained in Art. 84.

Examples xlv.

Find the value of the following expressions:—

1. $2.5\dot{7} + .\dot{0}4\dot{3} + 13.\dot{2}$.
2. $14.7\dot{6}\dot{2} + 3.\dot{5}4\dot{9} + 2.20\dot{4}$.
3. $15.0\dot{2}\dot{5} - 13.\dot{2}4\dot{7}$.
4. $.02\dot{4}\dot{6} - .00\dot{3}9\dot{7}$.
5. $3.\dot{7} \times 5.\dot{4}\dot{9}$.
6. $.007\dot{2} \times .\dot{4}\dot{5}$.
7. $3.\dot{4} \div 4.0\dot{9}$.
8. $.0\dot{7}\dot{4} \div .5\dot{9}$.

95. When vulgar and decimal fractions are combined in the same expression, it may *usually* be simplified in the neatest and easiest way by reducing the vulgar fractions to a decimal form.

Thus, if we have to find the sum of $476\frac{1}{4}$, $13\frac{3}{8}$, and 10.375, we should proceed thus,

$$\begin{array}{rr} 476\frac{1}{4} = & 476.25 \\ 13\frac{3}{8} = & 13.375 \\ & 10.375 \\ \hline \text{Sum} = & 500.000 \end{array}$$

EXAMINATION PAPERS.

I.

1. Show that any decimal is multiplied by 1000 by removing the decimal point in the multiplicand three places towards the right.

2. Enunciate the general rules for the division of decimals. In cases when the division does not terminate, explain how to determine the place of the decimal point in the quotient.

3. Which of the following statements is more nearly correct?

$$\frac{10}{9.009} = 1.11 \text{ or } \frac{10}{1.111} = 9.009.$$

4. How many times can .0087 be taken from 2.291? What fraction will the remainder be of the former?

5. Whence does it appear that a vulgar fraction may always be reduced either to a terminated or a circulating decimal?

Calculate the limits of the error made in taking $\frac{355}{113}$ as an approximate value of 3.1415926 to seven places of decimals.

II.

1. Explain what vulgar fractions can be expressed as finite decimals.

Which of the following fractions can be thus expressed?

$\frac{5}{32}$, $\frac{77}{1100}$, $\frac{1820}{2912}$, $\frac{231}{288}$, $\frac{79}{405}$, $\frac{91}{560}$.

2. If a pound of sugar cost .0093125 of $8, find the value of .0625 of 16 barrels of 200 pounds each.

3. Whether is 3.714535 more accurately represented by 3.715 or 3.714, and why?

4. What vulgar fraction is equivalent to the sum of 14.4 and 1.44 divided by their difference?

5. Find a decimal which shall not differ from $\frac{5}{7}$ by a ten-thousandth.

III.

1. What are the advantages and disadvantages of working with decimals instead of vulgar fractions?

2. If a business produces an annual return of $6,000, and of three partners one has .475 and another $.3\dot{8}$ share of the profits, how much money falls to the share of the third partner?

3. A man who owns $\frac{3}{5}$ of a steamboat sells $.\dot{7}$ of his share for $1,400. What decimal part of the boat does he still own, and what was the boat worth?

4. A man paid $120 for a horse; for a buggy 36\frac{5}{16}$ more than $.\dot{3}$ of the cost of the horse; for harness $.\dot{1}8\dot{5}$ of the cost of horse and buggy. Find his entire outlay.

5. The product of three vulgar fractions is $\frac{4}{7}$; two of them are expressed by the decimals, $.\dot{6}\dot{3}$ and $.13\dot{6}$. By what fraction will the third one be expressed?

IV.

1. How do Decimals differ from Vulgar Fractions?

2. A storekeeper buys 140 yards of cloth at $.36 per yard. In selling, he uses a measure which is $\frac{1}{36}$ of a yard too short, and charges $.50 per yard. What is his net gain?

3. One vessel contains a mixture of 18 pints of brandy and 7 of water; another contains 34 pints of brandy and 13 of water. If the strength of the first mixture is represented by 423, what number will represent that of the second?

4. A person settling his bills paid $.\dot{3}$ of his money to one; .6 of the remainder to another, and $.\dot{5}7142\dot{8}$ of the rest to a third. If he had $1 remaining, how much had he at first?

5. A piece of cloth was said to contain 84 yards, but it was found that the so-called yard-measure with which it was measured was $.0208\dot{3}$ of a yard too short. What was the correct length of the cloth?

V.

1. When a vulgar fraction is changed to a decimal, explain how many figures there will be in the decimal if it does not repeat; if it is a repeating decimal, explain when it will consist of a part which does not repeat, and how many figures there will be in this part.

2. The French metre is 39.371 inches in length. Express the length of 25 metres as a fraction of an English mile, there being 5280 feet in it, and 12 inches in a foot.

3. If a steamer makes a passage from New York to Liverpool (say 2700 miles) in 230 hours, and a train goes from London to Edinburgh (say 405 miles) in 18 hours; how much does the one go faster than the other?

4. Given that the sum of the divisor and quotient is 7.5; and that the divisor is $\frac{3}{7}$ of the quotient; also that the remainder is $\frac{20}{27}$ of the divisor. Find the dividend.

5. Divide $448.71½ among *A*, *B*, and *C*, so as to give *B* $46.70 less than *A*, and $34.59 more than *C*.

VI.

1. What vulgar fractions must be represented by mixed repetends, and what by pure repetends?

2. Show that no recurring decimal can have more places in the period than there are units in the denominator less one.

3. A man spent $2.50 more than $.7\dot{9}$ of his money at one time, and $\$1.\dot{1}\dot{5}$ less than $\frac{960}{1441}$ of the remainder at another, and now has $\$2.6\dot{0}\dot{9}$. How much had he at first?

4. Divide 7.19641 into two parts such that the quotient arising from dividing the less by the greater is .0244.

For additional examples see page 299.

CHAPTER VI.

INVOLUTION AND EVOLUTION.

Involution.

96. When a number is multiplied *by itself* once, twice, three times, . . . the resulting products are called the second, third, fourth, . . . **Powers** of the number. The process is called *Involution*, and the Power to which the number is raised is expressed by the number of times the number has been employed as a factor in the operation.

The notation 3^2, 3^3, 3^4, 3^5 expresses the second, third, fourth, and fifth powers of 3. The small figure which indicates the power is called the **exponent**, or **index.**

The term *square* is usually employed instead of *second* power.

The term *cube* is usually employed instead of *third* power.

Thus, 144 is the square of 12, because $12 \times 12 = 144$.
64 is the cube of 4, because $4 \times 4 \times 4 = 64$.
81 is the fourth power of 3, because $3 \times 3 \times 3 \times 3 = 81$.

Examples xlvi.

Find the squares of

1. 705.
2. 978.
3. 78.9.
4. .568.
5. .365.
6. $71\frac{1}{4}$.
7. $70\frac{5}{8}$.
8. .583.
9. $.75\frac{1}{4}$.

Find the cubes of

10. 37.
11. 135.
12. .016.
13. .736.
14. $21\frac{1}{3}$.
15. $34\frac{2}{5}$.

Find the value of

16. $30^3 + 10^3$.
17. $2^3 \times 5^2 \times 3^4$.
18. $(.625)^2 - (.375)^2$.
19. $75^4 - 35^4$.
20. $8^4 - 4^5 + 3^6 - 2^7$.
21. $\dfrac{(4.5)^3 - (3.4)^3}{4.5 - 3.4}$

Square Root.

97. When a number is multiplied by itself, the result is called the **Square** of the number. Thus 144 is the square of 12, and 225 is the square of 15.

98. The **Square Root** of a given number is that number whose square is equal to the given number.

Thus the square root of 144 is 12, because the square of 12 is 144.

The symbol $\surd$, placed before a number denotes that the square root of that number is to be taken: thus $\surd 25$ is read "the square root of 25."

99. A number which has an Integer for its square root is called a **Perfect Square.**

100. For Perfect Squares not greater than 100 we know the square roots, thus we know that the square root of 81 is 9; and for many Perfect Squares greater than 100 we know the square roots by experience, as, for instance, we know that the square root of 169 is 13, and the square root of 400 is 20, and the square root of 10000 is 100. But we have rules for finding the Square Root of any number, as we shall now explain.

First, suppose we have to find the Square Root of 1225.

We draw a line separating the two figures on the right from the other two, thus,

12|25.

The figures 12 make what is called the *first period.*

The figures 25 make what is called the *second period.*

We then take the nearest perfect square not greater than 12, that is 9, and place it under the 12 and put its square root, that is 3, as the first figure of the square root we have to find, thus,

```
12 25 ( 3
 9
```

We subtract 9 from 12, and annex to the remainder, 3, *the second period,* 25, to make a dividend, and we double the first figure of the root, and set down the result as the first term of a divisor; thus our process up to this point will stand thus,

$$
\begin{array}{r|l}
\multicolumn{1}{r}{} & 12|25\ (\ 3 \\
\multicolumn{1}{r}{} & \ \ 9 \\
\cline{2-2}
6 & 325
\end{array}
$$

Now we shall have to annex another figure to the 6, and we must therefore reckon the 6 as *six tens*, or 60, and then we seek the number of times 60 is contained in 325, and this being *five* times, we set down 5 as the second figure of the root, and annex 5 to the 6, so that our process up to this point will stand thus,

$$
\begin{array}{r|l}
\multicolumn{1}{r}{} & 12|25\ (\ 35 \\
\multicolumn{1}{r}{} & \ \ 9 \\
\cline{2-2}
65 & 325
\end{array}
$$

We then multiply 65 by 5, and set the product down under the 325; and subtracting the product from the 325, we have no remainder, and we conclude that 35 is the square root of 1225, the full process being,

$$
\begin{array}{r|l}
\multicolumn{1}{r}{} & 12|25\ (\ 35 \\
\multicolumn{1}{r}{} & \ \ 9 \\
\cline{2-2}
65 & 325 \\
 & 325 \\
\cline{2-2}
\end{array}
$$

∴ 35 is the root required.

Next to find the Square Root of 622521.

Drawing a line to mark off the two figures on the right, and another line to mark off the next two figures, our process for finding the first two figures of the root will be the same as that explained in the first example, and it will stand thus,

$$
\begin{array}{r|l}
\multicolumn{1}{r}{} & 62'25|21\ (\ 78 \\
\multicolumn{1}{r}{} & 49 \\
\cline{2-2}
148 & 1325 \\
 & 1184 \\
\cline{2-2}
 & \ \ 14121
\end{array}
$$

We now annex to the remainder the *third* period 21, and we double the part of the root already found, 78, and set down the result 156 as a partial divisor, and proceed, as before, to divide 14121 by 1560, and annex the quotient 9 to the root and to the divisor; and multiplying 1569 by 9 we set the product under the 14121: thus our process in full will be

$$\begin{array}{r|l}
 & 62\vert 25\vert 21\ (\ 789 \\
 & 49 \\ \hline
148 & 1325 \\
 & 1184 \\ \hline
1569 & 14121 \\
 & 14121 \\ \hline
\end{array}$$

∴ 789 is the root required.

NOTE.—In practice, instead of dividing 1325 by 140, it is usual to divide 132 by 14, and instead of dividing 14121 by 1560, to divide 1412 by 156. The quotient thus obtained is, however, sometimes too great, as will be seen in the next examples.

We now give two examples in which the first period has only *one* figure, which must always be the case when the proposed square has an *odd* number of figures in it.

To find the Square Root of 189475225.

Marking off the figures by pairs, commencing from the right, we have

$$\begin{array}{r|l}
 & 1\vert 89\vert 47\vert 52\vert 25\ (\ 13765 \\
 & 1 \\ \hline
23 & 89 \\
 & 69 \\ \hline
267 & 2047 \\
 & 1869 \\ \hline
2746 & 17852 \\
 & 16476 \\ \hline
27525 & 137625 \\
 & 137625 \\ \hline
\end{array}$$

NOTE.—In dividing 89 by 20 the quotient is 4, but if we added this to complete the divisor, it would become 24, which, being multiplied by 4, would give 96, a number larger than 89.

To find the Square Root of 39601.

$$\begin{array}{r|l} \multicolumn{2}{l}{\quad 3|96'01\ (\ 199} \\ \multicolumn{2}{l}{\quad 1} \\ \hline 29 & 296 \\ & 261 \\ \hline 389 & 3501 \\ & 3501 \\ \hline \end{array}$$

NOTE I.—The division of 296 by 20 illustrates the remarks made on the last example.

NOTE II.—The second remainder, 35, is greater than the divisor, 29, a result not uncommon in this operation.

Examples xlvii.

Find the Square Roots of

1. 196.
2. 529.
3. 1024.
4. 5625.
5. 88209.
6. 119025.
7. 106929.
8. 751689.
9. 193600.
10. 697225.
11. 36372961.
12. 22071204.
13. 550183936.
14. 5256250000.
15. 4124961.
16. 546121000000.
17. 32239684.
18. 191810713444.

19. Resolve the number 300155625 into prime factors and from these determine its square root.

101. To find the Square Root of a Decimal Fraction.

When the given number has an *even* number of decimal places, we proceed to find the Square Root as if the number were an integer, and mark off in the root a number of decimal places equal to *half the number in the square.*

Thus, if the square be a decimal of the *sixth* order,
the root will be a decimal of the *third* order.

For example, to find the Square Root of 5.322249.

```
       5.|32|22'49 ( 2.307
       4
     |----
  43 |  132
     |  129
     |------
  46 |    322
```

Since 46 is not contained in 32, we annex a 0 to the divisor, and also to the root, and bring down the next period thus,

```
  4607 | 32249
       | 32249
       |------
```

Examples xlviii.

Find the Square Roots of

1. 16.81.
2. .9025.
3. .2601.
4. .0625.
5. .000729.
6. 17242.3161.
7. 1.002001.
8. 44415.5625.
9. 18947.5225.

102. In finding the Square Root of a Decimal Fraction we must be careful to make the decimal such that the index of its order is an *even* number.

Thus, if we have to find the Square Root of .4, we change the decimal into an equivalent decimal of the *second*, *fourth*, *sixth* .... order, thus, .40, .4000, .400000 ....

This is done in order that the denominator of the equivalent fraction may be a perfect square, which is the case in the fractions

$$\frac{40}{100}, \frac{4000}{10000}, \frac{400000}{1000000} \dots$$

but not in the fractions

$$\frac{4}{10}, \frac{400}{1000}, \frac{40000}{100000} \dots$$

Also, since for every *pair* of figures in the square we have *one* figure in the root, we shall have to take a number of figures in the decimal part of the square double the number of decimal places we are to have in the root.

Suppose, for example, we have to find the Square Root of .144 to *four* places of decimals.

We must have *eight* decimal places in the square, thus, .14400000, and we mark off these and proceed as in the extraction of the root of whole numbers, the root being a decimal of the *fourth* order, thus,

$$
\begin{array}{r|l}
 & .14|40|00.00\,(.3794\ldots. \\
 & 9 \\
\hline
67 & 540 \\
 & 469 \\
\hline
749 & \;7100 \\
 & \;6741 \\
\hline
7584 & \;\;35900 \\
 & \;\;30336 \\
\hline
 & \;\;\;5564
\end{array}
$$

NOTE.—The Square Root of a decimal of an *odd* order is a non-terminating decimal.

Examples xlix.

Extract to four places of decimals the Square Roots of

1. 20.
2. 30.
3. .9.
4. .121.
5. .169.
6. .016.
7. .00064.
8. .00121.
9. 16.245.
10. $.\dot{9}$.
11. $.\dot{2}\dot{5}$.
12. $42.0\dot{3}$.

103. If we have to find the Square Root of a Vulgar Fraction, we can always, by multiplication, make the denominator a perfect square, if it be not already so, multiplying the numerator by the same number.

We then find the Square Root of the denominator, and find, exactly or approximately, the square root of the numerator, and make the results respectively the denominator and numerator of a fraction, which is the root required, exactly or approximately.

Ex. 1. $\sqrt{\frac{25}{36}}=\frac{\sqrt{25}}{\sqrt{36}}=\frac{5}{6}$.

Ex. 2. $\sqrt{\frac{2}{3}}=\sqrt{\frac{2\times3}{3\times3}}=\frac{\sqrt{6}}{\sqrt{9}}=\frac{\sqrt{6}}{3}$.

We can now extract the square root of 6 to, say, *three* places of decimals.

$$\sqrt{6}=2.449\ldots.$$

$$\therefore\sqrt{\frac{2}{3}}=\frac{2.449\ldots}{3}=.816\ldots.$$

Or, we might have reduced $\frac{2}{3}$ to a decimal, thus: .666666 . . ., and then have extracted the square root of this decimal.

Ex. 3. $\sqrt{8\frac{17}{64}} = \sqrt{\frac{529}{64}} = \frac{\sqrt{529}}{\sqrt{64}} = \frac{23}{8} = 2\frac{7}{8}$.

Ex. 4. To find the Square Root of $\frac{1.28}{12.5}$.

Here we can reduce the fraction to lower terms;

$$\text{Thus, } \sqrt{\frac{1.28}{12.5}} = \frac{\sqrt{.64}}{\sqrt{6.25}} = \frac{.8}{2.5} = .32.$$

104. An integer can always be changed into a perfect square by multiplying by a number equal to or less than the proposed integer.

For example,

7 is changed into a perfect square if multiplied by 7,
18 is changed into a perfect square if multiplied by 2.

Examples l.

Find the Square Roots of

1. $\frac{36}{49}$.
2. $\frac{64}{121}$.
3. $\frac{289}{625}$.
4. $\frac{1369}{7569}$.
5. $\frac{15129}{182329}$.
6. $5\frac{1}{16}$.
7. $5\frac{19}{25}$.
8. $3\frac{22}{169}$.
9. $65\frac{64}{81}$.
10. $38\frac{11}{25}$.
11. $17\frac{16}{25}$.
12. $11\frac{37}{49}$.

and find to four places of decimals the Square Roots of

13. $\frac{5}{8}$.
14. $\frac{10}{24}$.
15. $6\frac{2}{5}$.
16. $9\frac{1}{2}$.
17. $76\frac{14}{17}$.
18. $16\frac{25}{49}$.

Examples li.

1. The product of two equal numbers is 731025. Find one of them.

2. The product of two numbers, one of which is twice the other, is 1270418. Find the smaller number.

3. One number is three times as large as another and their product is 1647243. Find the larger number.

4. One number is $\frac{1}{4}$ of another, and their product is 139876. Find the numbers.

5. One number is $\frac{2}{3}$ of another, and their product is 109350. Find the numbers.

Cube Root.

105. When a number is multiplied by itself twice, the result is called the **Cube** of the number. Thus 27 is the cube of 3, and 216 is the cube of 6.

106. The **Cube Root** of a given number is that number whose cube is equal to the given number.

Thus the Cube Root of 343 is 7, because the cube of 7 is 343.

The symbol ∛, placed before a number, denotes that the cube root of that number is to be taken; thus ∛ 125 is read "the cube root of 125."

107. A number which has an integer for its cube root is called a **Perfect Cube.**

The numbers, less than 1000, which are perfect cubes should be committed to memory; they are

1, 8, 27, 64, 125, 216, 343, 512, 729;

and the Cube Roots of these numbers are respectively

1, 2, 3, 4, 5, 6, 7, 8, 9.

108. To find the Cube Root of a perfect cube, greater than 1000, we proceed by a rule which we shall now explain.

Ex. To find the Cube Root of 91125.

	4		91|125
			64
12	5	4800	27125
		625	
		5425	27125

First divide the number 91125 into two periods by drawing a line marking off *three* figures on the right.

Then take the nearest perfect cube not greater than 91, which is 64, and set down its cube root, which is 4, in a line with 91125, and some way to the left. This is the first figure of the root.

Then subtract 64 from 91, and to the remainder attach the second period, 125.

Now place three times the first figure of the root, 12, to the extreme left, and three times the square of the first figure of the root, 48, with two zeros annexed to it, just on the left of the 27125.

Divide 27125 by 4800, and set the quotient, 5, midway between 12 and 4800. Then read 12 5 as 125; multiply this by 5; put the result, 625, under the 4800; add to it the 4800; this gives 5425; multiply this by 5; put the result, which is 27125, under the first remainder; subtract, and as there is no remainder, the process is complete, and the root is 45.

Examples lii.

Find the Cube Roots of

1. 4096.
2. 32768.
3. 74088.
4. 493039.
5. 614125.
6. 262144.
7. 39304.
8. 389017.
9. 778688.
10. 970299.
11. 59319.
12. 250047.

Next, let us take the case in which the cube root has three figures, and extract the cube root of 428661064.

```
         7              428|661|064
                        343
                       ──────────
  21     5    14700  │   85661
               1075⎫ │
             ─────── ⎬
              15775⎪ │   78875
                 25⎭ │ ──────────
             ─────── │
 225     4  1687500  │  6786064
               9016  │
            ──────── │
            1696516  │  6786064
```

We separate the number 428661064 into three periods, and take the nearest perfect cube not greater than 428, which is 343, and we set down its cube root, which is 7. We then subtract 343 from 428, and annex to the remainder 661, the second period.

Then we set down three times 7, which is 21, and three times the square of 7, which is 147, and annex two zeros to it.

Then we divide 85661 by 14700, which gives the quotient 5, and this we put down midway between 21 and 14700.

Then we multiply 215 by 5, which gives 1075; we add this to 14700; we multiply the result, 15775, by 5; and subtract the product, 78875, from 85661; and to the remainder we annex the third period, 064.

We then set down three times 75, which is 225,
and three times the square of 75, which is 16875.

N. B.—This last result can be obtained by setting the square of 5, the second figure of the root, under the second divisor, and adding the three numbers coupled by the bracket.

We then annex two zeros to 16875 and repeat the process explained above to find 4, the third figure of the cube root, which is in this case 754.

Next, take the case in which the root has *four* figures and find the Cube Root of 14832537993.

```
            2                 14|832|537|993
                               8
                             ----------
      6     4     1200       | 6832
                   256 ⎫     |
                  -----⎪     |
                  1456 ⎬     |
                    16 ⎭     | 5824
                  -------    |-----------
     72     5     172800     | 1008537
                    3625 ⎫   |
                  -------⎪   |
                  176425 ⎬   |  882125
                      25 ⎭   |-------------
                  ---------  |
    735     7     18007500   |  126412993
                     51499   |
                  ---------  |
                  18058999   |  126412993
```

Hence the root required is 2457.

Note.—In dividing 6832 by 1200, the quotient is 5, but if we took this for the second figure of the root we should find that the addition of 5 times 65, or 325, to 1200 would give 1525, and this multiplied by 5 would give 7625, a number too large to be subtracted from 6832.

Examples liii.

Find the Cube Roots of

1. 14706125.
2. 149721291.
3. 28934443.
4. 300763000.
5. 2097152.
6. 5735339.
7. 99252847.
8. 1092727.
9. 16777216.
10. 194104539.
11. 84027672.
12. 130323843.
13. 322828856.
14. 354894912.
15. 700227072.
16. 134217728.
17. 122615327232.
18. 673373097125.

109. To Extract the Cube Root of a Decimal Fraction.

In order that a Decimal Fraction may be a Perfect Cube, it must be of the 3rd, 6th, 9th . . . . order, the index of the order being some multiple of 3.

We then proceed in the following way:—

Ex. 1. To find the Cube Root of .343.

$$\sqrt[3]{.343}=\sqrt[3]{\frac{343}{1000}}=\frac{7}{10}=.7.$$

Ex. 2. To find the Cube Root of .039304.

$$\sqrt[3]{.039304}=\sqrt[3]{\frac{39304}{1000000}}=\frac{34}{100}=.34.$$

Ex. 3. To find the Cube Root of .012812904.

$$\sqrt[3]{.012812904}=\sqrt[3]{\frac{12812904}{1000000000}}=\frac{234}{1000}=.234.$$

110. To extract the cube root of an integer or decimal expression to a particular place of decimals, in the given expression, we must take *three times the number* of decimal places required.

Thus, to find the cube root of 4.23 accurately to three places of decimals, we extract the cube root of 4.230000000, making the given expression a decimal of the *ninth* order. In working this example, we find the cube root of 4.230000000, *regarded as a whole number*, and mark off three decimal places in the result.

111. The Cube Root of a *Vulgar Fraction* may be found by taking the roots of the numerator and denominator, or by reducing the fraction to a decimal of the 3rd, 6th, 9th . . . . order, and proceeding as in Art. 110.

Examples liv.

Find the Cube Root of

1. .389017.
2. .048228544.
3. 27054.036008.
4. $\frac{1331}{1728}$.
5. $\frac{250}{686}$.
6. $5\frac{13}{343}$.
7. $405\frac{28}{125}$.

Find to three places of decimals the Cube Roots of

8. 5.
9. 576.
10. .121861281.
11. 15.926972504.
12. $\frac{5}{9}$.
13. $\frac{3}{4}$.
14. $\frac{1}{3}$.
15. $7\frac{3}{5}$.
16. $3\frac{1}{5}$.

112. The *fourth* root of a number is found by taking the square root of the square root of the number.

Thus $\sqrt[4]{4096}=\sqrt{64}=8$.

The *sixth* root of a number is found by taking the cube root of the square root of the number.

Thus $\sqrt[6]{64}=\sqrt[3]{8}=2$.

Examples lv.

Find the Fourth Roots of

1. 531441.
2. 4100625.
3. 1575.2961.

Find the Sixth Roots of

4. 4826809.
5. 24794911296.
6. 282429.536481.

Examples lvi.

1. The product of three equal numbers is 679151439. Find one of them.

2. There are three numbers, the second is twice the first, and the third is twice the second. The product of the three is 5000211000. Find the largest number.

3. Of three numbers the second is $\frac{1}{3}$ of the first and the third is twice the second. Their product is 178746. Find the numbers.

4. There are four numbers, the second being twice the first, the third twice the second, and the fourth twice the third. Their continued product is 585640000. Find the numbers.

5. Write down the squares of 8, 9, 10, and 11, and from these squares derive a rule for finding the square of any number when the square of the number next greater or next less to it is known.

6. The square of 5987 is 35844169. From this find the square of 5988 and of 5986.

7. Resolve the number 3456649728 into prime factors, and from these determine the cube root of the given number.

8. Find the ninth root of 387420489.

113. The extraction of the cube root, by the ordinary rule, is a troublesome process, seldom used and easily forgotten. The following process is much simpler and more easily remembered.

Let a be an approximate value of the cube root of N, so that $\sqrt[3]{N} = a + x$, x being very small ;

then $N = (a+x)^3 = a^3 + 3a^2x + 3ax^2 + x^3$

$= a^3 + 3ax(a+x)$, nearly, since x is small.

First suppose $N = a^3 + 3a^2x$;

$$\therefore x = \frac{N - a^3}{3a^2} ;$$

$$\text{and } \therefore \; a + x = \frac{N + 2a^3}{3a^2} ;$$

therefore more nearly, $N = a^3 + 3ax\dfrac{N + 2a^3}{3a^2}$,

and $$x = \frac{N - a^3}{N + 2a^3}a,$$

and, therefore, $$\sqrt[3]{N} = a + x$$
$$= \frac{2N + a^3}{N + 2a^3}.a.$$

Suppose we want to find the cube root of any number N. In the first place we find some number a whose cube is somewhere near the given number. Then the fraction,

$$\frac{2N + a^3}{N + 2a^3}.a$$

will be a nearer approximation to the cube root than a itself was. When we have found this value, we can take this as a and repeat the process.

Thus, to find the cube root of 241.804367, we observe that 216, the cube root of 6, is nearest to 241. Hence the first value of a is 6.

Therefore, $$\frac{2N + a^3}{N + 2a^3}.a$$
$$= \frac{699.608734}{673.804367} \times 6$$
$$= \frac{4197.652404}{673.804367}$$
$$= 6.23, \text{ very nearly.}$$

On trying 6.23, we find it is correct.

Ex. 1. Find the Cube Root of 47.

The nearest cube to 47 is that of 4.

Hence,

$$\begin{aligned}&\frac{2N+a^3}{N+2a^3}.a\\ &=\frac{94+64}{47+128}\times 4\\ &=\frac{632}{175}\\ &=\frac{2528}{700}\\ &=3.61\text{, nearly.}\end{aligned}$$

Next, take 3.61 for a, and substitute in the formula, and we get 3.6088261, which is correct to seven places of decimals.

Ex. 2. Find the Cube Root of 10.

In this case,

$$\begin{aligned}&\frac{2N+a^3}{N+2a^3}.a\\ &=\frac{20+8}{10+16}\times 2\\ &=2.153.\end{aligned}$$

Next substitute 2.15 instead of 2, and we get

$$\begin{aligned}&\frac{20+9.938375}{10+19.876750}\times 2.15\\ &=2.1544346,\end{aligned}$$

which is correct as far as six places of decimals. This method has also the practical advantage that an error of work gets corrected at the next trial.

Examples lvii.

Find to four places of decimals the Cube Root of

1. 12.
2. 30.
3. 225.
4. 375.
5. .9.
6. .08.
7. 9.27.
8. .587.
9. 8.76.

CHAPTER VII.

COMPOUND NUMBERS.

Measures.

114. **STERLING MONEY.**

4 farthings (q.) - - =1 penny, or 1d.
12 pence - - - =1 shilling, or 1s.
20 shilling - - - =1 pound, or £1.
21 shillings - - - =1 guinea.

115. **CANADIAN MONEY.**

100 cents (c.) - - - - = 1 dollar, or $1.
Ten *mills* make one cent. The mill is not coined.

116. **TABLE OF CANADIAN AND UNITED STATES COINS.**

CANADIAN COINS.	UNITED STATES COINS.	
GOLD.	**GOLD.**	
British Sovereign, worth $4.86⅔.	Double Eagle, or - -	$20
British Half-Sovereign.	Eagle, or - - - - - -	10
	Half Eagle, or - - -	5
	Three Dollar Piece.	
	Quarter Eagle, or - -	2½
SILVER.	**SILVER.**	
	Dollar.	
50-cent piece, answers to - - - - -	Half dollar.	
25-cent piece, answers to - - - -	Quarter dollar.	
10-cent piece, answers to - - - - -	Dime.	
	NICKEL.	
5-cent piece answers to - - - - - -	5-cent piece.	
	3-cent piece.	
BRONZE.	**BRONZE.**	
1 cent - - - - - - - - - - - - - -	1 cent.	
Mill, not coined. - - - - - - - -	Mill, not coined.	

The English gold coinage consists of $\frac{11}{12}$ pure metal and of $\frac{1}{12}$ alloy.

The gold and silver coinage of the United States consists of $\frac{9}{10}$ pure metal and $\frac{1}{10}$ alloy.

The silver coin in Canada and Great Britain is $\frac{37}{40}$ pure metal and $\frac{3}{40}$ copper.

Gold and silver thus alloyed are called *standard.* The gold or silver before it is coined is called *bullion.*

The term *carat* is employed to denote the fineness of gold. Perfectly pure gold is said to be 24 carats fine; a mixture of eighteen parts pure gold and six parts of some other metal, is said to be 18 carats fine. This latter is termed jewellers' gold.

The copper coins in use in Great Britain are the Farthing, the Halfpenny, and the Penny.

The silver coins in use are the Crown (5s. the Half-crown (2s. 6d.), the Florin (2s.), the Shilling, the Sixpence, the Four-penny piece (or Groat) and the Threepenny piece.

The gold coins in use are the Sovereign or Pound, and the Half-sovereign. The Guinea (21s.) and the Half-guinea (10s. 6d.) are not in use, but reference is frequently made to them.

117. TIME.

60 seconds (sec.) - - - = 1 minute (min.
60 minutes - - - - = 1 hour (hr.)
24 hours - - - - - - - = 1 day (da.)
7 days - - - - - - = 1 week (wk.)
365 days - - - - - - - = 1 common year.
366 days - - - - - - = 1 leap year.

In rough calculations a year is taken to consist of 365 days.

In rough calculations a month is taken to consist of 30 days.

A *Lunar* Month, or the time between two new moons, is rather more than 29½ days.

The 12 months into which we divide the year are called *Calendar* Months: they are of variable length, for 7 of them contain 31 days, 4 contain 30 days, and February has 28 days (and in Leap-year 29).

The names of the 4 months which have 30 days are given in the old verse:—

Thirty days have September,
April, June and November.

To find whether a particular year is a Leap-year, we divide the number of the year by 4; if no remainder be left, the year is Leap-year, but to correct an error in our present Calendar, the *centuries* which are not exactly divisible by 400, as 1900, 2100 . . . are to be taken as common years, and not as leap-years.

118. LENGTH.

12 inches (in.) - - - - = 1 foot (ft.)
3 feet - - - - - - - - = 1 yard (yd.)
5½ yards - - - - - - - - = 1 pole (po.)
40 poles - - - - - - - = 1 furlong (fur.)
8 furlongs - - - - - - = 1 mile (mi.)
3 miles - - - - - - - = 1 league (lea.)

1 mi. = 320 po. = 1760 yds. = 5280 ft. = 80 chains.
A hand, used in measuring horses = 4 in.
A knot, used in navigation = 6086 ft.
A fathom, used in measuring depth at sea = 6 ft.

119. SURFACE.

144 square inches (sq. in.) - = 1 square foot (sq. ft.)
9 square feet - - - - = 1 square yard (1 sq. yd.)
30¼ square yards - - - - = 1 square pole (1 sq. po.)
40 square poles - - - - = 1 rood (ro.)
4 roods - - - - - - - - = 1 acre (ac.)

1 ac. = 160 sq. po. = 4840 sq. yd. = 10 sq. chains.
Land surveyors make use of a Chain 22 yards in length, divided into 100 equal parts, called Links.

120. VOLUME.

1728 cubic inches (cu. in.) - = 1 cubic foot (cu. ft.)
27 cubic feet - - - - - - = 1 cubic yard (cu. yd.)

A cord is equal to a pile 8 ft. long, 4 ft. wide, and 4 ft. high.
Firewood and rough stone are measured by the cord.

121. CAPACITY.

2 pints (pt.) - - = 1 quart (qt.)
4 quarts - - - - = 1 gallon (gall.)
2 gallons - - - = 1 peck (pk.)
4 pecks - - - - = 1 bushel (bu.)
8 bushels - - - = 1 quarter (qr.)

A cubic foot of water weighs 1000 ounces, or 62½ pounds, and contains 6¼ gallons. Hence, a gallon of water weighs 10 pounds.

122. TROY WEIGHT.

24 grains (gr.) - - - = 1 pennyweight (dwt.)
20 pennyweights - - = 1 ounce (oz.)
12 ounces - - - - = 1 pound (lb.)

Chiefly used in weighing the precious metals.

480 grains = 1 oz. Troy ; 5760 grains = 1 lb. Troy.

123. AVOIRDUPOIS WEIGHT.

16 ounces (oz.) - = 1 pound (lb.)
100 pounds - - = 1 cental or hundredweight (cwt.)
20 hundredweight - = 1 ton (t.)
14 pounds - - - = 1 stone.

The pound Avoirdupois contains 7000 grains.
The pound Troy contains 5760 grains.
In Great Britain 112 lb. make 1 cwt.

124. APOTHECARIES' WEIGHT.

i. MEASURES OF WEIGHT.

437½ grains = 1 ounce.
16 ounces = 1 pound.

The grain is the same as the grain Troy.
The ounce is the same as the ounce Avoirdupois.
This is the table given in the British Pharmacopœia.
The Avoirdupois ounce and pound are taken in preference to the ounce and pound Troy of the old table, because the former are used by wholesale dealers in drugs and medicines. In prescribing, many physicians still employ the scruple (℈) of 20 grains, and the dram (ʒ) of 60 grains.

ii. MEASURES OF CAPACITY.

60 minims = 1 fluid dram, written fl dr.
8 fluid drams = 1 fluid ounce, " fl oz.
20 fluid ounces = 1 pint, " O.
8 pints = 1 gallon, " C.

NOTE.—O is a contraction of *Octavus* or *eight*, and C for *Congius*, a Roman liquid measure.

The relation of the measures of capacity to those of weight in these tables is given by the definition that

1 Minim is the measure of .91 Grain of Water.

The connection may be better remembered by the old rhyme:—

A Pint of Water
Weighs a Pound and a Quarter.

125. ANGLES.

60 seconds (″) = 1 minute (′).
60 minutes = 1 degree (°).
30 degrees = 1 sign (S.)
12 signs = 1 circumference (C.)

A degree of the circumference of the earth at the equator contains 60 geographical miles, or 69.16 statute miles.

126. MISCELLANEOUS UNITS.

12 units or things make 1 dozen.
12 dozen " 1 gross.
12 gross " 1 great gross.
20 units " 1 score.
24 sheets " 1 quire.
20 quires " 1 ream.

127. Certain articles are sold not by measure, but by weight. The following table gives the weight of a bushel of a number of these:—

Oats,	34 lbs.	Indian Corn,	56 lbs.	Wheat,	60 lbs.
Barley,	48 lbs.	Rye,	56 lbs.	Potatoes,	60 lbs.
Buckwheat,	48 lbs.	Beans,	60 lbs.	Turnips,	60 lbs.
Timothy Seed,	48 lbs.	Peas,	60 lbs.	Onions,	50 lbs.
Flax Seed,	56 lbs.	Clover Seed,	60 lbs.	Fine Salt,	56 lbs.

128. Fractional Measures.

Ex. 1. How many shillings and pence are there in $\frac{5}{8}$ of a pound?

$$\begin{aligned}\tfrac{5}{8}\text{ of a pound} &= \tfrac{5}{8}\text{ of 20 shillings.}\\ &= \tfrac{5\times 20}{8}\text{ shillings.}\\ &= \tfrac{100}{8}\text{s.}\\ &= 12\text{s. }6\text{d.}\end{aligned}$$

Ex. 2. Find the value of $\frac{3}{7}$ of £15 5s. 8d.

$$\begin{aligned}\tfrac{3}{7}\text{ of £15 5s. 8d.} &= 3\text{ times }\tfrac{1}{7}\text{ of £15 5s. 8d.}\\ &= 3\text{ times £2 3s. 8d.}\\ &= \text{£6 11s.}\end{aligned}$$

Or thus,

$$\begin{array}{r|rrr} & \text{£} & \text{s.} & \text{d.}\\ & 15 & 5 & 8\\ & & & 3\\ \hline 7 & 45 & 17 & 0\\ \hline & \text{£}6 & 11 & 0\end{array}$$

Ex. 3. Find the value of $2\frac{3}{4}$ of $\frac{3}{22}$ of 5 acres.

$$\begin{aligned}2\tfrac{3}{4}\text{ of }\tfrac{3}{22}\text{ of 5 acres} &= \tfrac{11}{4}\text{ of }\tfrac{3}{22}\text{ of 5 acres.}\\ &= \tfrac{11\times 3}{4\times 22}\text{ of 5 acres.}\\ &= \tfrac{3}{8}\text{ of 5 acres.}\\ &= \tfrac{15}{8}\text{ ac.}\\ &= 1\text{ ac. 3 ro. 20 po.}\end{aligned}$$

Examples lviii.

Find the value of the following:—

1. $\frac{2}{3}$ of £1; $\frac{5}{8}$ of £2 10s.; $\frac{3}{7}$ of £5 18s. 5d.

2. $\frac{4}{5}$ of a mile; $\frac{3}{16}$ of an acre; $\frac{5}{6}$ of a cwt.

3. $2\frac{4}{5}$ of £54 9s. 8d.; $3\frac{3}{14}$ of half-a-guinea; $\frac{2}{3}$ of $3\frac{3}{8}$ of a mile.

4. $\frac{4}{5}$ of $\frac{1}{7}$ of $1\frac{5}{19}$ of $1\frac{3}{8}$ of 2470 guineas; $\frac{2}{3}$ of $\frac{1}{4}$ of $4\frac{1}{2}$ guineas.

5. $\frac{3}{5}$ of £1 + $\frac{2}{3}$ of 1s. + $\frac{5}{8}$ of 16s. 4d.

6. $\frac{3}{24}$ of £1 + $\frac{4}{5}$ of 2s. 6d. + $\frac{3}{7}$ of a guinea.

7. $\frac{3}{4}$ of 5 ac. 3 ro. + $\frac{5}{8}$ of 7 ac. 2 ro. 20 po. + $\frac{3}{5}$ of 3 ro. 15 po.

8. $\frac{3}{73}$ of a year + $\frac{9}{56}$ of a week + $\frac{7}{12}$ of an hour.

9. $\frac{3}{16}$ of a mile + $\frac{2}{3}$ of a furlong + $\frac{3}{8}$ of a yard.

10. $\frac{3}{4}$ of 2 cwt. 3 qr. + $\frac{3}{7}$ of 5 cwt. 3 qr. 14 lb. + $\frac{2}{5}$ of $7\frac{1}{2}$ lb.

129. The following are examples of an operation which is the *converse* of that just explained:—

Ex. 1. Express 14s. 7d. as the fraction of £5.

14s. 7d. = 175d., and £5 = 1200d.
Now 1d. = $\frac{1}{1200}$ of 1200d.
∴ 175d. is $\frac{175}{1200}$d. of 1200d.
Hence the fraction required is $\frac{175}{1200}$, or $\frac{35}{240}$, or $\frac{7}{48}$.

Ex. 2. Express 6 lb. 5 oz. avoird. as the fraction of 3 lb. 12 oz.

6 lb. 5 oz. = 101 oz., and 3 lb. 12 oz. = 60 oz.;
∴ the fraction required is $\frac{101}{60}$.

Ex. 3. Express $\frac{2}{3}$ of 5s. 9d. as the fraction of 4s. 7d.

5s. 9d. = 69d., and 4s. 7d. = 55d.
∴ 5s. 9d. is $\frac{69}{55}$ of 4s. 7d.
∴ $\frac{2}{3}$ of 5s. 9d. is $\frac{2}{3}$ of $\frac{69}{55}$ of 4s. 7d.
∴ the fraction required is $\frac{2\times69}{3\times55}$ or $\frac{46}{55}$.

Ex. 4. Express $\frac{3}{7}$ of $2\frac{4}{5}$ of 5 ac. 3 ro. as the fraction of $\frac{3}{5}$ of 14 ac. 2 ro.

5 ac. 3 ro. = 23 roods, and 14 ac. 2 ro. = 58 roods;

$\therefore$ fraction required is ($\frac{3}{7}$ of $\frac{14}{5}$ of 23) ÷ ($\frac{3}{5}$ of 58);

or $\frac{3\times14\times23\times5}{7\times5\times3\times58}$, or $\frac{2\times23}{58}$, or $\frac{23}{29}$.

NOTE.—There are several modes of demanding the operation explained in the foregoing examples. Thus the demand.

Express 3 *shillings as the fraction of* 6 *shillings*,

may be put in the following terms:—

1. Reduce 3 shillings to the fraction of 6 shillings.
2. What part of 6 shillings is 3 shillings?
3. What fraction of 6 shillings is 3 shillings?
4. If 6 shillings be the unit, what is the measure of 3 shillings?

Examples lix.

1. Express $1\frac{3}{4}$d. as the fraction of 6s. $8\frac{1}{2}$d.
2. Express £10 5s. 4d. as the fraction of £11 6s. 5d.
3. Express 5s. 6d. as the fraction of a guinea.
4. Reduce 9s. $10\frac{1}{2}$d. to the fraction of 13s. $2\frac{1}{4}$d.
5. Reduce 2 days 3 hrs. 5 min. to the fraction of a week.
6. Reduce 2 roods 20 poles to the fraction of an acre.
7. What fraction is 8 lb. 1 oz. 19 dwt. 9 gr. of 13 lb. 7 oz. 5 dwt. 15 gr.?
8. What part of 2 qr. 10 lb. 7 oz. is 1 qr. 7 oz.
9. What fraction of 4 lb. 1 oz. 8 dwt. 15 gr. is 1 lb. 1 oz. 9 dwt. 15 gr.?
10. If the unit of measurement be $2\frac{1}{2}$ yd., what is the measure of $2\frac{1}{2}$ feet?
11. If the unit of measurement be 5 inches, what is the measure of $\frac{5}{374}$ of a mile?
12. What fraction of 2 ac. 37 po. is 3 ac. 2 ro. 1 po.?

Decimal Measures.

130. Reduction of Decimals.

Ex. 1. Find the value of 3.16875 of £1.

$$\begin{array}{rr} & \pounds 3.16875 \\ & 20 \\ \hline \text{s.} & 3.37500 \\ & 12 \\ \hline \text{d.} & 4.50000 \\ & 4 \\ \hline \text{q.} & 2.00000 \end{array}$$

$\therefore$ £3.16875 = £3 3s. $4\frac{1}{2}$d.

Ex. 2. Find the value of .4256 of 12s. 8d.

.4256 of 12s. 8d. = .4256 of 152d. = (.4256 × 152)d.

$$\begin{array}{r} .4256 \\ 152 \\ \hline 5812 \\ 21280 \\ 4256 \\ \hline 64.6912 \end{array}$$

$\therefore$ value required is 64.6912d.

Ex. 3. Multiply 27 ac. 3 ro. 14 po. by .235.

27 ac. 3 ro. 14 po. = 4454 po.

.235 × 4454 po. = 1046.690 po.

$$\begin{array}{r|l} 40 & 1046.690 \\ \hline 4 & 26 \text{ ro. } 6.69 \text{ po.} \\ \hline & 6 \text{ ac. } 2 \text{ ro. } 6.69 \text{ po.} \end{array}$$

Ex. 4. Find the value of $.2\dot{5}$ of £1.

$.2\dot{5}$ of £1 = $\frac{25-2}{90}$ of £1 = $\frac{23}{90}$ of £1 = $\frac{460}{90}$*s.* = 5*s.* $1\frac{1}{3}$*d.*

Or thus,

$$\begin{array}{rl} & \pounds .255\dot{5} \ldots . \\ & 20 \\ \hline s. & 5.111\dot{1} \ldots . \\ & 12 \\ \hline d. & 1.333\dot{3} \ldots . \end{array}$$

$\therefore$ value required is 5*s.* $1\frac{1}{3}$*d.*

Examples lx.

Find the value of

1. .625 of £1.
2. £15.275.
3. £.009765.
4. .9375 of a cwt.
5. .046875 of 1 lb. avoir.
6. 2.003125 of £8.
7. .425 of 3s. 4d.
8. 2.46875 of £1 3s.
9. $.8\dot{3}$ of 5s.
10. $4.1\dot{3}$ of 12s. 6d.
11. .35 of 2 qr. 14 lb.
12. 2.125 of 3½ guineas.
13. 2.1372 of 2 tons 5 cwt.
14. $5.24\dot{7}$ of £5 2s. 6d.
15. .45 of £3 10s. + .75 of 4s. 8d. + 3.245 of 3s. 4d.
16. $.\dot{7}$ of £1 + $.\dot{8}$ of 7s. 6d. − 2.45 of 1s. 8d.
17. $.\dot{2}8571\dot{4}$ of £3 3s. + $.\dot{1}4285\dot{7}$ of £3 17s. + $.\dot{3}\dot{4}$ of 16s. 6d.

131. The following examples illustrate the operation which is the *converse* of that already explained.

Ex. 1. Express 5s. 6d. as the decimal of £1.

$$\begin{gathered}\text{5s. 6d.} = \text{66d., and } £1 = \text{240d.;}\\ \therefore \text{5s. 6d.} = \tfrac{66}{240} \text{ of } £1.\\ \text{Now } \tfrac{66}{240} = \tfrac{11}{40} = .275;\\ \therefore \text{5s. 6d.} = .275 \text{ of } £1.\end{gathered}$$

Or more briefly thus

$$\begin{array}{r|l} 12 & \underline{6.0 \quad \text{d.}} \\ 20 & \underline{5.5 \quad \text{s.}} \\ & £.275. \end{array}$$

Where we first express 6d. as the decimal of a shilling, *i.e.*, .5, and then express 5.5s. as the decimal of a pound, *i.e.*, .275.

Ex. 2. Express £7 15s. 10½d. as the decimal of £1.

$$\begin{array}{r|l} 4 & \underline{2.0} \\ 12 & \underline{10.5} \\ 20 & \underline{15.875} \\ & £7.79375 \end{array}$$

Ex. 3. Express £3 5s. 9d. as the decimal of £5 7s. 6d.

£	s.	d.	£	s.	d.
3	5	9	5	7	6
20			20		
65			107		
12			12		
789			1290		

Now $\frac{789}{1290}=\frac{263}{430}=\frac{26\cdot3}{43}=.611\ldots\ldots$

$\therefore$ £3 5s. 9d. is .611......of £5 7s. 6d.

Ex. 4. Express $\frac{2}{3}$ of 5s. $9\frac{1}{4}$d. as the decimal of $\frac{3}{5}$ of 6s. 2d.

5s. $9\frac{1}{4}$d. = 277q., and 6s. 2d. = 296q.

$\therefore$ $\frac{2}{3}$ of 5s. 9d. is $\frac{\frac{2}{3}\times277}{\frac{3}{5}\times296}$ of $\frac{3}{5}$ of 6s. 2d.

Now $\frac{\frac{2}{3}\times277}{\frac{3}{5}\times296}=\frac{2\times277\times5}{3\times3\times296}=\frac{1385}{1332}=1.039\ldots\ldots$

Examples lxi.

1. Express 6 cwt. 2 qr. 7 lb. as the decimal of a ton.
2. Express 12 grains as the decimal of a lb. Troy.
3. What decimal of 10 guineas is £1 19s. $4\frac{1}{2}$d.
4. Express $\frac{3}{5}$ of 14s. 4d. as the decimal of £1.
5. Reduce 3.45 of half a guinea to the decimal of 2s. 6d.
6. Express $\frac{3}{8}$ of 2 qr. 14 lb. as the decimal of a cwt.
7. Express $4\frac{2}{5}$ of 7 oz. 4 dwt. as the decimal of a lb. Troy.
8. Reduce $3\frac{7}{8}$ of $1\frac{1}{4}$ of 5 cwt. 2 qr. 21 lb. to the decimal of a ton.
9. What decimal of a lb. Troy is $\frac{2}{3}$ of a dwt. ?
10. Reduce $3\frac{3}{4}$ guineas to the decimal of £2 15s.
11. Reduce 2s. 6d. to the decimal of $\frac{5}{12}$ of £1.
12. Express 18s. $4\frac{1}{2}$d. as the decimal of £1000.
13. Reduce £24.25 + 3.4125s. + 9.25d. to the decimal of £10.
14. Express $.\dot{4}\dot{3}$ of 8s. 3d. as the decimal of $.0\dot{1}$ of £9.
15. Express $.0\dot{4}$ of £2 5s. + $.\dot{2}\dot{3}$ of 3s. 9d. as the decimal of $.\dot{2}4\dot{5}$ of £4 3s. 3d.

EXAMINATION PAPERS.

I. MEASURES OF TIME.

1. A sidereal day is less than a solar day by 3 min. 56 sec. In how many days will the difference amount to 24 hr.?

2. If Sirius, one of the brighest of the fixed stars, which is probably 592200 times farther from the earth than the sun, were suddenly extinguished, for how long would it appear to shine to the inhabitants of the earth, supposing the sun's mean distance from the earth to be 91713000 mi., and that light from the sun reaches the earth in 8 min. 18 sec.?

3. The exact length of the year being 365 da. 5 hr. 48 min. 49.7 sec., and computing time as at present, find the error in 12000 yr.

4. The *Globe* newspaper of Monday, July 16, 1900, bears the number 15729. Supposing the paper to have been published every week day without intermission, and numbered consecutively, give the day of the week, month, and year when No. 1 was published.

5. There was a full moon on June 26, 1858, at 9 hr. 13 min. a.m. The interval between successive full moons has since been on the average 29 da. 12 hr. 47 min. 30 sec. How many full moons happened until December 31, 1873, and when did the last take place within that period?

II. MEASURES OF LENGTH.

1. Reduce 9 mi. 7 fur. 39 per. 5 yd. 1 ft. 9 in. to inches, and show that the work is correct by changing it to miles, etc.

2. The fore-wheel of a carriage, which is 11 ft. in circumference, makes 718 revolutions more than the hind one in going 7 mi. Find the circumference of the hind-wheel.

3. A train, which travels at the uniform rate of 66 ft. a second, leaves Toronto for Montreal at 6.25 a.m. When will it reach Montreal, the distance being 333 mi.? At what distance from Montreal will it meet a train which leaves Montreal for Toronto at 8 a.m., and travels one-third faster than it does?

4. From Ephesus to Cunaxa, Xenophon, with the army of Cyrus, marched 16050 stadia of 202 yd. 9 in. each in 93 da. Find the average length of a day's march in miles and yards.

5. How many strokes of his legs must a person on a bicycle give in going 26 mi., supposing each wheel to have a circumference of $2\frac{4}{9}$ yd., and that two strokes turn the wheel five times round?

III. MEASURES OF SURFACE.

1. If the magnitude of the lineal unit be given, what are the corresponding units of area and volume? Exemplify when the lineal unit is 12 in.

2. If a halfpenny piece be one inch in diameter, how many can be laid in rows touching each other on a table which is 7 ft. 6 in. long and 3 ft. 4 in. wide; and what is their amount?

3. Divide 17 ac. 2 ro. 38 per. 19 yd. 7 ft. 45 in. among *A*, *B*, and *C*, giving to *B* as much again as to *A*, and to *C* $\frac{3}{4}$ of what *A* and *B* got.

4. If 68 bales of linen contain 67048 yd., and each bale contains 34 pieces, and each piece the same number of yards, how many yards are there in each piece?

5. If the pressure of the atmosphere at the surface of the earth, when the barometer stands at 30 in., be 15 lb. on the square inch, what is the pressure in pounds on the surface of the human body, supposing it to be 15 sq. ft.? What would be the difference of the pressure when the barometer stands at 29 in.?

IV. MEASURES OF CAPACITY.

1. What will 2 bu. 3 pk. 3 qt. of strawberries amount to at $12\frac{1}{2}$c. per qt.?

2. A laborer dug 130 ro. 4 yd. $2\frac{1}{2}$ ft. of ditching at $\$2\frac{1}{2}$ per ro., for which he is to take $100 in cash and wheat at $87\frac{1}{2}$c. per bu. To what quantity of wheat will he be entitled?

3. A grocer exchanged 29 gal. 3 qt. 1 pt. of brandy, at $43\frac{3}{4}$c. per gal., for rye at $31\frac{1}{4}$c. per bu. What quantity of rye did he thus obtain?

4. I wish to put 111 bu. 2 pk. 4 qt. of grain into bags that shall contain 2 bu. 1 pk. 4 qt. each. How many bags will be required?

5. A farmer had a field of corn consisting of 129 rows, and each row contained 95 hills, and each hill had on an average $4\frac{1}{2}$ ears of corn. If it takes 8 ears of corn to make a quart, what is the produce of the field worth at 45c. per bu.?

V. MEASURES OF WEIGHT.

1. If John buy, by Avoirdupois weight, 12 lb. of opium at $37\frac{1}{2}$c. per oz., and sell by Troy weight, at 40c. per oz., should he gain or lose by so doing, and how much?

2. A person purchases goods at the rate of $1.80 per lb., Troy weight, and sells them again by Avoirdupois weight. At what rate must he sell per ounce so as exactly to reimburse himself?

3. By multiplying a certain weight by a whole number the result is 8 lb. 20 grains Avoirdupois weight, and by multiplying the same weight by another whole number the result is 8 lb. 11 oz. 16 dwt. 16 gr. Find the largest weight.

4. A row of cent pieces is laid from Toronto to Hamilton. Find their weight, the distance being 39 mi. 1 fur. 1 per. 9 in.

5. Find the value of 500 times the difference between an eighty-fourth-part of $2\frac{1}{2}$ cwt. and a thirtieth part of 1 cwt. 0 qr. 3 lb. (28 lb. to the quarter).

VI.

1. A cask of wine containing 38 gal. was bought for £25. By leakage 8 gal. were lost. At what price per gallon must the remainder be sold to gain £2 10s. on the whole?

2. Find the H. C. F. and the L. C. M. of 49 ac. 3 ro. 38 po. $2\frac{3}{4}$ yd. and 63 ac. 2 ro. 19 po. $11\frac{3}{4}$ yd.

3. What is the greatest unit of time with which 2 da. 14 hr. 50 min. and 2 da. 19 hr. 10 min. can be both expressed as integers.

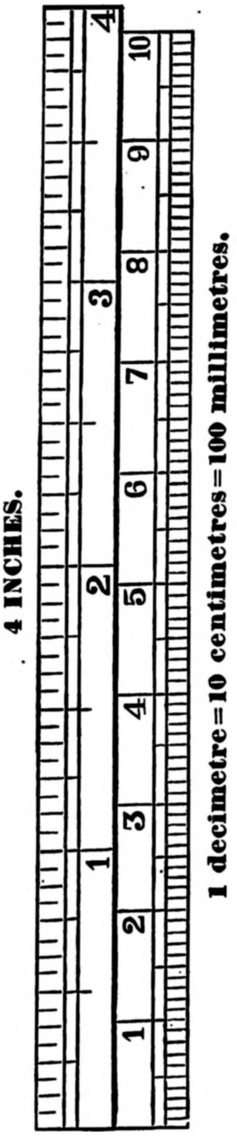
4. A person has 4988 francs worth $9\frac{3}{4}$d. each, the same number of dollars worth 4s. $2\frac{1}{2}$d. each, and has as many rupees worth 2s. $1\frac{1}{2}$d. each, and one-fourth as many Spanish reals worth $2\frac{1}{2}$d. each. If he receive £1500 for all of them, how much does he gain or lose?

5. Standard gold is coined at the rate of £3 17s. $10\frac{1}{2}$d. per ounce. Find the least number of ounces that can be coined into an exact number of half-sovereigns (a sovereign is £1).

For additional examples see page 301.

CHAPTER VIII.

THE METRIC SYSTEM OF MEASURES.



132. The Metric System of Measures was established in France at the end of the 18th century. It has been adopted by most of the nations of Europe and South America. It is almost universally used in scientific treatises.

133. The fundamental unit is the **Metre,** a measure of length supposed to be equal to the ten-millionth part of the distance from the North Pole of the Earth to its Equator, and is defined by law to be the distance between the ends of a rod of platinum made by Borda, the temperature being that of melting ice. It has since been found that Borda's rod is not exactly the ten-millionth part of the distance between the Equator and the North Pole, so the Metric Standard is **Borda's Rod** and not the terrestrial globe.

134. The advantages of the Metric System may be briefly enumerated as follows:—

i. It does away with the Reduction, Addition, Subtraction, Multiplication, and Division of Compound Numbers.

ii. All arithmetical operations are the same as for simple numbers.

iii. As it would give all nations a universal system of measures, its general introduction would greatly facilitate trade and exchange. Practically this is its greatest advantage.

135. UNITS OF METRIC MEASURES.

1. LENGTH.—The Metre.
2. SURFACE.—The Are =100 square metres.
3. SOLIDITY.—The Stere = 1 cubic metre.
4. CAPACITY.—The Litre = the cube of the tenth part of a metre.
5. WEIGHT.—The Gramme, which is the weight of a quantity of distilled water which fills the cube of the hundredth part of a metre.

The tables of Weights and Measures under the Metric System are constructed upon one uniform principle. Prefixes derived from Greek and Latin are attached to each of the units.

GREEK PREFIXES.

Deca stands for	10 times	the unit.
Hecto stands for	100 times	
Kilo stands for	1000 times	
Myria stands for	10000 times	

LATIN PREFIXES.

Deci stands for the	10th part	of the unit.
Centi stands for the	100th part	
Milli stands for the	1000th part	

Thus,

A decametre - - - - . = 10 metres.
A hectolitre - - - - - - = 100 litres.
A kilogramme - - - - - = 1000 grammes.
A myriametre - - - - - = 10000 metres.

Also,

A decilitre - - - - - = .1 litre.
A centimetre - - - - - = .01 metre.
A milligramme - - - - - = .001 gramme.

136. LINEAR MEASURE.

10 millimetres	(mm.) = 1 centimetre	=	.01	metre.
10 centimetres	(cm.) = 1 decimetre	=	.1	"
10 decimetres	(dcm.) = 1 metre	=	1.	"
10 metres	(m.) = 1 dekametre	=	10.	"
10 dekametres	(Dm.) = 1 hectometre	=	100.	"
10 hectometres	(Hm.) = 1 kilometre	=	1000.	"
10 kilometres	(Km.) = 1 myriametre	=	10000.	"

1 metre = 39⅜ inches nearly; 70 yd. = 64 metres nearly;
8 Km. = 5 miles nearly.

1 inch = 2.539954 centimetres.	1 yard = 0.914383 metres.
1 foot = 3.047945 decimetres.	1 mile = 1.609315 kilometres.

NOTE.—A rough rule for converting French metres into English yards is to add 10 per cent. to them. Thus 40 metres are nearly equal to 44 yards.

Examples lxii.

1. Read 798.465 m., giving the denomination of each figure.

2. Write 5 Km. 7 m. 3 cm. 9 mm. in the denomination of the prime unit (metre).

In the same way as in 2 write,

3. 17 Km. 8 Hm. 3 dcm. 8 mm.

4. 6 Mm. 7 Dm. 8 dcm. 7 mm.

5. Express 5.76 Km. in metres, in decimetres, in millimetres.

6. Express 8769 mm. in metres, in hectometres, in kilometres.

7. Add 7.96 m., 5.8 Km., 7 Dm., 19.6 dm. and 7896 mm.

8. From 1 Mm. take 7 Km. 5 dcm. 6 mm.

9. A train runs 36.84 Km. per hour. How far does it go in 3.25 hours ?

10. A merchant bought 1896 m. of cloth at $1.15 per metre. What did the cloth cost ?

11. Posts are placed 1.5 m. apart along a distance of 4.65 Km. from a stone wall. How many posts are there ?

12. How many m. of fence will enclose a rectangular field 625.7 m. long and 378.3 m. wide ?

13. How many times will a wheel, the circumference of which is 3.25 m., turn in a distance of 10.5 Km. ?

137. SQUARE OR SURFACE MEASURE.

100 sq. millimetres (sq. mm.) = 1 sq. centimetre = .0001 sq. metre.
100 sq. centimetres (sq. cm.) = 1 sq. decimetre = .01 "
100 sq. decimetres (sq. dcm.) = 1 sq. metre = 1. " = 1 centare.
100 sq. metres (sq. m.) = 1 sq. dekametre = 100. " = 1 are.
100 sq. dekametres (sq. Dm.) = 1 sq. hectometre = 10000. " = 1 hectare.
100 sq. hectometres (sq. Hm.) = 1 sq. kilometre = 1000000.

The centare, are, and hectare are used only in measuring land.

100 centares (ca.) - - - - - - - = 1 are (a.)
100 ares - - - - - - - - - - - - = 1 hectare (Ha.)

The hectare is rather less than 2½ acres.

1 square inch - - - - - - - = 6.4513669 sq. cm.
1 " foot - - - - - - - = 9.2899683 sq. dcm.
1 " yard - - - - - - - = 0.83609715 sq. m.
1 " acre - - - - - - - = 0.40467101 hectare.

Examples lxiii.

1. Express 5 Ha. 2 a. 5 ca. as hectares ; as ares ; as centares.
2. Express 19 sq. Km. 7 sq. Hm. 5 sq. m. as square metres.
3. Write the following as sq. m. 6 sq. Dm. 7 sq. m. 81 sq. dcm.
4. How many sq. m. are there in 2705608 sq. mm.
5. Express the difference between 1 Ha. and 1 a. in centares.
6. Reduce 78696 ares to hectares.
7. Find the cost of 56 Ha. of land at $.025 per centare.
8. Supposing your school lot to be a rectangle 150.6 m. long and 85.5 m. wide, and the buildings to occupy just 406 sq. m., what space is left for the play-ground ?

138. MEASURES OF CAPACITY.

1000 cu. millimetres (c.mm.) = 1 cu. centimetre = .000001 cu. metre.
1000 cu. centimetres (c.cm.) = 1 cu. decimetre = .001 " = 1 litre.
1000 cu. decimetres (c.dcm.) = 1 cu. metre = 1. " = 1 stere.

The units are obtained by cubing the units of Linear Measure.

The **Stere** is the unit used is measuring wood, excavations, etc.

1 stere - - - - - - - = $1\frac{4}{13}$ c. yd., nearly.
$3\frac{7}{10}$ steres - - - - - = 1 cord, nearly.

In measuring liquids, the **Litre** is used, and in measuring grains, fruits, etc., the **Hectolitre.** The same numerical prefixes are used with the litre as with the metre.

1 cubic inch - - - - - = 16.386176 cu. cm.
1 " foot - - - - = 28.315312 cu. dcm.
1 gallon - - - - - - = 4.54345797 litres.

Examples lxiv.

1. Write 715 cu. m. 7 cu. dcm. 78 cu. cm. as cu. metres ; as cu. dcm.; as cu. cm.
2. Express 7 Kl. 4 Dl. 7 l. 8 cl. as litres ; as millilitres.
3. How many steres of earth must be removed from a cellar 3.5 m. deep, 20.4 m. long, and 12.6 m. wide ?
4. How many steres of wood are there in a pile 16.6 m. long, 4.5 m. wide and 3.4 m. high ?

5. How many loads of earth, each measuring 3.25 cu. m., will fill a rectangular hole 14.3 m. long, 6.5 m. wide, and 5 m. 5 dcm. wide?

6. A cistern is 4 m. long, 24 dcm. wide, and 80 cm. deep. Find its capacity in litres.

7. How many times must 3 l. 4 dcl. be taken from 2125 cl. to leave 11 l. 5 cl.?

139. MEASURES OF WEIGHT.

10 milligrams (mg.) = 1 centigram = .01 gram.
10 centigrams (cg.) = 1 decigram = .1 "
10 decigrams (dcg.) = 1 gram = 1. "
10 grams (g.) = 1 dekagram = 10. "
10 dekagrams (Dg.) = 1 hektogram = 100. "
10 hektograms (Hg.) = 1 kilogram = 1000. "

1 cubic centimetre of distilled water at 4°C. at the sea's level in the latitude of Paris is 1 gram.

1000 cubic centimetres of distilled water weighed under the same conditions............1 kilogram (Kg.).

1 grain - - - - - - - - = 0.06479895 gram.
1 Troy oz. - - - - - - - = 31.103496 grams.
1 oz. Avd. - - - - - - - = 28 grams.
1 lb. Avd. - - - - - - - = 0.45359265 kilo.
1 gram - - - - - - - - = 15.432 grains.
1 kilogram - - - - - - - = $2\frac{1}{5}$ lb. Avoirdupois, nearly.

In weighing heavy articles two other weights, the *quintal* (100 Kg.) and the *tonneau or ton* (1000 Kg.) are used. The ton is a little less than 2205 lb.

Examples lxv.

1. Name the denomination of each figure in 3471.0868 grams.

2. How many centigrams are there in 43 Kg. 3 Dg. 5 g. 7 dcg.?

3. By how many grams do 21 Kg. exceed 14 Kg. 7 Hg. 7 Dg.?

4. Multiply 3 Dg. 7 g. 5 mg. by 1000 and express the result as kilograms.

5. A vessel contains 71 Dl. 5 dcl. of water. Express the weight of this water in grams; in mg.; in dcg.

6. Find the weight of 45 hectolitres of water.

7. Find the weight of the water in a rectangular box 1.4 m. long, .8 m. wide, .75 m. deep.

8. Neglecting the weight of the vessel, how heavy is a vessel holding 78 dekalitres of water.

9. A vessel weighs 17.4 Kg. and holds 7865 litres of water. How much does the whole weigh?

10. Mercury weighs 13.5 times as much as water. Find the weight of 17 litres of mercury.

11. A flask weighing 10 g. when empty, weighs 10.6465 g. when filled with air, and 510 g. when filled with water. Find the relative weight of air to water.

12. A flask when empty weighs 60 g., when filled with alcohol it weighs 180 g., and when filled with water it weighs 210 g. Find the relative weight of alcohol to water.

Examples lxvi.

1. How many decimetres are equivalent to 106725 millimetres?

2. Required, the number of milligrams in 15 cu. cm. of water measured at 4°C.?

3. How many millimetres and centimetres are respectively contained in 0.437 of a decimetre?

4. How many square centimetres are there in 15.5 square metres?

5. How many square decimetres are contained in 108642 square centimetres?

6. Define the gram and litre. How many grams are contained in 1.725 kilograms?

7. How many milligrams are there in a decigram? How many decigrams in a kilogram?

8. How many centigrams are there in 2.567 kilograms?

9. Required, the number of milligrams contained in 5 cubic centimetres of water measured at 4°C.

10. A gallon is equal to 4.543 litres. How many cubic centimetres are contained in one pint?

11. Three pipes furnish respectively 30 litres, 45 litres, and 80 litres an hour. What quantity of water do they supply together in 24 hours?

12. If 1 metre (39.3708 inches) is the ten-millionth part of a quadrant of a meridian, how many miles are there in the circumference of the earth?

13. If air is .00129206 times as heavy as water, find the weight in grams of the air in a room 25 m. long, 16 m. wide, and 10 m. high.

For additional problems, see page 303.

CHAPTER IX.

PRACTICE AND ACCOUNTS.

140. Practice is the name given to a method by which we find the cost of any number of articles of the *same* kind when the price of one is given, or the cost of any quantity of goods of *mixed* denominations, when the cost of a single unit of any denomination is given.

141. The fractions of a Unit, which have for their numerator *Unity*, are called **Aliquot Parts** of the unit. Thus, 5s., being $\frac{1}{4}$ of £1, is an aliquot part of a pound; and 5 lb., being $\frac{1}{5}$ of 1 qr., is an aliquot part of a quarter.

Ex. 1. Find the cost of 456 articles at $33\frac{1}{3}$c. each.

$$\begin{array}{r|l} 33\tfrac{1}{3}\text{c.} = \$\tfrac{1}{3} & \$456 = \text{cost at } \$1 \text{ each.} \\ \cline{2-2} & \$152 = \text{`` `` } 33\tfrac{1}{3}\text{c. each.} \end{array}$$

Ex. 2. Find the cost of 245 articles at $\$2.12\frac{1}{2}$ each.

$$\begin{array}{lrl} 12\tfrac{1}{2}\text{c.} = \$\tfrac{1}{8} & \$245 & = \text{cost at } \$1.00 \text{ each.} \\ & 2 & \\ \hline & 490 & = \text{cost at } \$2 \text{ each.} \\ & 30.62\tfrac{1}{2} & = \text{cost at } 12\tfrac{1}{2}\text{c. each.} \\ \hline & 520.62\tfrac{1}{2} & = \text{entire cost.} \end{array}$$

Ex. 3. Find the cost of 5 ac. 3 ro. 30 po. at \$60 an acre.

$$\begin{array}{r|lrl} 4 & 5 \times \$60 = & \$300 & = \text{cost of 5 ac.} \\ \cline{2-2} 40 & 3 \times \$15 = & 45 & = \text{cost of 3 ro.} \\ \cline{2-2} & 30 \times \$0.37\tfrac{1}{2} = & 11.25 & = \text{cost of 30 po.} \\ \cline{3-3} & & \$356.25 & = \text{entire cost.} \end{array}$$

Examples lxvii.

Find the price of

1. 789 articles at 75c.
2. 456 articles at $37\frac{1}{2}$c.
3. 450 articles at \$1.25.
4. 366 articles at $\$5.16\frac{2}{3}$.
5. 4321 articles at £1 17s. $3\frac{3}{4}$d.
6. 2175 articles at £2 15s. $4\frac{1}{2}$d.

7. 5 t. 13 cwt. 40 lb. of hay at $12 per ton.
8. 9 oz. 15 dwt. 6 gr. of gold at $16 per oz.
9. 2 mi. 6 fur. 28 rd. of railroad at $15000 per mile.
10. 5 ac. 3 ro. 4 po. 4½ yd. at £10 per rood.
11. 12 cwt. 3 qr. 22 lb. 12 oz. at £3 18s. 2d. per cwt.
12. 10 ac. 3 ro. 26 po. at £2 18s. 10⅔d. per acre.
13. 6 tons 12 cwt. 3 qr. 10½ lb. at £3 14s. 8¼d. per cwt.
14. 63 cwt. 3 qr. 17½ lb. at 12 guineas per cwt.

142. An **Invoice** is a statement in detail, sent by a Seller to the Buyer at the time the goods are delivered to the Buyer, of the quantity, description, and price of the goods.

An **Account** is a statement sent by the Seller to the Buyer at the end of a term of credit, showing the totals and dates of each Invoice and the sum total of the whole.

Each separate article or amount in an Invoice or an Account is called an **Item.**

A **Detailed Account** is a full statement, sent by the Seller to the Buyer at the end of a term of credit, showing the dates of delivery, the quantities, description, prices, and sum total of the goods delivered by the Seller to the Buyer during that term of credit.

When an account has been made out it is *rendered, i.e.*, sent to the Buyer.

SPECIMEN OF AN INVOICE.

TORONTO, June 20, 1900.

John Smith, Esq.,

Bought of J. Jones & Co., 21 Front St.

		cts.
5 lb. of Tea..........@ 75c..........................	3	75
8 lb. of Loaf Sugar...@ 12½c..........................	1	00
2½ lb. of Butter......@ 30c..........................	0	75
	5	50

SPECIMEN OF AN ACCOUNT.

TORONTO, July 21, 1900.

John Smith, Esq., Dr.

To J. Jones & Co., 21 Front St.

1900		$	cts.
June 20	To Goods, as per invoice................	5	50
June 23	To " "	7	80
July 3	To " "	3	60
July 12	To " "	2	27
		19	17

SPECIMEN OF A DETAILED ACCOUNT.

TORONTO, July 21, 1900.

John Smith, Esq., Dr.

To J. Jones & Co., 21 Front Street.

1900			$	cts.
June 20	5 lb. of Tea	@ 75c	3	75
" 20	8 lb. of Loaf Sugar	@ 12½c..........	1	00
" 20	2½ lb. of Butter.......	@ 30c...........	0	75
" 23	1 bbl. of Flour........	@ $6............	6	00
" 23	18 lb. of Cheese.......	@ 10c...........	1	80
July 3	12 lb. of Biscuit	@ 15c...........	1	80
" 3	6 jars of Pickles	@ 30c...........	1	80
" 12	1 gal. of Coal Oil......	@ 37c...........	0	37
" 12	8 lb. of Sugar.........	@ 11c....... ..	0	88
" 12	8½ lb. of Raisins.. ...	@ 12c...........	1	02
			19	17

Examples lxviii.

1. Make out Invoices of the following sales, supplying names and dates of your own selection :—

(*a*) 100 yd. of broadcloth @ $3.25 per yd. ; 2500 yd. of sheeting @ 12c. per yd. ; 3000 yd. of prints @ 18c. per yd. ; 300 yd. of French silk @ $1.75 per yd.

(*b*) 5 lb. of black tea @ 70c. ; 2¼ lb. of green tea @ 90c. ; 15½ lb. of lump sugar @ 12c. ; 17 lb. of brown sugar @ 9c. ; 7½ lb. of raisins @ 20c.; 4 lb. of currants @ 13c.

2. Make out Accounts of the following sales, supplying names and dates of your own selection :—

(*a*) 39½ yd. of Brussels carpet, @ $1.50 ; 62¾ yd. of Kidderminster carpet, @ $1.10 ; 27 yd. of matting, @ 23c. ; 34½ yd. of drugget, @ 65c. ; 43½ yd. of India matting, @ 18c.

(*b*) 23 yd. of black silk, @ $2.15 ; 17 yd. of ribbon, @ 23c. ; 13½ yd. of silk velvet, @ 25c. ; 1½ doz. pairs of stockings, @ 45c. a pair ; 5 pairs of gloves, @ $1.25 ; 18 yd. of muslin, @ 17c.

(*c*) 6 pairs of blankets, @ $5.50 ; 12½ yd. of merino, @ 45c. ; 15¾ yd. of cloth, @ $3.25 ; 5⅓ yd. of flannel, @ 30c. ; 2 counterpanes, @ $4.25 each ; 25¼ yd. of calico, @ 15c.

(*d*) Mrs. James Jones bought of John Burns, of Toronto, the following articles : Feb. 17, 1899, ¾ doz. linen napkins, @ $1.75 ; 2¼ doz. damask towels, @ $4.50 ; 3 bath towels, @ $2.40 a doz. ; Feb. 21, 1899, 2 table-cloths, @ $5.50 ; 1 piano-cover, @ $5.00 ; 7 yd. cambric, @ $0.12½ ; 2 pr. lace curtains, @ $2.50 a pair.

3. James Lock, Toronto, sold Wm. Hogg, May 15, 1899, 3½ bushels of canary-seed, @ $3.62½ ; 7¼ bushels of flax-seed, @ $2.95 ; 5 bushels of hemp-seed, @ $3.15 ; 163 lb. of clover-seed, @ $8.75 per hundred ; 4375 lb. crude brimstone, @ $36 per ton ; 420 lb. indigo, @ $128 per hundred ; 5090 lb. Rio Grande shinbones, @ $39 per ton. Hogg gives his due-bill for the amount. Make out bill and receipt it accordingly.

4. Tyndale & Co., Montreal, sold to Mrs. John Smith, Dec. 31, 1899, the following articles : 1 soup tureen, @ $3.50 ; 2 sauce tureens, @ $1.25 ; 1 doz. tulip goblets, @ $1.40 ; 1 doz. individual salts, @ 50c. ; 2 glass pitchers, @ 62½c. ; 3 oval glass dishes, @ 50c. ; 1 glass nappy, @ 75c. ; 4 covered dishes, @ $1.25 ; 3 doz. stoneware plates,—1 doz. 6 in., @ $1.25, 1 doz. 7 in. @ $1.40, 1 doz. 8 in., @ $1.60. Required, the bill, receipted.

5. John Brown delivered the following, June 12, 1899, at the Queen's Hotel : 15 salmon, @ 30c. ; 37 lobsters, @ 10c. ; 5 fresh mackerel, @ 12c. ; 7 Spanish mackerel, @ 20c. ; 2 shad, @ 40c. ; 12 soles, @ 10c. ; 9 sea-bass, @ 10c. ; 15 sheepheads, @ 12c. ; 3 doz. soft crabs, @ 50c. ; 1½ doz. frogs, $1.75. Extend the items and find the footing of the bill.

6. James May and Co., Toronto, bought of Bausch & Lomb, Rochester, March 17, 1899, 3 doz. 8-in. thermometers on polished walnut, @ $10 ; 1½ doz. 8-in. parlor thermometers, @ $4 ; 6 doz. tin-case thermometers, @ $4 ; 9 Aneroid barometers, @ $5. Paid by note at 3 months. Write the note.

CHAPTER X.

PROBLEMS.

Simple Problems.

Ex 1. If 23 bullocks cost \$483, what is the cost of 1 bullock?

Since 23 bullocks cost \$483,
1 bullock will cost $\$\frac{483}{23}$, or \$21.

Ex. 2. If 7 men do a piece of work in 12 days, how long will it take one man to do it?

Since 7 men can do the work in 12 da.,
1 man can do the work in (7 × 12) da., or 84 da.

Ex. 3. If 28 men do a piece of work in 42 days, in how many days can 21 men do it?

Time for 28 men to do the work = 42 da.
" 1 man " " = 28 × 42 da.
" 21 men " " = $\frac{28 \times 42}{21}$ da.
= 56 da.

Ex. 4. If 75 men finish a piece of work in 12 days, how many men will finish it in 20 days?

In 12 da. the work is done by 75 men.
In 1 da. the work is done by (12 × 75) men.
In 20 da. the work is done by $\frac{12 \times 75}{20}$ men, or 45 men.

Ex. 5. A bankrupt's debts are \$2520, and his assets (that is, the value of his property) are \$1890. What can he pay in the dollar?

In the place of \$2520, he can pay \$1890.
In the place of \$1 he can pay $\$\frac{1890}{2520}$, or $\$\frac{3}{4}$, or 75 cents;
∴ he pays 75 cents in the dollar.

Ex. 6. A bankrupt's debts are £4264, and he pays 12s. 6d. in the pound. What are his assets?

That which he has to meet a debt of £1 is $12\frac{1}{2}$s.
That which he has to meet a debt of £4264 is $(4264 \times 12\frac{1}{2})$s.;
∴ his assets are $\frac{4264 \times 25}{2}$s., or £2665.

Ex. 7. If 27 men can do a piece of work in 14 days, working 10 hours a day, how many hours a day must 12 men work to do the same in 45 days?

Since 27 men can do the work in (14 × 10) hr., or 140 hr.
∴ 12 men can do the work in $\frac{27 \times 140}{12}$ hr., or 315 hr.

Now 315 hr. have to be distributed equally over 45 da.;
∴ the number of hours they work each da. $= \frac{315}{45}$, or 7.

Ex. 8. If 7 lb. of tea cost $5.60, what will be the cost of 12 lb.?

Since 7 lb. of tea cost $5.60,
1 lb. of tea costs $$\frac{5.60}{7}$, or 80c.
12 lb. of tea cost 12 × 80c., or $9.60.

Ex. 9. If 9 horses can plough 46 acres in a certain time, how many acres can 12 horses plough in the same time?

Since 9 horses can in the given time plough 46 ac.,
1 horse can in the given time plough $\frac{46}{9}$ ac.
∴ 12 horses can in the given time plough $\frac{12 \times 46}{9}$ ac.,
or $61\frac{1}{3}$ ac.

Ex. 10. If 15 horses can plough a certain quantity of land in 5 days, how many horses will be required to plough it in 3 days?

In 5 da. the land can be ploughed by 15 horses;
In 1 da. the land can be ploughed by (5 × 15) horses;
In 3 da. the land can be ploughed by $\frac{5 \times 15}{3}$, or 25 horses.

143. In simple questions of this kind we have a *supposition* and a *demand.* Each contains two kinds of things. In the *supposition* the magnitudes of *both* kinds are given. In the *demand* a magnitude of one kind is given, and the appropriate corresponding magnitude of the other kind has to be found. The first line of the solution contains the magnitudes of the supposition so arranged that *at the end of the line we have that kind of thing of which the magnitude is required in the demand.*

Thus, in Ex. 10 the order of the supposition is changed, and the magnitude, 15 horses, put at the end of the line, because we have to find how many horses will be required in the demand.

Examples lxix.

1. If a man walk 62 mi. in 4 da., in how many da. will he walk 93 mi.?

2. If 12 men reap a field in 4 da., in what time will 32 men reap it?

3. If 350 ac. of land cost $61250, what will 273 ac. cost?

4. How many men can perform in 12 da. a piece of work which 15 men can perform in 20 da.?

5. The rent of 47 ac. is $297. What is the rent of 86 ac.?

6. If a man walk 116 mi. in 8 da., how far will he walk in 14 da.?

7. A farmer sells a flock of 270 sheep at $240 a score. What does he get for them?

8. A servant's wages being $108 per annum, how much ought she to receive for 7 wk.?

9. A clerk's salary is £191 12s. 6d. per annum. What ought he to receive for 60 days' service?

10. A ship performs a voyage in 63 da., sailing at the rate of 6 knots an hr. How long would it take her if she sailed at the rate of 7 knots an hr.?

11. A bankrupt's effects are worth $860, and his debts are $4300. What does he pay in the dollar?

144. To one of the magnitudes in a supposition there is a corresponding magnitude of the same kind in the demand, and these magnitudes must be expressed in units of the same denomination.

Ex. A man walks 1 mile 1 furlong 7 poles in 20 minutes. How long will he take to walk 41 miles 2 furlongs 12 poles?

Here 1 m. 1 fur. 7 po. = 367 po.,
and 41 m. 2 fur. 12 po. = 13212 po.

Then he walks 367 po. in 20 min.;
he walks 1 po. in $\frac{20}{367}$ min.;
he walks 13212 po. in $\frac{13212 \times 20}{367}$ min., or 720 min.

12. If 3 bu. of wheat are worth $3.50, what is the worth of 43 qr. 6 bu.?

13. If 15 yd. of silk cost \$6.75, how much will 20 yd. 1 ft. cost?

14. If 3 cwt. 3 qr. cost \$27, what will be the cost of 2 cwt.?

15. If 2 cwt. 3 qr. 7 lb. cost £5 17s. $8\frac{1}{2}$d., what is the cost of 9 cwt.?

Problems Involving Fractions.

Ex. If $\frac{3}{7}$ of an estate be worth \$1500, what is the value of $\frac{4}{5}$ of the estate?

Since $\frac{3}{7}$ of the estate is worth \$15000,
$\frac{1}{7}$ of the estate is worth $\$\frac{1500}{3}$;
$\therefore$ the estate is worth $\$\frac{7\times1500}{3}$ or \$3500.
Hence $\frac{4}{5}$ of the estate is worth $\$\frac{4\times3500}{5}$ or \$2800.

Examples lxx.

1. If $\frac{3}{5}$ of an estate be worth \$7520, what is the value of $\frac{5}{8}$ of the estate?

2. A person owns $\frac{3}{8}$ of a ship and sells $\frac{2}{3}$ of his share for \$1260. What is the value of the ship?

3. If $3\frac{2}{5}$ lb. of tea cost 15s. 3d., how much can I buy for £4 3s. $10\frac{1}{2}$d.?

4. If $\frac{2}{11}$ of a piece of work be done in 25 days, how much will be done in $11\frac{2}{3}$ da.?

5. A man walks 18 mi. 2 fur. 26 po. $3\frac{2}{3}$ yd. in $5\frac{1}{2}$ hr. How long does he take to walk a mile and a half?

6. A gentleman possessing $\frac{3}{14}$ of an estate sold $\frac{2}{7}$ of $\dfrac{1}{3\frac{1}{5}}$ of his share for \$603.12½. What would $\frac{1}{5}$ of the $\frac{3}{16}$ of the estate sell for at the same rate?

7. If the carriage of 15.5 cwt. of goods for 60 mi. cost \$3.10, how far ought 3.25 cwt. be carried for the same money?

8. What is the value of $\frac{1}{11}$ of $\frac{1}{12}$ of a vessel, if a person who owns $\frac{3}{11}$ of it sell $\frac{1}{9}$ of $\frac{7}{8}$ of his share for \$1400?

9. When the ounce of gold is worth £3.89, what is the cost of .04 lb.?

10. If the price of candles $8\frac{1}{2}$ in. long be 9d. per half-dozen, and that of candles of the same thickness and quality $10\frac{1}{4}$ in. long be 11d. per half-dozen, which kind do you advise a person to buy?

11. If the carriage of 60 cwt. for 20 mi. cost £$14\frac{1}{2}$, what weight can be carried the same distance for £$5\frac{7}{16}$?

Complex Problems.

145. We now proceed to cases in which the supposition, expressed in the simplest form, contains *more than two magnitudes*, the demand containing the same number of magnitudes, all of which are given, except one, which has to be found.

Ex. 1. If 12 horses can plough 96 acres in 6 days, how many horses will plough 64 acres in 8 days?

In 6 da. 96 ac. can be ploughed by 12 horses.
In 1 da. 96 ac. can be ploughed by 6×12 horses.
In 1 da. 1 ac. can be ploughed by $\frac{6\times 12}{96}$ horses.
In 8 da. 1 ac. can be ploughed by $\frac{6\times 12}{8\times 96}$ horses.
In 8 da. 64 ac. can be ploughed by $\frac{64\times 6\times 12}{8\times 96}$ horses.
∴ the number of horses required is 6.

Ex. 2. If 35 bushels of oats last 7 horses for 20 days, how many days will 96 bushels last 18 horses?

35 bu. last 7 horses for 20 da.
1 bu. lasts 7 horses for $\frac{20}{35}$ da.
1 bu. lasts 1 horse for $\frac{7\times 20}{35}$ da.
96 bu. last 1 horse for $\frac{96\times 7\times 20}{35}$ da.
96 bu. last 18 horses for $\frac{96\times 7\times 20}{18\times 35}$ da.
∴ the number of days is $21\frac{1}{3}$.

Examples lxxi.

1. If 40 ac. of grass are mowed by 8 men in 7 da., how many ac. will be mowed by 24 men in 28 da.?

2. If $60 will pay 8 men for 5 days' work, how much will pay 32 men for 24 days' work?

3. If a regiment of 939 soldiers consume 351 qr. of wheat in 168 da., how many soldiers will consume 1404 qr. in 56 da.?

4. If two horses eat 8 bu. of oats in 16 da., how many horses will eat 3000 qr. in 24 da.?

5. If a carrier receive $12 for the carriage of 3 cwt. for 150 mi., how much ought he to receive for the carriage of 7 cwt. 3 qr. 14 lb. for 50 mi.?

6. If I pay $1.50 for the carriage of 2 t. for 6 mi., what must I pay for the carriage of 12 t. 17 cwt. for 34 mi.?

7. If 3 men earn $15 in 4 da., what sum will 18 men earn in 16 da.?

8. How many bu. of wheat will serve 72 people 8 da., when 4 bu. serve 6 people 24 da.?

9. If a man travel 150 mi. in 5 da. when the days are 12 hr. long, in how many days of 10 hr. each will he travel 500 mi.?

10. If the carriage of goods weighing 5 cwt. 2 qr. 12 lb. for 150 mi. come to $15.70, what will be the charge for carrying four wagon-loads of the same, each weighing 7 cwt. 0 qr. 2 lb., the same distance, there being 112 lb. in the cwt.

11. If $120 pay 16 laborers for 6 da., now many laborers at the same rate will $270 pay for 8 da.?

12. If the gas for 5 burners, 5 hr. every day, for 10 da. cost $1.20, how many burners may be lighted 4 hr. every evening for 15 da. at a cost of $21.60.

13. If a traveling party of 3 spend $190 in 4 wk., how long will $475 last a traveling party of 5 at the same rate?

14. If it cost $120 to keep 2 horses for 5 mo., what will it cost to keep 3 horses for 11 mo.?

15. If it cost £29 7s. 6d. to keep 5 horses for 6 wk., how long may 3 horses be kept for £20 11s. 3d.?

16. If 5 men can reap a field of 12½ ac. in 3½ da., working 16 hr. a day, in what time can 7 men reap a field of 15 ac., working 12 hr. a day?

17. If 858 men in 6 mo. consume 234 qr. wheat, how many quarters will be required for the consumption of 979 men for 3½ mo.?

18. The wages of 5 men for 6 wk. being $315, how many weeks will 4 men work for $231?

19. If 7 men mow 22 ac. in 8 da., working 11 hr. a day, in how many days, working 10 hr. a day, will 12 men mow 360 ac.?

20. If 10 horses consume 7 bu. 2 pk. of oats in 7 da., in what time will 28 horses consume 3 qr. 6 bu. at the same rate?

21. If 44 cannon, firing 30 rounds an hour for 3 hr. a day,

consume 300 barrels of powder in 5 da., how long will 400 barrels last 66 cannon, firing 40 rounds an hour for 5 hr. a day?

22. If the wages of 29 men for 54 da. amount to £80 9s. 6d., how many men must work 12 da. to receive £407?

23. What must I pay for the hire of 4 horses for 5 mo., if I pay £18 for the hire of 3 horses for a month?

Problems Relating to Work Done in a Certain Time.

NOTE I.—If a man can do a piece of work in 7 hours, the part of the work which he can do in 1 hr. will be represented by $\frac{1}{7}$.

Ex. 1. *A* can do a piece of work in 5 days, and *B* can do it in 12 days. How long will *A* and *B*, working together, take to do the work?

Here $\frac{1}{5}$ represents the part *A* does daily,
and $\frac{1}{12}$ represents the part *B* does daily;
$\therefore \frac{1}{5}+\frac{1}{12}$ represents the part *A* and *B* do daily;
$\therefore$ they do $\frac{17}{60}$ in 1 daily;
$\therefore$ they do $\frac{1}{60}$ in $\frac{1}{17}$ da.;
$\therefore$ they do the whole work in $\frac{60}{17}$ da., or $3\frac{9}{17}$ da.

Ex. 2. *A* can do a piece of work in 50 days, *B* in 60 days, and *C* in 75 days. In what time will they do it, all working together?

Here $\frac{1}{50}+\frac{1}{60}+\frac{1}{75}$ represents the part they do daily:
$\therefore$ they do $\frac{6\times5\times4}{300}$, or $\frac{15}{300}$, or $\frac{1}{20}$ daily;
$\therefore$ they do the work in 20 da.

Ex. 3. *A* can reap a field in $4\frac{1}{5}$ days, and *B* can reap it in $5\frac{2}{3}$ days. How long will they take to reap it, working together?

$$A \text{ does } \frac{1}{4\frac{1}{5}}, \text{ or } \frac{5}{21} \text{ daily.}$$

$$B \text{ does } \frac{1}{5\frac{2}{3}}, \text{ or } \frac{3}{17} \text{ daily,}$$

$\therefore$ together they do $\frac{5}{21}+\frac{3}{17}$, or $\frac{148}{357}$ daily;
$\therefore$ they do $\frac{1}{357}$ of the work in $\frac{1}{148}$ da.;
$\therefore$ they do the work in $\frac{357}{148}$ da., or $2\frac{61}{148}$ da.

Ex. 4. *A* and *B* do a piece of work in 4 hours; *A* and in $3\frac{3}{5}$ hours; *B* and *C* in $5\frac{1}{7}$ hours. In what time can *A* do it alone?

A and *B* can do $\frac{1}{4}$ in an hour;
A and *C* can do $\frac{5}{18}$ in an hour;
∴ two men of *A*'s strength, assisted by *B* and *C*, can do $\frac{1}{4}+\frac{5}{18}$ in an hour;
Now *B* and *C* can do $\frac{7}{36}$ in an hour;
∴ two men of *A*'s strength can do $\frac{1}{4}+\frac{5}{18}-\frac{7}{36}$ in an hour,
or $\frac{19}{36}-\frac{7}{36}$, or $\frac{12}{36}$, or $\frac{1}{3}$ in an hour;
∴ *A* can do $\frac{1}{6}$ in an hour;
∴ *A* can do the work in 6 hr.

NOTE II.—If a tap can fill a vessel in 5 hours, the part filled by it in 1 hour will be represented by $\frac{1}{5}$.

Ex. 1. A vessel can be filled by three taps, running separately, in 20, 30, and 40 minutes, respectively. In what time will they fill it when they all run at the same time?

They fill $\frac{1}{20}+\frac{1}{30}+\frac{1}{40}$ of the vessel in 1 min.;
∴ they fill $\frac{6\times4\times3}{120}$, or $\frac{13}{120}$ in 1 min.;
∴ they fill the vessel in $\frac{120}{13}$ min., or $9\frac{3}{13}$ min.

Ex. 2. A bath is filled by a pipe in 40 minutes. It is emptied by a waste pipe in an hour. In what time will the bath be full if both pipes be opened at once?

One pipe fills $\frac{1}{40}$ of the bath in a minute.
The other empties $\frac{1}{60}$ of the bath in a minute.
∴ when both are running, $\frac{1}{40}-\frac{1}{60}$ or $\frac{1}{120}$ of the bath is filled in a minute;
∴ the bath is filled in 120 min.

Examples lxxii.

1. *A* can do a piece of work in 6 hr.; *B* can do it in 9 hr. In what time will they do it if they work together?

2. *A* can do a piece of work in 35 da.; *B* can do it in 40 da.; *C* can do it in 45 da. In what time will they do it, all working together?

3. *A* and *B* can reap a field of wheat in 3 da.; *A* and *C* in $3\frac{1}{2}$ da.; *B* and *C* in 4 da. In what time could they reap it, all working together?

4. If three pipes fill a vessel in 6, 8, and 12 min. respectively, in what time will the vessel be filled when all three are open at once?

5. *A* does $\frac{7}{10}$ of a piece of work in 14 da. He then calls in *B*, and they finish the work in 2 da. How long would *B* take to do the whole work by himself?

6. *A* does a piece of work in 3 hr., which is twice the time *B* and *C* together take to do it; *A* and *C* could together do it in $1\frac{1}{3}$ hr. How long would *B* alone take to do it!

7. *A* can do a piece of work in 27 da., and *B* in 15 da.; *A* works at it alone for 12 da., *B* then works alone 5 da., and then *C* finishes the work in 4 da. In what time could *C* have done the work by himself?

8. A cistern is filled by two pipes in 18 and 20 min., respectively, and emptied by a tap in 40 min. What part of it will be filled in 10 min. when all are opened at the same instant?

9. *A* can do a work in 3 da., *B* can do it in 4 da. and *C* in 5. They all begin to work together, but *A* stops 1 da. and *B* $\frac{1}{2}$ da. before the work is finished. *C* finishes it. How long did each one work?

10. If 5 men, 4 boys, and 3 girls can clear a field of stones in 9 da., and 8 boys, 6 girls, and 3 men can do it in 8 da., and 10 boys, 9 girls, and 4 men can do it in 6 da., how long will it take 3 men, 5 boys and 4 girls to do it?

Problems Relating to Clocks.

146. The minute-hand moves 12 times as fast as the hour-hand, and therefore in 12 minutes the minute-hand gains 11 minute-divisions on the hour-hand.

Ex. 1. Find the time between 3 and 4 o'clock when the hands of a watch are together.

At 3 o'clock there are 15 minute-divisions between the hands; we have therefore to find how long it will take the minute-hand to gain 15 minute-divisions on the hour-hand.

The minute-hand gains 11 minute-divisions in 12 min.;

1 minute-division in $\frac{12}{11}$ min.;

15 minute-divisions in $\frac{15 \times 12}{11}$ min.;

$\therefore$ the time required is $\frac{15 \times 12}{11}$ min., or $16\frac{4}{11}$ min. past 3.

Ex. 2. At what time between 2 and 3 are the hands of a clock at right angles to each other?

When the hands are at right angles there is a space of 15 minute-divisions between them.

Hence, since at 2 o'clock there are 10 minute-divisions between the hands, we have to find how long it will take the minute-hand to gain $10+15$, or 25 minute-divisions on the hour-hand.

The minute-hand gains 11 minute-divisions in 12 min.;
1 minute-division in $\frac{12}{11}$ min.;
25 minute-divisions in $\frac{25\times 12}{11}$ min.;

$\therefore$ the time required is $\frac{25\times 12}{11}$ min., or $27\frac{3}{11}$ min. past 2.

Ex. 3. At what *times* between 6 and 7 are the hands of a clock at right angles to each other?

Twice between 6 and 7 this will occur; first, before the minute-hand has overtaken the hour-hand; secondly, after the minute-hand has passed the hour-hand.

Now, since at 6 o'clock there are 30 minute-divisions between the hands, we have to find:

First, how long it will take the minute-hand to gain $30-15$, or 15 minute-divisions on the hour-hand.

Secondly, how long it will take the minute-hand to gain $30+15$, or 45 minute-divisions on the hour-hand.

The process in each case will be similar to that in the preceding examples, and the results are $16\frac{4}{11}$ min. and $49\frac{1}{11}$ min. past 6.

Ex. 4. Find the time between 7 and 8 o'clock when the hands of a watch are opposite to each other.

When the hands are opposite there is a space of 30 min. between them, and at 7 o'clock there is a space of 35 min. between the hands.

Hence in this case we have to find how long it will take the minute-hand to gain a space of $35-30$, or 5 min. on the hour-hand.

The process will be similar to that in the preceding examples, and the result is $5\frac{5}{11}$ min. past 7.

Examples lxxiii.

1. At what time are the hands of a watch together between the hours of

(*a*) 4 and 5? | (*b*) 6 and 7? | (*c*) 9 and 10?

2. At what hour are the hands of a watch at right angles to each other between

(*a*) 4 and 5? | (*b*) 7 and 8? | (*c*) 11 and 12?

3. At what time are the hands of a watch opposite to each other between

(*a*) 1 and 2? | (*b*) 4 and 5? | (*c*) 8 and 9?

4. At what times are the hands of a watch 3 minute-spaces apart between

(*a*) 3 and 4? | (*b*) 7 and 8? | (*c*) 10 and 11?

5. At what times are the hands of a watch equally distant from the figures

(*a*) 2? | (*b*) 7? | (*c*) 9?

6. At what times between 10 and 11 do the hands of a clock form an angle of

(*a*) 48°? | (*b*) 50°? | (*c*) 72°?
(*d*) 130°? | (*e*) 150°? | (*f*) 159°?

7. It is between 5 and 6 and the hands of a clock form with a line joining their extremities an isosceles triangle each of whose angles at the base is one-third of the third angle. Find the time?

8. It is between 6 and 7 and the hands of a clock form with a line joining their extremities an isosceles triangle each of whose angles at the base is double the third angle. Find the time?

9. It is between 2 and 3 o'clock and the minute hand is $6\frac{1}{2}$ minute spaces in advance of the hour hand. Find the time?

Problems Relating to the Sum and Difference of Two Rates.

147. The rate of a man rowing *down* stream is the sum of his rate in still water and the rate of the stream. His rate *up* the stream is his rate in still water diminished by the rate of the stream.

Ex. 1. *A* can row 10 miles down stream in 1 hour and 6 miles, up stream in the same time. Find his rate of rowing in still water.

A's rate in still water + rate of stream = 10 mi. per hour.
A's rate in still water − rate of stream = 6 mi. per hour.
∴ twice *A*'s rate in still water = 16 mi. per hour.
A's rate in still water = 8 mi. per hour.

Ex. 2. *A* can row a distance down stream in 20 minutes, and the same distance up in 30 minutes. What is the distance, the rate of the stream being $\frac{1}{2}$ mile per hour?

$$\begin{aligned}\text{Rate in still water} + \text{rate of stream} &= 3 \text{ distances.}\\ \text{Rate in still water} - \text{rate of stream} &= 2 \text{ distances.}\\ \therefore \text{twice rate of stream} &= 1 \text{ distance.}\\ \therefore \text{distance} &= 1 \text{ mi.}\end{aligned}$$

Ex. 3. *A* can row 6 miles down stream and back again in 2 hours 40 minutes, and his rate of rowing in still water is twice the rate of the stream. Find his rate of rowing in still water per hour.

Since his rate down is three times that of the stream and his rate up is once the rate of the stream.

$\therefore$ he is three times as long in rowing up the distance as in rowing down it.

$\therefore$ time in rowing down $= \frac{1}{4}$ of 2 hr. 40 min. $=$ 40 min.;
and " " up $= \frac{3}{4}$ of 2 hr. 40 min. $=$ 2 hr.

In 40 min. he rows 6 mi. down stream, ∴ rate down per hour is 9 mi.
In 2 hr. he rows 6 mi. up stream, ∴ rate up per hour is 3 mi.
Hence, as in Ex. 1, his rate is $4\frac{1}{2}$ mi. per hour.

Examples lxxiv.

1. A steamboat can sail 60 mi. down stream in 5 hr. and takes 6 hr. to return. Find the rate of the stream per hour.

2. A steamer sails 15 mi. per hour in still water. How far may it go up a stream which flows at the rate of 3 mi. per hour so that 5 hr. may be spent on the round trip?

3. *A* rows 30 mi. and back in 12 hr. He finds he can row 5 mi. with the stream in the same time as 3 mi. against it. Find the rate of the stream per hour.

4. A crew can row 10 mi. down stream in an hour. Without the aid of the stream it would have taken $1\frac{1}{4}$ hr. How long will it take to return?

5. *A* floats down stream a certain distance in 10 min. and rows back in 5 min. If his rate of rowing in still water is 6 mi. per hour, find the distance.

6. Two trains, 92 ft. and 84 ft. long, respectively, move with uniform velocities on parallel rails in opposite directions, and pass each other in $1\frac{1}{2}$ sec. When moving in the same direction, with the same velocities as before, the faster train passes the other in 6 sec. Find the rate at which each moves.

7. A train 88 yd. long overtook *A* walking along the track at the rate of 4 mi. per hour, and passed him completely in 10 sec. It afterwards overtook *B* and passed him in 9 sec. At what rate per hour was *B* walking?

8. A train 352 ft. long overtakes a man going in the same direction at 4 mi. per hour and passes him in 15 sec. Shortly after it passes a man walking in the opposite direction in 9 sec. At what rate per hour is the second man walking?

EXAMINATION PAPERS.

I.

1. A man's income is reduced from $2720 to $2640.66 when he has paid his income tax. What is his tax on the dollar?

2. If 10 horses and 132 sheep can be kept 8 days for $202, what sum will keep 15 horses and 148 sheep for the same time, supposing 5 horses to eat as much as 84 sheep?

3. A man receives 75c. in the dollar of what was due to him and thereby loses $602.10. What was due to him?

4. If 15 men can perform a piece of work in 22 days, how many men will finish another piece of work 4 times as large in a fifth part of the time?

5. If 72 men dig a trench in 63 da., in how many days will 42 men dig another trench three times as great?

II.

1. The wages of *A* and *B* together for $7\frac{1}{2}$ da. amount to the same sum as the wages of *A* alone for $12\frac{6}{7}$ days. For how many days will the sum pay the wages of *B* alone?

2. If 100 men can perform a piece of work in 30 da., how many men can perform another piece of work thrice as large in one-fourth of the time?

3. If 5 men or 7 women can do a piece of work in 37 da., how long will a piece of work twice as great occupy 7 men and 5 women?

4. Two persons, *A* and *B*, finish a work in 20 da., which *B* by himself could do in 50 da. In what time could *A* finish it by himself? How much more of the work is done by *A* than *B*?

5. A cistern when full of water can be emptied in 15 min. by a pipe, and when empty can be filled by another in 20 min. If the cistern be full, in what time can it be emptied by both pipes being opened at the same time?

III.

1. *A* and *B* can do a piece of work alone in 15 and 18 da. respectively; they work together at it for 3 da., when *B* leaves, but *A* continues, and after three days is joined by *C*, and they finish it together in 4 da. In what time would *C* do the work by himself?

2. If a man can do treble, and a woman double, the work of a boy in the same time, how long would 9 men, 15 women, and 18 boys take to do double the work which 7 men, 12 women, and 9 boys complete in 250 da.?

3. *A* and *B* walk to meet each other from two places 100 mi. distant. *A* walks 6 mi. an hour and *B* 4 mi. an hour. At what point on the road do they meet, and at what two times are they fifty miles apart from each other?

4. A watch which is 10 min. too fast at noon on Monday loses 3 min. 10 sec. daily. What will be the time indicated by the watch at a quarter past ten on the morning of the following Saturday?

5. A watch set accurately at 12 o'clock indicates 10 min. to 5 at 5 o'clock. What is the exact time when the watch indicates 5 o'clock? If it indicated 10 min. past 5 at 5 o'clock, what would be the exact time when the hands indicated 5 o'clock?

IV.

1. A laborer agreed to work for 60 da. on this condition: that every day he worked he should receive $2, and for every day he was idle, he should pay $1.50 for his board. At the expiration of the time he received $92. How many days did he work?

2. A piece of work can be done in a day of $11\frac{1}{2}$ hr. by 2 men, or 5 women, or 12 boys; in what time could it be done by 1 man, 2 women, and 3 boys together?

3. A cistern has two supplying pipes, *A* and *B*, and a tap, *C*. When the cistern is empty, *A* and *B* are turned on, and it is filled in 4 hr.; then *B* is shut and *C* turned on, and the cistern is quite emptied in 40 hours; when, lastly, *A* is shut and *B* turned on, and in 60 hr. afterwards the cistern is again filled. In what time could the cistern be filled by each of the pipes, *A* and *B*, singly?

4. A clock is set at 12 o'clock on Saturday night, and at noon on Tuesday it is 3 min. too fast. Supposing its rate regular, what will be the true time when the clock strikes four on Thursday afternoon?

For additional examples see page 304.

CHAPTER XI.

AGGREGATES AND AVERAGES.

148. The **Aggregate** of a number of quantities of the same kind is their sum.

149. The **Average** of two or more groups of numbers is found by dividing their sum by the number of groups.

Thus, to find the average of 13, 15, 74, 23, 6, and 31, we find the sum of the numbers to be 162, and as the number of groups is 6, the average will be $162 \div 6$, or 27.

NOTE.—Express any remainder, which may occur, *decimally.*

Examples lxxv.

1. Find the aggregate and the average of the following :—

   (*a*) 14, 26, 9, 18, 13, 24, 27, 39.
   (*b*) 1600, 276, 974, 0, 236, 845, 1239.
   (*c*) 34729, 46238, 87296.
   (*d*) $15\frac{1}{2}$, $36\frac{3}{4}$, $17\frac{5}{8}$, 0, $10\frac{3}{8}$, $74\frac{1}{5}$, $28\frac{1}{4}$, 33.
   (*e*) $12\frac{22}{25}$, 21, $7\frac{3}{4}$, .034, $3\frac{1}{8}$, 0, $24\frac{1}{2}$, $12\frac{7}{20}$.

2. Five men made the following scores at a rifle match: 97, 94, 91, 91, and 87. Find the average per man.

3. The average weight of the 8 oarsmen in a boat is increased $1\frac{1}{2}$ lb. when one of the crew who weighs 162 lb. is replaced by a fresh man. What is the weight of the new man?

4. In a class of 30 boys, 7 are 9 yr. old, 15 are 10 yr. old, 2 are 11 yr. old, 1 is 12 yr. old, and the rest are 13 yr. old. What is the average age of the class?

5. In a factory a certain number of men receive $13 per week, 4 times as many receive $9 per week, and 10 times as many receive $5 per week. What is the average wage per man per week?

6. A grocer mixed together 7 lb. of tea worth 25c. per lb., 11 lb. @ 35c., 19 lb. @ 40c., and 3 lb. @ 50c. What was the average price of the mixture?

For additional examples see page 310.

CHAPTER XII.

PERCENTAGE.

150. Percentage is the process of computation in which the basis of comparison is a hundred.

151. The phrase Per Cent. means *for* or *by the hundred.*

The symbol % is used for the phrase per cent.

When we speak of an agent getting 3 per cent. as a commission on the management of an estate, we mean that from every \$100 collected he deducts \$3 to remunerate himself for the trouble of collection.

When we read that the population of a town has increased 15 *per cent.* since the last census, we mean that if the number of inhabitants *then* had been divided into groups of 100, and the number of inhabitants *now* into groups of 115, the number of groups would be the same in both cases.

Ex. 1. How much is 3 per cent. on \$1479?

Since \$100 yields \$3,
\$1 yields $\$\frac{3}{100}$,
\$1479 yields $\$\frac{1479\times 3}{100}$, or \$44.37.

Examples lxxvi.

1. Find the value of

(*a*) 5 % of \$2400.
(*b*) 8 % of 3475 horses.
(*c*) $22\frac{1}{2}$% of \$900.
(*d*) $13\frac{1}{3}$ % of 30 ac.
(*e*) $37\frac{1}{2}$ % of 576 books.
(*f*) $11\frac{1}{9}$ % of 270 yd.

2. A farmer having 150 sheep sold 18 % of them. How many had he left?

3. A man with a salary of \$1,500 per year pays $12\frac{1}{2}$ % of this for rent. What is his yearly rent?

4. The population of a certain city is 28773. What will it be in one year from this time if it increase $11\frac{1}{9}$ per cent.?

5. The number of boys in a school is 85 % of the number of girls. The number of girls is 80. How many pupils are there in the school?

6. If milk yields 18 % of cream, and cream yields 24 % of butter, and a quart of milk weighs $2\frac{1}{2}$ lb., how many quarts of milk will yield 486 lb. of butter?

7. A farmer raised 850 bu. of wheat. He sold 18 % of it @ 84c. per bu., 48 % @ 80c. per bu., and the remainder @ 85c. a bu. How much did he receive for the wheat?

Ex. 2. The number of boys in a school increases in a certain period from 125 to 180, what is the increase per cent.?

On 125 the increase is 55.
On 1 the increase is $\frac{55}{125}$.
On 100 the increase is $\frac{100\times 55}{125}$, or $\frac{220}{5}$, or 44;
∴ the increase is 44 per cent.

Examples lxxvii.

1. Find how much per cent. is

(*a*) 25 parts out of 75.
(*b*) 178 parts out of 890.
(*c*) 24 oz. of 1 cwt.
(*d*) $\frac{1}{5}$ part out of $\frac{1}{2}$.
(*e*) 5c. of $2.
(*f*) $3.75 of $$33\frac{1}{3}$.

2. A drover sold 240 sheep and had 960 left. What per cent. of his sheep did he sell?

3. The population of London proper decreased from 113,387 in 1861 to 75,844 in 1871. Find the decrease per cent. during this period.

4. How much per cent. is 9d. in the pound; $12\frac{1}{2}$c. in the dollar; $3 in every $20?

5. A city of 20,000 inhabitants lost 550 of its population by deaths and gained 900 by births in a year. Find the increase per cent. in the population during this year.

6. A brick kiln contained 34785 bricks, and, after burning, it was found that only 30920 were in good condition. What per cent. had been spoiled in burning?

7. If 24 bu. of wheat are raised from $1\frac{1}{2}$ bu. of seed, what per cent is the increase?

8. The population of a town in a certain year was 5675. Five years afterward it was 9307. What was the per cent. of increase during the interval?

Ex. 3. *A* has \$56 left after spending 80 per cent. of his salary for the month. What was his salary for the month?

$\$100 - \$80 = 20$

\$20 is left out of \$100 received.
\$1 " " " " $\$\frac{100}{20}$ "
\$56 " " " " $56 \times \$\frac{100}{20}$, or \$280 received;
∴ *A* received \$280 per month.

Examples lxxviii.

1. Find the number of which 21 is 7 %; 750 is $3\frac{1}{5}$ %; 215 is .005 %.

2. What number increased by 40 % of itself becomes 2625?

3. Smith drew $62\frac{1}{2}$ % of his money from the bank and paid $33\frac{1}{3}$ % of it for a house worth \$4500. How much money had he remaining in the bank?

4. 3 % of a certain number, together with 5 % of half the number is 55. Find the number.

5. 17 % of a certain number, together with 7 % of three times the number, make 76. Find the number.

6. What number diminished by $13\frac{1}{3}$ % of itself becomes 286?

7. A farmer's crop of oats this year is $7\frac{1}{2}$ % greater than last year. What was last year's crop if in the two years he raised 747 bu.?

8. The population of a town in 1895 was 15624. This is 56 % of its present population. What is its present population?

9. 40 % of a mixture of wine and water is wine, but when 4 gal. of water are added the wine is only $37\frac{1}{2}$ % of the whole. How many gallons are there in the mixture at first?

10. A drover bought 320 sheep at a certain price per head. He sold $\frac{3}{8}$ of them at a gain of 20 %; $\frac{3}{10}$ of them at a gain of $12\frac{1}{2}$ %, and the remainder at a loss of 25 %, gaining on the whole \$112. How much did he pay for the 320 sheep?

For additional examples, see page 311.

CHAPTER XIII.

APPLICATIONS OF PERCENTAGE.

Trade Discount.

152. When a bill, debt, or note is paid a sum is frequently deducted and the remainder is accepted as payment in full. The sum deducted is called **Discount.**

153. Trade or Commercial Discount is the sum deducted from the catalogue or list price of goods.

Sometimes several discounts are allowed. In this case, the first discount is to be deducted, and then the second one is to be reckoned upon the remainder and then deducted, and thus on for each discount.

Ex. 1. If knives of a certain quality are sold at $18 per dozen, with discounts of $33\frac{1}{3}$ per cent. and 5 per cent., what is the net price?

The discount on $18 at $33\frac{1}{3}\,\% = \$6$.
$\$18 - \$6 = \$12$.
The discount on $12 at $5\,\% = \$0.60$.
$\$12 - \$0.60 = \$11.40$.
$\therefore$ net price $= \$11.40$.

Examples lxxix.

1. Find the net price of goods listed as follows:—
   (*a*) $245 subject to 20 % off.
   (*b*) $360 subject to 25 % and 5 % off.
   (*c*) $3280 subject to 25 %, 10 %, and 5 % off.

2. After a discount of 25 % had been deducted, the purchaser paid $186. Find the list price of the goods.

3. After a discount of $84 had been taken off a bill of goods the trader paid $166, paying the bill in full. What was the rate of discount allowed?

4. An article cost $24. How must it be marked so that after making a deduction of 10 % the merchant may make a gain of 25 %?

5. What single discount is equivalent to discounts of 25 % and 10 % off?

6. At what price must goods which cost 76c. be listed to give $12\frac{1}{2}$ % gain after deducting discounts of 20, 10, and 5 %?

7. A baker gives thirteen loaves for a dozen. What rate per cent. discount is equivalent to this?

8. A merchant marked his goods $33\frac{1}{3}$ % above cost and allowed a discount of 25 %. Find his gain or loss per cent.

9. A bill of hardware at list prices amounts to $240. The discounts are 40, $12\frac{1}{2}$, and 10 %. What is due on this bill?

10. At what price must a suit of clothes which cost $10 be marked so that after a discount of 5 % is allowed the merchant may gain $5.20?

11. Goods were marked 50 % above cost and discounts of 25, 20, and 5 % allowed. Find the gain or loss per cent.

12. A bookseller gives a discount of 5 % for cash and allows teachers a second discount of 10 % on all cash prices. A teacher bought some books and paid $6.84. What was the marked price of the books?

Profit and Loss.

154. If I sell for $105 that for which I gave $100, I gain $5 on an outlay of $100.

If I sell for $95 that for which I gave $100, I lose $5 on an outlay of $100.

The following Examples will show the method of solving questions relating to Profit and Loss, the principles laid down in Chapter X. being followed.

Ex. 1. I sell for $6 that for which I gave $5. What is my gain per cent.?

On an outlay of $5 my gain is $1;
On an outlay of 1 my gain is $$\frac{1}{5}$;
On an outlay of 100 my gain is $$\frac{100}{5}$, or $20;
∴ I gain 20 %.

Ex. 2. I bought some goods for $17. How must I sell them to gain $17\frac{11}{17}$ per cent.?

That for which I gave $100 I must sell for $$117\frac{11}{17}$;
That for which I gave $1 I must sell for $$\frac{2000}{100\times17}$;
That for which I gave $17 I must sell for $$\frac{17\times2000}{100\times17}$, or $20.

Ex. 3. By selling goods for \$7.20 I made a profit of 20 per cent. What did I give for them?

That which I sold for \$120 I bought for \$100;
That which I sold for \$1 I bought for $\$\frac{100}{120}$;
That which I sold for \$7.20 I bought for $\$\frac{7.20\times100}{120}$, or \$6.

Ex. 4. If by selling coffee at 1s. 7d. per pound I lose 5 per cent., what must I sell it at to gain 5 per cent.?

That which I sell at 95d. I bought for 100d.;
that which I sell at 1d. I bought for $\frac{100}{95}$d.;
that which I sell at 19d. I bought for $\frac{19\times100}{95}$d., or 20d.

Having thus found the *cost* price, we proceed thus,

To gain 5 %,
that for which I gave 100d. I must sell for 105d.;
that for which I gave 1d. I must sell for $\frac{105}{100}$d.;
that for which I gave 20d. I must sell for $\frac{20\times105}{100}$d., or 1s. 9d.

Or thus,

In the first case,
that which costs 100d. sells for 95d.
In the second case,
that which costs 100d. sells for 105d.;
∴ that which sells for 95d. must bring 105d.;
" " 1d. must bring $\frac{105}{95}$d.;
" " 19d. must bring $\frac{19\times105}{95}$d.,
or 1s. 9d., as before.

Ex. 5. A quantity of tea is sold for $83\frac{1}{3}$ cents per pound; the gain is 10 per cent., and the total gain is \$48. What is the quantity of the tea?

That which sells for \$110 costs \$100;
" " \$1 " $\$\frac{100}{110}$;
" " $83\frac{1}{3}$c. " $\$\frac{0.83\frac{1}{3}\times100}{110}$;
∴ the cost price per lb. $=\frac{10}{11}$ of $\$0.83\frac{1}{3}$;
∴ the gain on 1 lb. $=\frac{1}{11}$ of $\$0.83\frac{1}{3}$.
Quantity on which the gain is $\frac{1}{11}$ of $\$0.83\frac{1}{3}=1$ lb.;
∴ " " " " " " \$48 $=\frac{48\times1}{\frac{1}{11}\text{ of }.83\frac{1}{3}}$ lb;
$=633\frac{3}{5}$ lb.

∴ $633\frac{3}{5}$ lb. is the quantity sold.

155. When tea, spirits, wine, and such commodities are *mixed* it must be observed that

quantity of ingredients = quantity of mixture,
cost of ingredients = cost of mixture.

Thus, if a mixture is made of 1 gallon of ale at 8 cents a gallon, 3 at 15 cents, 4 at 20 cents, and 12 at 7 cents.

quantity of ingredients = (1 + 3 + 4 + 12) gal., or 20 gal.
cost of ingredients = (8 + 45 + 80 + 84)c., or $2.17.

If I want to know what gain per cent. I shall make by selling this mixture at 26 cents a gallon, I reason thus,

20 gal. @ 26c. will sell for $5.20;
∴ that for which I gave $2.17, I sell for $5.20;
∴ $2.17 gains ($5.20 − $2.17), or $3.03;
∴ $1 gains $\$\frac{3.03}{2.17}$;
∴ $100 gains $\$\frac{100\times 3.03}{2.17}$, or $139.63.
∴ I gain 139.63 %.

156. In solving questions on Profit and Loss the student must be very careful to notice whether the gain is calculated on the selling price or cost price. Thus, it is sometimes said that a retailer's profit is 25 per cent., meaning that he gave 75 cents for an article which he sells for $1. His profit in this case is 33⅓ per cent. on his outlay. Care must be taken to express distinctly what is meant. The profit on a single transaction, or set of transactions, by no means represents a net profit, as it is not charged with a variety of expenses which belong to the business in general rather than to the set of transactions in question.

Ex. 6. If 100 articles of a given kind can be made in a week out of $40 worth of raw materials, cost of labor, etc., being $10, fixed charges for rent, etc., being $250 a year, find (*a*) the cost price of each article; (*b*) the invoice price in order that a profit of 30 per cent. on the cost price may be realized, the following allowances being necessary, viz.: 10 per cent. commission to agents on money received for sales, and 12 per cent. for bad debts; and (*c*) the amount of profit in a year.

(*a*) The fixed charges must be referred to the same unit of time as the rest of the estimate, viz.: 1 wk. ∴ these = $\$\frac{250}{52}=\$\frac{125}{26}$.

Cost of 100 articles $=\$50+\$\frac{125}{26}=\$54.8077$;

∴ cost of 1 article $=0.548077$.

(*b*) The profit on capital may be regarded as part of the cost of production. It would be so, in fact, if the money were borrowed at 30 % interest. 30 % added to \$0.548077 gives $\$\frac{130\times.548077}{100}$.

Again, the commission is paid on the money actually received; to provide for it we must take the $\frac{10}{9}$ of $\$\frac{130\times.548077}{100}$,

or $\$\frac{10\times130\cdot548077}{9\times100}$.

Next: 12 % on bad debts means that 12 do not pay for 88 who do. To provide for it we must take $\frac{100}{88}$ of the selling price. The invoice price will therefore be

$$\$\frac{100\times10\times130\times.548077}{88\times9\times100}, \text{ or } \$.899.$$

(*c*) To find the profit we must take $\frac{30}{100}$ of the cost price and multiply by 100×52.

Annual profit $=\$\frac{30\times100\times52\times.548077}{100}=\$855$.

Examples lxxx.

1. If I buy an article for \$3.20 and sell it for \$4, what is my gain per cent.?

2. A retail dealer sold 50 pair of boots for \$200. They cost him \$2.75 a pair. What is his gain per cent.?

3. Bought a house for \$4500, expended \$1500 in repairs, \$1500 for furniture, and \$125 for taxes. I sold the whole for \$8000. Find my gain per cent.

4. The manufacturer will supply a certain article @ 1½d. If a tradesman charges 2d., what profit per cent. will he make?

5. A newsboy buys papers @ 8c. per dozen and sells them @ 1c. each. Find his gain per cent.

6. What per cent. is made by buying coal by the long ton and selling it at the same price by the short ton?

7. A man buys goods @ £23 5s. 5d. and sells them @ £22 2s. 1¾d. How much does he lose per cent.?

8. A man buys goods @ £15 6s. 3d. and sells them again @ £11 15s. 9¾d. How much does he lose per cent.?

9. A man buys goods at the rate of $96 per cwt. and sells 2 t. 14 cwt. 3 qr. 12 lb. for $6000. How much has he gained or lost per cent. on his outlay?

10. A merchant bought tea @ 36c. a pound and sold it to gain 11⅑ %. Find the selling price.

11. If 125 overcoats cost $1000, for what must they be sold apiece to gain 12½ %?

12. How many pounds of sugar costing 3⅓c. a pound must be sold for a dollar to gain 25 %?

13. If I buy broadcloth @ $2 a yard and silk @ $2.80 a yard and sell the cloth @ $2.50 a yard, at what price must I sell the silk to make the same gain per cent. on one as on the other?

14. *A* bought a house for $3500 and sold it at a loss of 20 %. The buyer sold it at a gain of 25 %. What did the latter receive for it?

15. If I sell goods for $2240 and gain 12 %, what was the cost price?

16. If 375 yd. of silk be sold for $1960, and 20 % profit be made, what did it cost per yard?

17. If, by selling wine @ 17s. 5d. a gallon, I lose 5 %, at what price must I sell it to gain 15 %?

18. If, by selling goods for $544, I lose 16 %, how much per cent. should I have lost or gained if I had sold them for $672?

19. I sold a lot for $425, thereby losing 15 %. For what ought I to have sold it to gain 20 %?

20. If 15 % is gained by selling 40 sheep for $552, at what price apiece should they have been sold in order to make 25 % gain?

21. A tradesman's prices are 20 % above cost price. If he allows a customer 10 % on his bill, what profit does he make?

22. A tradesman's prices are 25 % above cost price. If he allows a customer 12 % on his bill, what profit does he make?

23. If 8 % be gained by selling a piece of ground for $4125.60, what would be gained per cent. by selling it for $4202?

24. If 3 % more be gained by selling a horse for $333 than by selling him for $324, what must his original price have been?

25. A grocer mixes 12 lb. of tea @ 2s. 6½d. per pound with 4 lb. @ 3s. 2¼d. At what price must he sell the mixture so as to gain 33⅓ % upon his outlay?

26. How many pounds of tobacco @ $1.05 per pound must a tobacconist mix with 4 lb. @ $1.30, that he may sell the mixture @ $1.56⅔ per pound, and gain 33⅓ % upon his outlay?

27. A spirit merchant buys 80 gal. of whisky @ $3.60 per gallon, and 180 gal. more @ $3.00 per gallon, and mixes them. At what price must he sell the mixture to gain 8⅓ % upon his outlay?

28. I mix 80 gal. of gin @ $3.10 per gallon with 96 gal. @ $3.41⅔, and sell the mixture so as to gain 10 %. At what price per gallon do I sell it?

29. A grocer buys two sorts of tea @ 55c. and 61⅔c. per pound, respectively. He mixes them so as to have 3 lb. of the dearer for every 1 lb. of the cheaper sort, and sells the mixture @ 80c. per pound. What does he gain per cent.?

30. A grocer buys 150 bbl. of apples each containing 2 bu. 1 pk. @ $1.50 a barrel. He paid $14.40 for carriage. If the loss by decay amounts to 20 %, what is the least price per bushel at which he can sell them to clear 12½ % on his outlay?

31. What must I ask for a watch costing me $30 in order to take off 10 % for cash and yet make a profit of 20 %?

32. *A* sells a lot to *B* at a gain of 25 %; *B* sells it to *C* for $322 at a gain of 15 %. What did the lot cost *A*?

33. *A* sold a lot to *B* at a gain of 20 %; *B* sold it to *C* at a gain of 25 %. *C* paid $180 more for it than *A*. What did the lot cost *A*?

34. A merchant marked goods at an advance of 40 %, but in selling them he used a false balance, by means of which he gave 14 oz. to the pound. His total gain being $240, find the cost of the goods?

35. I bought goods and sold ¼ of them at a loss of 20 %. By what increase per cent. must I raise this selling price so as to gain 20 % on the entire transaction?

36. I sold two lots at the same selling price. On one I gained 33⅓ %. On the other I lost 33⅓ %. My total loss was $60. Find the cost of each lot.

37. I bought an article and sold it to gain 20 %. If the cost had been 10 % less and the selling price $24 less, I would have gained 30 %. Find the cost price.

38. *A* sold oranges so that ⅔ of his gain on 12 is the selling price of 4. Find his gain per cent.

Commission.

157. Commission is the charge made by an agent for buying or selling goods, and is generally a percentage on the *money engaged in the transaction.*

In computing Commissions, care must be taken to calculate it on the money actually employed in the business.

Ex. 1. My agent has purchased wheat, on my account, to the amount of \$18768. What is his commission at $1\frac{3}{4}$ %?

$$\begin{aligned}
\text{The Commission on } \$100 &= \$1.75\,;\\
\therefore \qquad \text{“} \qquad \$1 &= \$\frac{1.75}{100}\,;\\
\therefore \qquad \text{“} \qquad \$18768 &= \$\frac{18768\times1.75}{100}\\
\text{Commission required} &= \$328.44.
\end{aligned}$$

Hence the following rule may be used:—

Multiply the given sum by the rate per cent. and divide the product by 100, *and the result is the Commission.*

Ex. 2. I send my agent \$1827 with instructions to deduct his commission at $1\frac{1}{2}$ per cent., and invest the balance in silk. How much did he invest?

$$\begin{aligned}
\text{Since the Commission on } \$100 &= \$1.50\,;\\
\therefore \text{ sum invested out of } \$101.50 &= \$100\,;\\
\therefore \quad \text{“} \quad \text{“} \quad \text{“} \quad \text{“} \quad \$1 &= \$\frac{100}{101.50}\,;\\
\therefore \quad \text{“} \quad \text{“} \quad \text{“} \quad \text{“} \quad \$1827 &= \$\frac{1827\times100}{101.50}\\
&= \$1800\,;\\
\therefore \text{ sum invested} &= \$1800.
\end{aligned}$$

If in the above question the Commission is required, we reason as follows:—

$$\begin{aligned}
\text{On } \$101.50 \text{ the Commission} &= \$1.50\,;\\
\text{“} \quad \$1 \qquad \text{“} \qquad &= \$\frac{1.50}{101.50}\,;\\
\text{“} \quad \$1827 \qquad \text{“} \qquad &= \$\frac{1827\times1.50}{101.50}\\
&= \$27\,;\\
\therefore \text{ the Commission required} &= \$27.
\end{aligned}$$

Examples lxxxi.

1. Find the Commission on

(*a*) \$7600 @ $1\frac{2}{5}$ %. | (*b*) \$5600 @ $12\frac{1}{2}$ %.

2. An agent sold 618 bbl. of flour @ \$6.25 per barrel. What is his commission @ $4\frac{1}{3}$ % ?

3. What is the ready money-payment of an account amounting to \$7680, allowing a discount of $2\frac{1}{2}$ % ?

4. A lawyer collected 86 % of a debt of \$775. What is his commission @ 4 % ?

5. An agent collected rents to the amount of \$578, and his commission amounted to \$26.01. What was the rate ?

6. What is the rate of commission when \$36 is paid for selling \$1600 worth of goods ?

7. An agent sold 630 bu. of oats @ 45c. per bushel and his commission amounted to \$15.12. What rate was charged ?

8. A house and lot were sold for \$7850 and the owner received \$7732.25 as the net proceeds. What was the rate of commission ?

9. What amount of money was invested when the agent's charges @ $1\frac{1}{2}$ % amounted to \$576 ?

10. An agent's commission @ $4\frac{1}{2}$ % for selling goods was \$352.80. What was the value of the goods sold ?

11. A real estate agent sold a house on a commission of 4 % and sent the owner \$3600 as the net proceeds. For what was the house sold ?

12. A lawyer collected 60 % of a debt of \$1250 and charged 5 % on the sum collected. How much did the creditor receive ?

13. I sent \$3377 to my agent to invest after deducting his commission @ $2\frac{1}{3}$ %. What was his commission ?

14. I sent my agent \$4840 to expend in flour @ \$7 per barrel after deducting a commission @ $3\frac{1}{3}$ %. How many barrels were purchased ?

15. *A* receives a consignment of wheat from *B*. He is to sell it on a commission of 2 %, and invest the proceeds in silk, after deducting his commission on this new transaction @ 4 %. *A*'s total commission was \$600. What sum did he invest ?

16. An agent sold a consignment of lumber for \$7650 and invested the proceeds, less his commission, in flour. His total commission on the two transactions amounted to \$300. What rate did he charge, the rates on both sale and purchase being the same ?

Insurance.

158. Insurance is security guaranteed by one party, on being paid a certain sum, to another against any loss.

The **Premium** is the sum paid for Insurance. It is always a certain per cent. of the sum insured.

The **Policy** is the written contract of Insurance.

NOTE.—As the Premium is always so much per cent. of the sum insured, it is found by the same rule as Commission.

Ex. What sum should be insured at 4 per cent., on goods worth \$2940, that the owner may receive, in case of loss, the value of both goods and premium?

Since the premium on \$100 @ 4 % is \$4,
\$96 worth of goods would be covered by \$100;
$\therefore \$1 \quad " \quad " \quad " \quad \$\frac{100}{96};$
$\therefore \$2940 \quad " \quad " \quad " \quad \$\frac{2940\times100}{96}$
$=\$3062.50$, sum required.

Examples lxxxii.

1. What will be the premium of insurance on the furniture of a house valued at \$2500 @ $\frac{1}{8}$ %?

2. What is the premium for insuring a cargo, valued at \$21350, @ $3\frac{1}{2}$ %?

3. For what sum should goods worth £4384 0s. 3d. be insured @ $2\frac{1}{3}$ % that the owner may recover, in case of loss, the value of both goods and premium?

4. A person at the age of 40 insures his life in each of two offices for \$5500, the premiums being at the rate of $3\frac{3}{4}$ and $3\frac{4}{5}$ %, respectively. Find his annual payment?

5. What sum must be paid to insure a cargo worth \$26400, the premium being $1\frac{1}{2}$ %, policy duty $\frac{1}{6}$ %, and brokerage $\frac{1}{8}$ %?

6. A trader gets 500 bbl. of flour insured for 75 % of its cost @ $2\frac{1}{5}$ %, paying \$80.85 premium. At what price per barrel did he purchase the flour?

7. A company took a risk @ $2\frac{1}{2}$ %, and re-insured $\frac{4}{5}$ of it in another company @ 3 %. The premium received exceeded that paid by \$10. What was the amount of the risk?

8. A shipment of apples was insured @ $2\frac{3}{8}$ % to cover $\frac{5}{8}$ of its value. The premium was \$71.25. What were the apples worth?

9. A shipment of goods is insured for $10000, which sum covers the value of the goods, the premium @ .65 % and $5 for expenses. What was the value of the goods?

Taxes.

159. A tax is a sum of money assessed on a person in proportion to the value of his property, amount of income, etc., for public purposes.

In order to levy a tax, persons, called assessors, are first employed to ascertain or appraise the value of all the property taxed. When this has been done, the sum to be levied is apportioned amongst the property-owners according to the value of the property of each.

Ex. A certain town has property valued at $1560000 and levies a tax of $23400. What should *B* pay whose property is valued at $7500?

Since $1560000 pays $23400;
∴ $1 pays $\$\frac{23400}{1560000}$;
∴ $7500 pays $\$\frac{7500 \times 23400}{1560000}$
$= \$112.50$, tax required.

Examples lxxxiii.

1. In a school section containing property valued at $100000 a tax has to be levied to pay the teacher's salary of $800, and $250 which had been expended in purchasing maps, etc. Find *A*'s tax, who owns property, real and personal, worth $5400.

2. A man who owns $8500 worth of property pays a tax of $144.50. Find the rate on the dollar.

3. If the property of Toronto be valued at $125000000, and *B*, who pays tax on $80000 worth of property, pays $1560. Find the total tax levied in Toronto.

4. In a certain village a school-house is to be built at an expense of $8400, to be defrayed by a tax upon property valued at $700000. What is the rate of taxation to cover both the cost of the school-house and the collector's commission @ 4 %?

5. My salary is $1800. My net income is $1781.85 after paying an income tax on all over $700. Find the rate.

6. A tax of $3900 is levied on a village, the assessed valuation being $280000. What tax does *A* pay on his income of $2500, $700 being exempted from taxation?

7. I paid \$25.60 income tax, \$700 of my salary being exempted. The rate was 16 mills on the dollar. What was my income?

Duties or Customs.

160. Duties or Customs are sums of money required by Government to be paid on nearly all imported goods.

The law requires that all goods entering Canada shall be landed at certain places where **Custom Houses** are established. These places are called **Ports of Entry,** and the duties levied are called **Customs Duties.**

Excise is a duty on articles manufactured in the Dominion itself, as on Spirits, Cigarettes, etc.

Duties are of two kinds, *ad valorem* and *specific.*

An **Ad Valorem** duty is a certain percentage on the cost of the goods in the country from which they are imported.

A **Specific** duty is the sum computed on the ton, yard, gallon, etc., without regard to the value of the goods.

NOTE.—As Ad Valorem duties are percentages, they are computed in the same manner as Commission, etc.

Ex. Find the Specific Duty on 760 lbs. of Sulphuric Acid at $\frac{1}{2}$ cent per lb.

Duty on 1 lb. is $\frac{1}{2}$c.
" 760 lb. is $\frac{760}{2}$c. = \$3.80, duty required.

Examples lxxxiv.

1. What is the duty on 7635 bu. of wheat, valued at \$4500, @ 12c. per bushel?

2. Find the ad valorem duty on an invoice of cottons which cost \$1760 @ 35 %.

3. Find the specific duty on 8750 gal. of crude petroleum @ $2\frac{1}{2}$c. per gallon.

4. Find the duty on 8400 lb. of confectionery worth $7\frac{1}{2}$c. per pound; the specific duty being $\frac{1}{2}$c. per pound and the ad valorem duty 35 %.

5. Paid \$1662.50 duty on an invoice of cotton at the rate of 35 %. What was the value of the cotton?

6. The duty on cabinet organs is 25 %. I paid \$196.50 on a consignment of these. For how much must they be sold to gain 20 % on my outlay?

7. If goods invoiced at $845 cost $1265.75 when laid down in the warehouse, the insurance, cartage, and freight amounting to $125, what was the rate of duty?

Storage.

161. Storage is a charge made by a person who stores movable property or goods for another. It is usually reckoned by the month of 30 days at a certain price per bushel, cask, box, bale, etc.

The owners of the goods pay for putting the goods in store, stowing away, and the expenses of delivery.

When goods are received and delivered at the pleasure of the consignor, the dues for storage are usually determined by an average.

Ex. What is the cost of storage, at 1 cent per bushel per month, of wheat received and delivered as per following:—

ACCOUNT CLOSED OCTOBER 2ND, 1899.

ACCOUNT OF STORAGE OF WHEAT, RECEIVED AND DELIVERED FOR ACCOUNT OF JOHN JONES, TORONTO.

DATE.		Received.	Delivered.	Balance.	Days	Products.
1899.						
July	2	200	......	200	9	1800
“	11	......	150	50	5	250
“	16	350	......	400	5	2000
“	21	......	300	100	20	2000
August	10	400	......	500	5	2500
“	15	......	450	50	5	250
“	20	......	50	0	0	000
September	5	200	......	200	5	1000
“	10	100	......	300	5	1500
“	15	......	200	100	17	1700
		1250	1150			30)13000
Bal. on hand Oct. 2..		......	100			
						433⅓
		1250	1250			

433⅓ × 1c. = $4.33⅓.

The storage of 200 bu. for 9 da. + 50 bu. for 5 da. + 400 bu. for 5 da. + 100 bu. for 20 da. + 500 bu. for 5 da. + 50 bu. for 5 da. + 200 bu. for 5 da. + 300 bu. for 5 da. + 100 bu. for 17 da. is the same as the storage of 13000 bu. for 1 da., or of 433⅓ bu. for a month of 30 da. The storage of 433⅓ bu. @ 1c. per bushel is $4.33⅓.

EXAMINATION PAPERS.

I.

1. If a grocer's pound weight is ⅝ oz. too light, find his gain per cent. from this source alone.

2. If a debt, after a deduction of 5 %, becomes $228, what should it have become if a deduction of 6¼ % had been made?

3. Find the value of the goods imported when an *ad valorem* duty of 17½ % produces $637.

4. The population of a city has increased by 5975 persons between 1890 and 1900. This increase is 12½ % of the population of 1900. What was the population in 1890?

5. In 1880 the population of a town was 7600. In 1900 it was found to be 9196. If the increase per cent. during the first decade was the same as during the last, what was this per cent.?

II.

1. *A*, after paying an income tax of 1½ % on all his salary over $700, has $1744.10 left. Find his salary.

2. A town has levied a tax of $7340, which sum includes the amount voted for building a bridge and the collector's fees @ 3 %. What was expended on the bridge?

3. The average of ten results was 17.5. That of the first three was 16.25, and of the next four 16.5. The eighth was 3 less than the ninth, and 4 less than the tenth. What was the tenth?

4. The gross receipts of a railway company in a certain year are apportioned thus: 40 % to pay the working expenses, 54 % to give the shareholders a dividend at the rate of 3½ % on their shares; and the remainder, $42525, is reserved. What was the paid up capital of the company?

5. *A* can do 5 % of a piece of work in 3 da. of 10 hr. each; *B* can do 7½ % of it in 5 da. of 8 hr. each. If both men work together and the whole work be worth $85, what does each get?

III.

1. A cargo is valued at \$7905.45. The premium of insurance is at the rate of $5\frac{1}{4}\%$. The policy duty @ $\frac{1}{5}\%$. Commission @ $\frac{7}{16}\%$. What sum must be insured to cover the cargo and the expenses of insurance ?

2. Received, and delivered, on account of James Smith, sundry bales of cotton, as follows: Received Jan. 1, 1899, 2310 bales; Jan. 16, 120 bales; Feb. 1, 300 bales; delivered Feb. 22, 1000 bales; March 1, 600 bales; April 3, 400 bales; April 10, 312 bales. Required, the number of bales remaining in store May 1, and the cost of storage up to that date, at the rate of 5c. a bale per month.

3. The increase in the number of male and female criminals is $2\frac{1}{2}\%$, while the decrease in the number of males alone is $7\frac{1}{2}\%$, and the increase in the number of females is $10\frac{1}{4}\%$. Compare the antecedent number of male and female prisoners.

4. A person takes a railway return-ticket for a month, paying 25 % more for it than he would have done for a single ticket. At the end of the month he obtains an extension of time for a week by paying 5 % on the monthly ticket. The whole sum paid is \$10.50. Find the price of the single ticket.

5. The paper duty was $1\frac{1}{2}$d. per pound, and the weight of a certain book $1\frac{1}{2}$ lb. The paper manufacturer realized 10 % on his sale, and the publisher 20 % on his outlay. What reduction might be made in the price of the book on the abolition of the paper duty, allowing to each tradesman the same rate of profit as before ?

IV.

1. A merchant bought 37 yd. 2 qr. of cloth @ \$$4.87\frac{1}{2}$ per yard, and 49 yd. $2\frac{1}{2}$ qr. of silk @ $93\frac{3}{4}$c. per yard. For what sum must the whole be sold to make a profit of $33\frac{1}{3}\%$?

2. A commission merchant is to sell 12000 lb. of cotton and invest the proceeds in sugar, retaining $1\frac{3}{4}\%$ on the sale and the same on the purchase. Cotton selling @ 7c., and sugar @ 5c. per pound. What quantity of sugar can the merchant buy ?

3. In an examination of 750 candidates, .22 of the whole do well, .34 barely pass, and the rest fail. How many do well, barely pass, and fail, respectively ?

4. Sold grain on commission @ 5 %. Invested net proceeds in groceries @ 2 % commission. My whole commission was \$70. What was the value of the grain and groceries ?

5. A commission merchant receives 125 bbl. of flour from *A*, 150 bbl. from *B*, 225 bbl. from *C*. He finds on inspection that *A*'s is 10 % better than *B*'s, and *C*'s $5\frac{5}{11}$ % better than *A*'s. He sells the whole lot @ $7 per barrel, and charges 4 % commission. How much does he remit to each?

V.

1. A broker charges me $1\frac{1}{3}$ % commission for purchasing some uncurrent bank bills @ 25 % discount. Of these bills three of $10 each and one of $50 became worthless. I dispose of the remainder at par, and thus make $985. What was the amount of bills purchased?

2. A wholesale merchant sent a quantity of goods into the country to be sold by auction, on a commission of $4\frac{1}{2}$ %. What amount of goods must be sold that his agent may buy produce with the avails to the amount of $1910, after retaining a commission of 2 %?

3. A factor receives $30056, and is directed to purchase cotton at $289 per bale. He is to receive 4 % commission. How many bales does he buy?

4. Sold goods to a certain amount on a commission of 5 %, and having remitted the net proceeds to the owner, received for prompt payment $\frac{1}{3}$ %, which amounted to $16.15. What was the amount of commission?

5. A man obtained an insurance for life at the age of 37, and died when 51 years old. The policy required annual payments during life @ $2.8674 per $100, and secured to the heirs $1709.69 more than the amount of all the premiums paid. What was the face value of the policy?

6. The manufacturer of an article charged 20 % profit. The wholesale dealer charged 25 % of an advance on the manufacturer's price, and the retail dealer charged 30 % of an advance on the wholesale price. Find the cost to the manufacturer of an article for which the retail dealer charged $91.

7. If the Roman Catholics are 3 to 1 of the population of Ireland, and the Protestant Dissenters bear the proportion of 2 to 3 to the members of the Church of England, find the per cent. which the Protestant Dissenters bear to the Roman Catholics.

For additional examples see page 313.

Simple Interest.

162. Interest is that which is paid by one, who borrows money, for the use of the money.

The money lent is called the **Principal.**

The borrower agrees to pay at what is called a certain **Rate** of interest, which is usually reckoned by the sum paid for the use of \$100 for 1 year. Thus, if I borrow \$500 for 1 year, and agree to pay \$25 for the use of the money, I am said to borrow at the *Rate of* 5 *per cent. per annum.* That is, I agree to pay \$5 for the use of every \$100 in the loan, at the end of the year.

The sum made up of the Principal and Interest added together, is called the **Amount** at the end of the time for which the money is borrowed.

163. The solution of questions relating to Interest depends on precisely the same principles as those explained in Chapter X.

Ex. 1. To find the Simple Interest on \$2675 for 3 years, at 5 per cent., we reason thus,

Interest on \$100 for 1 yr. $=\$5$;
" on \$1 for 1 yr. $=\$\frac{5}{100}$;
" on \$2675 for 1 yr. $=\$\frac{2675\times5}{100}$;
" on \$2675 for 3 yr. $=\$\frac{3\times2675\times5}{100}=\$401.25$;
$\therefore$ the interest $=\$401.25$.

Hence we derive the following rule:—

Multiply the Principal by the Rate per cent., and the result by the measure of the Time expressed in years, and divide the product by 100.

Ex 2. Find the interest on \$3200 for 2 years and 7 months at $5\frac{1}{2}$ per cent.

Since 7 mo. is $\frac{7}{12}$ of a year, the time is $2\frac{7}{12}$ yr.

Interest on \$100 for 1 yr. $=\$5.50$.
" \$1 for 1 yr. $=\$\frac{5.50}{100}$.
" \$3200 for 1 yr. $=\$\frac{3200\times5.50}{100}$.
" \$3200 for $2\frac{7}{12}$ yr. $=2\frac{7}{12}\times\$\frac{3200\times5.50}{100}$
$=\$\frac{31\times3200\times5.50}{12\times100}$
$=\$454\frac{2}{3}$;
$\therefore$ the interest $=\$454\frac{2}{3}$.

Ex. 3. Find the interest on \$101178 from January 28th, 1900, to Sept. 16th, 1900, at 6 per cent.

The number of days between January 28th and Sept. 16th is 231, and 231 da. is $\frac{231}{365}$ of a year.

$$\text{Interest} = \$\frac{101178\times 6}{100}\times\frac{231}{365} = \$3841.992.$$

NOTE I.—In calculating the number of days between two given days of the year, the rule is *to include one of them only* in the calculation. Thus, from Jan. 4 to Jan. 9 will be 5 days.

In the preceding example, if we multiply numerator and denominator by 2 we have

$$\frac{101178\times 12\times 231}{73000}.$$

Hence, in computing the interest for any number of days, we have the following Rule:—

Multiply the Principal by twice the rate, and the result by the measure of the number of days, and divide the product by 73000.

When the Principal is not very large the division is most readily effected by dividing the product by 3, the quotient by 10, and the new quotient by 10, and adding these quotients and the product together, and pointing off five places of decimals.

Ex. 4. Find the interest of \$1000 for 121 days at 8 per cent.

$$\begin{array}{r} \$1000 \\ 16 \\ \hline 16000 \\ 121 \end{array}$$

$$\begin{array}{r|r} \hline 3 & 1936000 \\ 10 & 645333\frac{1}{3} \\ 10 & 64533\frac{1}{3} \\ & 6453\frac{1}{3} \\ \hline & \$26.52320. \end{array}$$

Since 73000 increased by $\frac{1}{3}$ of itself, $\frac{1}{30}$ of itself, and $\frac{1}{300}$ of itself becomes 100010,

$$\begin{array}{r|r} \frac{1}{3} & 73000 \\ \frac{1}{10} \text{ of } \frac{1}{3} & 24333\frac{1}{3} \\ \frac{1}{10} \text{ of } \frac{1}{30} & 2433\frac{1}{3} \\ & 243\frac{1}{3} \\ \hline & 100010 \end{array}$$

and considering this as 100000, the reason for the above process is evident.

NOTE II.—In actual practice the time, when not an exact number of years, is always expressed in *days*, or in *years* and *days*.

Examples lxxxv.

1. Find the Simple Interest

(*a*) On $1160 for 11 mo. @ 6 % per annum.
(*b*) On $2750 for 3 yr. @ 5 % per annum.
(*c*) On $3625 for 4 yr. @ 6 % per annum.
(*d*) On $2700 for 6 yr. @ 4½ % per annum.
(*e*) On $8825 for 6½ yr. @ 6 % per annum.
(*f*) On $9125 for 78 da. @ 8 % per annum.
(*g*) On $5913 from Nov. 23, 1898, to April 7, 1899, @ 7½ % per annum.
(*h*) On £204 17s. 7d. from Aug. 3 to Jan. 9 @ 5 %.

2. I bought a house and lot for $4650, to be paid in 6 mo., with interest @ 6 %. Find the amount of the payment.

3. A man sold property for $11320. The terms were $3200 in cash, $3500 in **6** mo., $2500 in 10 mo., and the remainder in 12 mo., with interest @ 6 %. What was the whole amount paid?

4. A man who is paying $360 a year for house-rent borrows $5400 @ 6¼ %, and buys the house. Does he gain or lose, and how much?

164. We have explained how to find the Interest (and Amount) when the Principal, Rate, and Time are given. We shall now explain how to find the Rate, or Time or Principal, when the other two and also the Interest (or Amount) are given.

Ex. 1. At what Rate per cent. will $520 amount to $800.80 in 9 years?

As the rate is the interest on $100 for 1 yr., to find the rate we must find the interest on $100 for 1 yr.

Here interest = $880.80 − $520 = $280.80;
Thus, the interest on $520 for 9 yr. = $280.80;
∴ the interest on $520 for 1 yr. = $\$\frac{280.80}{9}$;
" on $1 for 1 yr. = $\$\frac{280.80}{520\times9}$;
" on $100 for 1 yr. = $\$\frac{100\times280.80}{520\times9}=\$6$;
∴ Rate required = 6 %.

Ex. 2. In what time will the Interest on $360 amount to $126 at 5 per cent.?

Interest on $360 for 1 year = $\$\frac{360\times5}{100}$, or $18.
Then, since $18 = the interest for 1 year,
$1 = the interest for $\frac{1}{18}$ year,
$126 = the interest for $\frac{126}{18}$ years, or 7 years;
∴ Time required = 7 years.

Ex. 3. What Principal will amount to $980 in 3 years at 7½ per cent.?

Interest on $100 for 3 years at 7½ % = $22.50;
∴ $122.50 is the Amount which has for its Principal $100;
$1 is the Amount which has for its Principal $\$\frac{100}{122.50}$;
$980 is the Amount which has for its Principal $\$\frac{980\times100}{122.50}$, or $800;
∴ Principal required = $800.

Ex. 4. At what rate will any sum triple itself in 20 years at simple interest?

Here the interest is twice the Principal, or $\frac{200}{100}$ of the Principal.
In 20 yr. the interest = $\frac{20}{100}$ of the Principal, @ 1 %;
∴ " " " " " = $\frac{1}{100}$ " " " @ $\frac{1}{20}$ %;
∴ " " " " " = $\frac{200}{100}$ " " " @ $\frac{200}{20}$ %, or 10 %;
∴ Rate required = 10 %.

Examples lxxxvi.

1. At what rate will be the interest on $326 for 15 yr. be $220.05?

2. In what time will $700 amount to $920.50 @ 6 %?

3. What sum will amount to $1395 in 8 mo. @ 5 %?

4. The interest on a sum of money for 12 yr. @ $4\frac{1}{2}$ % is $202.50. What is the sum ?

5. In what time will any sum double itself @ 5 %, simple interest ?

6. What must be the rate per cent. that the interest at the end of 16 yr. 8 mo. may be equal to seven-eighths of the sum lent?

7. A sum of money amounts in 10 yr. @ 7 % to $1275. In how many years will it amount to $1406.25?

8. The sum of $500 is borrowed at the beginning of the year at a certain rate per cent., and after 9 mo. $400 more is borrowed at double the previous rate. At the end of the year the interest on both loans is $35. What is the rate at which the first sum was borrowed ?

9. In how many days will the interest on £243 6s. 8d. be £4 0s. 10d. @ $6\frac{1}{4}$ % ?

10. If £556 17s. 6d. be loaned for 125 da. and then amount to £565 18s. 9d., what was the rate ?

11. The interest on $8000 for 1 da. is $2. Find the rate per cent. per annum.

12. Bought 5000 bu. of wheat @ $62\frac{1}{2}$c. a bushel, payable in 6 mo. I immediately realized for it @ 60c. cash, and put the money out at interest @ 8 %. At the appointed time I paid for the wheat. Did I gain or lose by the transaction, and how much ?

13. The interest on a sum of money at the end of $6\frac{1}{4}$ yr. is three-eighths of the sum itself. What rate per cent. was charged?

14. A sum of money at simple interest has in $4\frac{1}{2}$ yr. amounted to $735, the rate of interest being 5 % per annum. What was the sum at first, and in how many years *more* will it amount to $1140 ?

15. The interest on $1805, loaned on May 13th, @ $5\frac{1}{4}$ % per annum, is $37.905. On what day was the money returned?

16. A person borrowed money for 2 yr. For the first year he paid 5 %, and for the second 6 %. At the end of the time he paid back $355.20. How much did he borrow ?

17. The amount of a sum of money for 4 yr. is $310, and for $6\frac{1}{2}$ yr. at the same rate it is $347.50. Find the sum and the rate.

18. The amount of a principal @ 5 % is $405 for a certain time, and @ $6\frac{1}{4}$ % for the same time it is $416.25. Find the principal and the time.

Partial Payments.

165. A partial payment is the payment of a part of the amount due on a note or bond. When partial payments are made they are endorsed on the note or bond. To compute the interest on such a note proceed according to the following Rule:—

Compute the interest on the principal to the time of the first payment, and if this payment exceed the interest then due add the interest to the principal, and from the sum take the payment. The remainder will form a new principal, with which proceed as before.

But if the payment be less than the interest, compute the interest on the principal to the time when the sum of the payments shall first equal or exceed the interest due. Add the interest to the principal, and from the sum subtract the sum of the payments, and treat the remainder as a new principal.

This rule proceeds on the ground that in all cases the payment should be applied first to the interest due, then to the principal, and that the principal remains unchanged until the sum paid exceeds the accrued interest.

Ex. 1. $4000.

TORONTO, June 1, 1898.

Two years after date I promise to pay William Smith, or order, four thousand dollars, for value received, with interest at 7 %.

RICHARD PAYWELL.

On this note were the following endorsements:—

Sept. 15, 1898, four hundred and fifty dollars.
Dec. 15, 1898, fifty dollars.
Mar. 1, 1899, five hundred dollars.
Jan. 1, 1900, one thousand dollars.

What remained due June 4, 1900?

Principal on interest June 1, 1898	$4000 00
Interest to Sept. 15, 1898	80 89
Amount	$4080 89
Less 1st payment	450 00
Remainder for a new principal	$3630 89

{Interest from Sept. 15 to Dec. 15, 1898, is $63.44, which exceeds the payment.}	
Interest from Sept. 15, 1898, to March 1, 1899	117 20
Amount	$3748 09
Less the sum of the 2nd and 3rd payments	550 00
Remainder for a new principal	$3198 09
Interest from March 1, 1899, to Jan. 1, 1900	186 47
Amount	$3384 56
Less payment Jan. 1, 1900	1000 00
Remainder for a new principal	$2384 56
Interest from Jan. 1 to June 4, 1900	70 94
Balance due June 4, 1900	$2455 50

Examples lxxxvii.

1. $1500.

HAMILTON, Jan. 1, 1899.

One year after date, we promise to pay S. White, or order, fifteen hundred dollars, with interest. Value received.

GEORGE BROWN & Co.

The following payments were made on this note :—

March 16, 1899, $100; June 13, 1899, $400; Sept. 1, 1899, $200.

What was due Jan. 1, 1900, interest at 6 %?

2. $3500.

BELLEVILLE, March 15, 1899.

For value received, we jointly and severally promise to pay Wm. Smith, or order, three thousand five hundred dollars, with interest.

JAMES JONES & Co.

Endorsed as follows :—

June 1, 1899, $800; Sept. 1, 1899, $100; Jan. 1, 1900, $1560; March 1, 1900, $300.

What was due May 16, 1900, interest @ 6 %?

3. $1200.

TORONTO, Oct. 15, 1895.

One year from date we promise to pay James Smith, or order, twelve hundred dollars, for value received, with interest.

WILDER & SON.

Endorsed as follows :—

Oct. 15, 1896, $1000; April 15, 1897, $200.

How much remained due Oct. 15, 1897, interest @ 6 %?

4. A note was given for $5760, Sept. 20, 1898.

Endorsements:—Nov. 30, $200; February 2, 1899, $600; April 9, 1899, $350.

What was due Sept. 20, 1899, interest @ 6 % ?

5. A note was given for $2500, April 1, 1899.

Endorsements:—June 11, $200; July 5, $100; Sept. 9, $450.

What is due 6 mo. from date, @ 7 % ?

6. A note was given for $1750, May 11, 1899.

Endorsements:—July 1, $100; Aug. 12, $45; Sept, 30, $60; Jan. 19, 1900, $250; March 10, $150.

What was due April 1, 1900, @ 8 % ?

7. $4000. TORONTO, July 11, 1894.

Three months after date I promise to pay James Jones, or order, four thousand dollars, for value received, with interest @ 6 %.

WM. SMITH & Co.

Endorsed as follows:—

Dec. 1, 1894, $25; March 10, 1895, $50; July 14, 1895, $180; Jan. 1, 1896, $200; April 25, 1896, $450; Sept. 9, 1896, $75; Jan. 1, 1897, $300.

The note was paid Sept. 9, 1897. What was then due ?

Present Worth and True Discount.

166. Suppose *A* owes *B* $105, to be paid at the end of a year. If *A* be disposed to pay off the debt at once the sum which he ought to pay should be such that, if put out at interest by *B*, it will amount at the end of a year to $105. Suppose, further, that *B* can put out his money at 5 per cent. interest. Then, if he put out $100 at interest, this is the sum which will amount at the end of a year to $105.

Hence, $100 is the sum, which *A* ought to pay at once, and this is called the **Present Worth** of the debt, and is evidently such a sum as would, if put out to interest for the given time and rate, amount to the debt. The difference between the Debt and the Present Worth, which is in the case under consideration $5, is called the Discount. (See Art. 152).

This kind of Discount is known as **Mathematical or True Discount.**

Its computation falls under the case of Simple Interest in which the Amount, Rate, and Time are given to find the Interest or the Principal. In True Discount the Debt corresponds to the Amount, the Present Worth to the Principal, and the True Discount to the Interest.

There are, however, a variety of problems which may arise in connection with True Discount, some of which are illustrated below.

Ex. 1. Thus, to find the Present Worth of $1781.40, due 4 years hence, reckoning interest at 5 per cent.

The interest on \$100 for 4 yr. @ 5 % = \$20.
$\therefore$ \$120 has for its Present Worth \$100;
$\therefore$ \$1 has for its Present Worth $\$\frac{100}{120}$;
$\therefore$ \$1781.40 has for its Present Worth $\$\frac{1781 \cdot 40 \times 100}{120}$,
or \$1484.50
$\therefore$ Present Worth required = \$1484.50.

Ex. 2. Find the Discount on $1781.40, due 4 years hence, reckoning interest at 5 per cent.

The Present Worth = \$1484.50, as we have just shown;
the Discount = \$1781.40 − \$1484.50
= \$296.90.

When the Discount alone is required to be found, the following is the solution:—

The interest on \$100 for 4 yr. @ 5 % = \$20.
$\therefore$ \$120 has for its Discount \$20;
$\therefore$ \$1 has for its Discount $\$\frac{20}{120}$;
$\therefore$ \$1781.40 has for its Discount $\$\frac{1781.40 \times 20}{120}$
= \$296.90.

Ex. 3. What was the debt of which the Discount for 8 months at 9 per cent. was $44.46?

The interest on \$100 for 8 mo. @ 9 % = \$6.
$\therefore$ \$6 is the Discount on \$106;
$\therefore$ \$1 is the Discount on $\$\frac{106}{6}$;
$\therefore$ \$44.46 is the Discount on $\$\frac{44 \cdot 46 \times 106}{6}$
= \$785.46.

Ex. 4. The interest on a certain sum of money for two years is \$50, and the Discount for the same time and rate is \$45. Find the sum and the rate per cent. per annum.

Since \$50 is the interest on a sum of money which sum
= (its Present Worth + its Discount)
= (its Present Worth + \$45)
and \$45 is the interest on its Present Worth,
$\therefore$ \$5 is the interest on \$45;
$\therefore$ \$1 is the interest on $\$\frac{45}{5}$;
$\therefore$ \$50 is the interest on $\$\frac{50\times 45}{5}$, or \$450;
$\therefore$ \$450 is the sum required.
Again, the interest on \$45 for 2 yr. = \$5;
$\therefore$ the interest on \$45 for 1 yr. = $\$\frac{5}{2}$;
$\therefore$ the interest on \$1 for 1 yr. = $\$\frac{5}{45\times 2}$;
$\therefore$ the interest on \$100 for 1 yr. = $\$\frac{100\times 5}{45\times 2} = \$5\frac{5}{9}$;
$\therefore$ the Rate = $5\frac{5}{9}\%$.

NOTE I.—From the above it will be seen that the discount on any sum is the Present Worth of the interest of that sum for the same time and rate. Thus, \$45 is the Present Worth of \$50 for two years at a certain rate per cent.

Ex. 5. If \$20 be allowed off a bill of \$420 due in 6 months, how much shall be allowed off the same bill due in 12 months?

The discount off \$420 for 6 mo. = \$20;
$\therefore$ the interest on \$400 for 6 mo. = \$20;
$\therefore$ the interest on \$400 for 12 mo. = \$40;
$\therefore$ the discount off \$440 for 12 mo. = \$40;
$\therefore$ the discount off \$1 for 12 mo. = $\$\frac{40}{440}$;
$\therefore$ the discount off \$420 for 12 mo. = $\$\frac{420\times 40}{440}$
= $\$38\frac{2}{11}$;
$\therefore$ the Discount required = $\$38\frac{2}{11}$.

NOTE II.—The student will observe that the Discount is not proportioned to either the time or the rate.

Ex. 6. If \$15 be the Interest on \$115 for a given time, what should be the Discount off \$115 for the same time?

The interest on \$115 = \$15;
∴ the discount off \$130 = \$15;
∴ the discount off \$1 = $\$\frac{15}{130}$;
∴ the discount off \$115 = $\$\frac{115\times15}{130}$
= $\$13\frac{7}{26}$;
∴ the discount required = $\$13\frac{7}{26}$.

Ex. 7. If \$10 be allowed off a bill of \$110, due 8 months hence, what should be the bill from which the same sum is allowed as 4 months' discount?

\$10 is the discount off \$110 for 8 mo.;
∴ \$10 is the interest on \$100 for 8 mo.;
∴ \$10 is the interest on \$200 for 4 mo.;
∴ \$10 is the discount off \$210 for 4 mo.;
∴ the sum required = \$210.

Examples lxxxviii.

1. Find the Present Worth of
   (*a*) \$5520, due 4 yr. hence, @ 5 %.
   (*b*) \$84.70, due $2\frac{1}{3}$ yr. hence, @ 9 %.
   (*c*) \$615, due 1 yr. 4 mo. hence, @ 7 %.
   (*d*) \$1120, due 16 mo. hence, @ 5 %.
   (*e*) £618 2s. 6d., due $3\frac{3}{4}$ yr. hence, @ 4 %.

2. Find the discount on
   (*a*) \$636, due in 9 mo. @ 8 %.
   (*b*) \$1884.30, due in $3\frac{1}{5}$ yr. @ 10 %.
   (*c*) \$637.50, due in $5\frac{1}{2}$ yr. @ 5 %.
   (*d*) £1165 16s. 3d., due in $2\frac{1}{2}$ yr. @ 6 %.
   (*e*) £252, 19s. 3d., due in 9 mo. @ $4\frac{1}{3}$ %.

3. Find the Present Worth of \$6934.50, due in 3 yr. hence, @ 5 %.

4. Find the Discount on \$68.40, due $1\frac{1}{4}$ yr. hence, @ $5\frac{1}{2}$ %.

5. A tradesman accepts \$19.3125 in payment of a debt of $\$20\frac{267}{960}$, due in 12 mo., in consideration of being paid at once. What rate of discount does he allow?

6. Find the Present Worth of a bill for \$1127.10, drawn Jan. 1 @ 4 mo., and discounted Feb. 20 @ 10 % per annum.

7. The Discount on \$275 for a certain time is \$25. What is the Discount on the same sum (*a*) for twice that time, and (*b*) for half the time?

8. A tradesman marks his goods with two prices, one for cash and the other for credit of 6 mo. What relation should the two prices bear to each other, allowing interest @ $7\frac{1}{2}\%$? If the credit price of an article be $33.20, what is the cash price?

9. If $98 be accepted in present payment of $128, due some time hence, what should be a proper discount off a bill of $128 which has only half the time to run?

10. A certain sum ought to have $20.80 allowed as 8 mo. interest on it. But a bill for the same sum due in 8 mo. at the same rate should have $20 only allowed off as discount in consideration of present payment. What is the sum and the rate per cent.?

Bank Discount.

167. The Discount which was treated in the last Section is called Mathematical Discount, or True Discount, to distinguish it from Practical Discount, of which there are two kinds.

i. The deduction made by a trader, when an account is paid to him before the time when he proposes to demand payment. This was explained under the name Trade Discount, in Articles 152 and 153.

ii. The deduction made by a lender of money from the sum which he proposes to lend. Thus, if a borrower binds himself by a bill to pay $100 a year hence, and a discounter advances money on the security of this bill, at the rate of 5 per cent., he gives to the holder of the bill $95 and takes the bill. The difference, $5, is known as Bank Discount.

168. Bank Discount is the charge made by a bank or money lender for advancing the payment of a note, draft, or bill of exchange not yet due. It is the *Simple Interest* on the *face value* of the note, for the time between the date of buying the note and the time it falls due. The lender deducts the discount from the *face value* of the note and pays the balance to the borrower.

169. A Promissory Note, or simply a **Note,** is a written promise made by one person to another to pay a specified sum of money on demand or at a certain time, as

NOTE.

$75.35.

Toronto, Feb. 23, 1900.

Three months after date I promise to pay William Roe, or order, the sum of seventy-five $\frac{35}{100}$ dollars, with interest @ 7 % per annum, for value received.

John Jones.

In this case, John Jones is the **Maker** of the Note, and William Roe is the **Payee.**

The Face of the Note is the sum for which it is given, viz., $75.35.

The Face Value is what the note is worth at maturity, viz., $76.53.

The Maturity of a note is the time at which it becomes legally due, viz., May 26, 1900.

Three days, called *Days of Grace*, are always allowed after a bill of exchange or a promissory note is *nominally* due before it is *legally* due. Thus, a bill drawn on July 5, for 3 months, would be nominally due on Oct. 5, but legally on Oct. 8. Calendar months are always reckoned, so that a bill of 3 months, whether drawn on the 28th, 29th, or 30th of Nov., 1896, would be due on the 3rd of March, 1897. The banker or money-lender who discounts a note always charges *interest* on the note from the time it is discounted till it is legally due. Hence, in computing Bank Discount of this nature, interest must be calculated for 3 days more than the time the note has to run.

170. A Negotiable Note is one which is made payable to bearer, or to the order of the Payee. It can be sold or transferred to another. If payable to bearer, no endorsement is necessary. If payable to the order of the payee, it must be endorsed by him before being disposed of.

171. A Non-negotiable Note is made payable to the person named, and can only be transferred by assignment, which carries all offsets and legal defences that may exist between the original parties.

172. An Endorsement in Blank is simply the signature of the endorser across the back of the note.

173. An Endorsement in Full is one in which the endorser states over his signature to whose order the note is payable.

A restrictive endorsement is one in which the payment is restricted to a particular person.

174. A Qualified Endorsement is one in which the endorser relieves himself of responsibility for payment by writing over his signature, "without recourse."

175. DRAFT.

$150.

TORONTO, June 15th, 1900.

Ten days after sight pay to the order of William Smith the sum of one hundred and fifty dollars, for value received, and charge the same to the account of

JAMES FORBES.

To JOHN JONES, Esq.,
Merchant, Winnipeg.

In this draft James Forbes is the drawer. John Jones, Esq., is the drawee. William Smith is the payee.

176. Notes differ from **Drafts** as follows:—

A Note is a promise to pay originating with the *debtor*.

A Draft is an order to pay originating with the *creditor* and addressed to the *debtor*.

There are three parties to a draft.

i. The *Drawer*, the person who orders the money to be paid.

ii. The *Payee*, the person in whose favor it is drawn.

iii. The *Drawee*, the person on whom it is drawn.

If the draft is accepted, the drawee becomes the acceptor.

The *Acceptor* of a draft stands in the same relation as the maker of a note.

The *Drawer* of a draft stands in the same relation as the first endorser of a note.

Ex. 1. What would a banker gain by discounting on Sept. 21, a bill of \$318.15, dated July 31, at 4 months at 5 per cent.?

The bill is legally due on Dec. 3.
The number of days from Sept. 21 to Dec. 3 = 73.
The interest on \$318.15 for 73 da. @ 5 % = \$3.1815.
The mathematical discount = \$3.15.
∴ the banker's gain = \$.0315.

Ex. 2. A merchant wishes to borrow \$96.91 on a bill made on July 5, for 3 months. What must be the face of the bill, interest being reckoned at $8\frac{1}{9}$ per cent.?

Time between July 5 and Oct. 8 = 95 da.
Interest on \$100 for 95 da. @ $8\frac{1}{9}$ % = $\$2\frac{1}{9}$.
∴ a note for \$100 would produce $\$97\frac{8}{9}$;
∴ face of note to produce $\$97\frac{8}{9}$ = \$100;

$$\therefore \quad " \quad " \quad " \quad \$96.91 = \$\frac{96.91 \times 100}{97\frac{8}{9}}$$

$$= \$99.$$

Examples lxxxix.

1. Find the date of maturity, the term of discount, the proceeds, and the discount of the following notes :—

(*a*) \$1000.

TORONTO, June 1st, 1898.

Six months after date I promise to pay John Smith, or order, the sum of one thousand $\frac{00}{100}$ dollars, at the Ontario Bank here. Value received.

JAMES BROWN.

Discounted Aug. 17th, @ 5 %.

(*b*) \$600.

TORONTO, Feb. 21st, 1899.

Four months after date I promise to pay William Rae, or order, the sum of six hundred $\frac{00}{100}$ dollars, with interest @ 6 % per annum, at the Dominion Bank here. Value received.

JOHN JOBB.

Discounted on April 12th, @ 8 %.

(c) $500.

LONDON, March 19, 1899.

Ninety days from date I promise to pay James Taylor, or order, the sum of five hundred $\frac{00}{100}$ dollars, with interest at 7 per cent. per annum. Value received.

CALEB SMITH.

Discounted April 15, @ 8 %.

2. Find the proceeds in the following cases :—

	Face.	Date.	Time.	Discounted.	Rate.
(a)	$400,	May 14,	4 mo.,	July 3,	8 %.
(b)	$1606,	Oct. 24,	4 mo.,	Dec. 19,	7 %.
(c)	$584,	June 21,	60 da.,	June 29,	7½ %.
(d)	$730,	Sept. 17,	6 mo.,	Dec. 30,	5 %.

3. I wish to receive $700 from the Ontario Bank, and give my note for 3 mo. (days of grace included), which, discounted @ 8 %, just produces this sum. Find the face of the note.

4. I discounted a note which is due in 3 mo. (days of grace included), @ 10 % at the Bank of Montreal, and received $1300. What was the face of the note ?

5. Discounted the following note 4 mo. (days of grace included), before it was due, at the Bank of Montreal, @ 9 % per annum. How much did I get for it ?

$500.

TORONTO, Feb. 15, 1898.

One year from date I promise to pay Alex. Hughes, or order, five hundred $\frac{00}{100}$ dollars, for value received.

ALFRED CONNOR.

6. The difference between the Interest and the True Discount on a sum for 6 mo. @ 8 % is $2. Find the sum.

7. *A* discounts a note due in 9 mo., so as to make 10 % per annum on his money. What per cent. does he exact on the face of the note ?

8. I have two notes, the face values of both together being $128. They are discounted 6 mo. before maturity, one at True and the other at Bank Discount @ 8 % per annum. The sum of their discounts is $5. Find the face of each note.

9. I bought a bill of goods for $864 on 4 mo. credit, but being offered 5 % off for cash, I borrowed the money at a bank, by giving my note, due in 125 da., discounted @ 6 %, and paid the bill. What was the face of the note, and how much did I gain ?

10. Find the discount @ $6\frac{1}{4}$ % on a note for $3500, due on May 15th, 1896, which is discounted on Jan. 30th, 1896.

11. What will a banker retain on discounting a note of $1275 drawn on the 4th of March, at 10 mo., and discounted on 14th of August, @ 5 % ?

12. A bill of $500 drawn on April 1st, at 6 mo., is discounted May 31st. What is the banker's discount @ $6\frac{1}{2}$ % ?

13. A note of $1460, discounted 60 da. before it was legally due, yielded $1442. At what rate was it discounted ?

14. The discount on a note of $1825, discounted 40 da. before it was legally due, was $$12\frac{1}{2}$. Find the rate of discount.

15. *A* received from a bank $990 for a note of $1000, the bank charging 5 % per annum. For how long before maturity was the note discounted ?

16. A note discounted 60 da. before maturity, @ $5\frac{1}{4}$ %, produced $289.48. Find the face of the note.

17. A banker buys a note for $3600, discounted 60 da. before maturity, the face being $3650. What rate was charged ?

18. On June 3, a bank gives me $715 for a note of $730, discount @ $7\frac{1}{2}$ %. When is the note due ?

19. *A* owes *B* $770, and gives him his note at 90 da. What sum must be on the face of the note to pay this debt, if discounted at $1\frac{1}{4}$ % per month ?

Compound Interest.

177. Compound Interest is that which is paid, not only for the use of the original sum lent, but also for *use of the interest* as it becomes due.

The interest on $500 for 1 year at 4 per cent. is $20.

If, then, $500 be lent at Compound Interest for 2 years, at 4 per cent., the interest for the *first* year is $20.

Now, as the borrower has to pay for the use of this $20, the interest for the *second* year must be calculated on $520.

$$\text{Hence, interest for second year} = \$\frac{520 \times 4}{100} = \$20.80.$$

To put the matter in a more simple way, we have supposed *the borrower to retain* the interest due at the end of the first year, but the reasoning will be the same if we suppose *the lender to receive* the interest at the end of the first year, and to put it out immediately at the same rate of interest.

178. We may calculate Compound Interest by the following rule:—

Find the interest for the first year. Add it to the original principal. Call the result the Second Principal. Find the interest on this for the second year. Add it to the second principal. Call the result the Third Principal. Find the interest on this for the third year, and so on.

Ex. 1. Find the Compound Interest on $7500, for 3 years, at 4 per cent.

The principal for the *first* yr. = $7500.00
The interest for the *first* yr., @ 4 % = $ 300.00

The principal for the *second* yr. = $7800.00
The interest for the *second* yr. = $ 312.00

The principal for the *third* yr. = $8112.00
The interest for the *third* yr. = $ 324.48

The amount at the end of the *third* yr. = $8436.48
∴ Compound interest = $(8436.48 − 7500)
= $ 936.48,

or Compound Interest required is
$300 + $312 + $324.48 = $936.48.

Ex. 2. What is the Compound Interest of $250, for 2 years, at 7 per cent.?

Principal for the *first* yr. = $250
Interest for the *first* yr., $250 × 0.07 = $ 17.50

Principal for the *second* yr. = $267.50
Interest for *second* yr., $267.50 × 0.07 = $ 18.725

Amt. at Compound Interest for 2 yr., = $286.225
First Principal, = $250.00

Compound Interest for 2 yr. = $ 36.225

Ex. 3. Find the present worth of $842.70 for two years, at 6 per cent. Compound Interest.

The compound interest on \$100 for 2 years at @ 6 % = \$12.36.

∴ \$112.36 has for its present worth \$100;

∴ \$1 has for its present worth $\$\frac{100}{112 \cdot 36}$;

∴ \$842.70 has for its present worth $\$\frac{842 \cdot 70 \times 100}{112 \cdot 36}$

= \$750;

∴ Present Worth required = \$750.

Examples xc.

1. Find the Compound Interest on

(*a*) \$375 for 3 yr. @ 5 %.
(*b*) \$564 for 4 yr. @ 6 %.
(*c*) \$1154.37 for 4 yr. @ 5 %.
(*d*) \$740 for 5 yr. @ 4 %.

2. Find the amount at Compound Interest of

(*a*) \$1000 for 3 yr. @ 4½ %.
(*b*) \$750 for 3 yr. @ 4 %.
(*c*) \$1200 for 4 yr. @ 5 %.
(*d*) \$1000 for 5 yr. @ 3 %.

NOTE I.—When the Compound Interest is required for 3½ years, it is usual to find the Compound Interest for the whole of the fourth year, and take half the result as the Compound Interest for the half year. This really implies that the interest is paid half-yearly, but the approximation does not differ much from the exact truth.

179. The process for finding the amount of a sum at Compound Interest may be presented in a very brief and neat form as follows:—

If the rate of interest be 4 %,
Amount of \$100 at the end of 1 yr. = \$104,
" of \$1 at the end of 1 yr. = $\frac{104}{100}$ of \$1.

Hence it follows that,

Amount of *any sum* @ 4 % in 1 yr. = $\frac{104}{100}$ of that sum.

Again,

Amount for *second* year = $\frac{104}{100}$ of amount for the first year;

∴ Amount of *any sum* @ 4 % in 2 yr.
= $\frac{104}{100}$ of $\frac{104}{100}$ of that sum.

Suppose, then, we have to find the amount of \$540 in 3 years, at 4 per cent., Compound Interest.

The amount = $\frac{104}{100}$ of $\frac{104}{100}$ of $\frac{104}{100}$ of \$540
= $\$\{540 \times (1.04^3)\}$
= \$607.426.

From the above example it will be noticed that the measure of the amount of \$1 for a year at 4 per cent. is raised to the power indicated by the number of years for which Compound Interest is to be calculated. Hence, we have the following Rule:—

To find the sum to which any principal will amount if put out to Compound Interest at a given rate in a given number of years, find the amount of \$1 *for a year at the given rate, raise the measure of that sum to the power which is denoted by the given number of years, and multiply the result by the number of dollars in the given principal.*

Ex. 1. Find the amount of \$850 in three years at 6 per cent., Compound Interest.

$$\begin{aligned} \text{The Amount} &= \$\{850 \times (1.06)^3\} \\ &= \$\{850 \times 1.191016\}, \text{ or} \\ &= \$1012.363. \\ \text{The Compound Interest} &= \$1012.36 - \$850. \\ &= \$162.36. \end{aligned}$$

NOTE II.—When the number of years is large, the student is recommended to employ the contracted method of multiplication, explained in Art. 87, page 80.

Interest may be payable either yearly, half-yearly, or quarterly, or at some other stated period.

In finding the Compound Interest on \$2000 in two years, when the interest is payable *half-yearly*, at 5 per cent., we reason thus,

5 % for 1 yr. = $2\frac{1}{2}$ % half-yearly;
" " 2 yr. = 4 half-years.

Hence, we have to find the Compound Interest on \$2000, *for four times of payment, at* $2\frac{1}{2}$ *per cent.*

$$\begin{aligned} \text{The Amount} &= \$\{2000 \times (1.025)^4\} \\ &= \$\{2000 \times 1.1038128\} \\ &= \$2207.625. \\ \text{The Interest} &= \$2207.625 - \$2000 \\ &= \$207.625. \end{aligned}$$

Ex. 2. What principal will amount to \$1012.363 in 3 years, at 6 per cent., Compound Interest?

$$\text{Principal} \times (1.06)^3 = \$1012.363$$
$$\therefore \text{ Principal} = \$\frac{1012.363}{(1.06)^3}$$
$$= \$850.$$

Examples xci.

1. What is the Compound Interest on $1000 for 2 yr., @ 6 %, payable half-yearly?

2. What is the amount of $200 for 3 yr., @ 6 %, payable half-yearly?

3. A man deposits $1500, on June 1st, 1896, in the Bank of Commerce, on which the interest, @ 3 % per annum, is to be added to the principal every 30th of Nov., and 31st of May. How much is at his credit on May 31st, 1900?

4. The Simple Interest of a sum of money for 3 yr., @ 5 %, is $126. What is the Compound Interest of the same sum for the same time and rate?

5. A sum of money lent at Simple Interest for $2\frac{1}{2}$ yr., @ 6 % per annum, amounted to $1150. To what would it have amounted if it had been lent at Compound Interest?

6. Find the Compound Interest on $675.75, for $3\frac{1}{2}$ yr., @ 6 % per annum.

7. A money dealer borrowed $1000, for 2 yr., @ 6 % interest, and loaned the same in such a manner as to *compound* the interest every 6 mo. What profit did he make in 2 yr. by this proceeding?

8. Find the difference in Compound Interest on £5000, for 2 yr., @ 4 %, according as it is reckoned yearly or half-yearly.

9. What is the difference between the Compound Interest on $40000, for 4 yr., and on $80000, for 2 yr., the rate in both cases being 5 %?

10. *A* and *B* lend each $248, for 3 yr., @ $3\frac{1}{2}$ %, one at Simple, the other at Compound Interest. Find the difference of the amount of interest which they respectively receive.

11. What sum @ 4 %, Compound Interest, will amount, in $2\frac{1}{2}$ yr., to $16989.7728?

12. What sum will amount to $27783, in 3 yr., @ 5 %, Compound Interest?

13. The difference between the Simple and the Compound Interest on a sum of money, for 2 yr., @ 5 %, is $3. Find the sum.

14. The Compound Interest on a sum of money during the 2nd yr. was $36.40, and during the 3rd yr. it was $37.856. Find the rate per cent.

15. The Compound Interest of a certain sum for the 2nd yr. is $41.60, and for the 3rd yr. it is $43.264. Find the sum and the rate per cent.

16. The Compound Interest of a sum for the 4th yr. is $2.205 greater than for the 3rd, and for the third it is $2.10 greater than for the 2nd. For the 1st year the interest is $40. Find the sum and the rate per cent.

17. What rate per cent. per annum, compounded yearly, is equivalent to $3\frac{1}{2}$ % per half-year, compounded half-yearly?

18. What rate per cent., payable half-yearly, is equivalent to 10 %, payable annually?

19. If at Compound Interest $5 be allowed off a bill of $125, due a certain time hence, what should be the discount allowed off, if the bill had twice as long to run?

20. If at Compound Interest $98 were accepted as present payment of $128, due a certain time hence, what should be a proper discount off a bill of $128 which has only half the time to run?

21. A debt of $648.27 is due in 3 yr. What sum is the debt worth now, money being worth 5 %, compounded annually?

22. A debt of $1323, bearing interest at the rate of 5 % per annum, due in two years, can be discharged by paying what sum now, money being worth 10 %, interest compounded half-yearly?

Equation of Payments.

180. When several sums of money are due from *A* to *B*, payable at different times, it is often required to find the time, called the **Equated Time,** at which all may be paid together, without injustice to *A* or *B*.

When great exactness is demanded, interest must be added to the sums paid after they are due, and discount subtracted from the sums paid before they are due. But in practice the following Rule is sufficiently accurate:—

Multiply each debt by the number of days (or months) after which it is due. Add the results together. Divide this sum by the sum of the debts. The quotient will be the number of days (or months) in the equated time.

Take the following examples:—

Ex. 1. If \$300 be due from *A* to *B* at the end of 5 months, and \$700 at the end of 9 months, when may both sums be paid in a single payment without unfairness to *A* or to *B*?

$$\begin{aligned}\text{Number of months in equated time} &= \frac{300\times5+700\times9}{300+700}\\ &= \frac{7800}{1000}\\ &= 7\tfrac{4}{5}\end{aligned}$$

∴ the whole amount of the debt should be paid at the end of $7\frac{4}{5}$ mo.

The principle on which this solution depends is, that the interest of the money, the payment of which is delayed beyond the time at which it is due, is equal to the interest of that which is to be paid before it becomes due.

In the above example \$300 is kept $2\frac{4}{5}$ months after it is due, and the interest on it for that time is the same as the interest on \$840, $\$(300 \times 2\frac{4}{5})$, for one month.

But \$700 is paid $1\frac{1}{5}$ months before it is due, and the interest on it for that time is the same as the interest on \$840, $\$(700 \times 1\frac{1}{5})$ for one month.

Ex. 2. *A* is indebted to *B* in the following amounts: \$500, due in 6 months; \$600, due in 7 months; \$800, due in 10 months. Find the time when all these payments should be made together.

$$\begin{array}{rcrcr} 500 & \times & 6 & = & 3000 \\ 600 & \times & 7 & = & 4200 \\ 800 & \times & 10 & = & 8000 \\ \hline 1900 & & 1900\,) & & 15200 \\ \cline{5-5} & & & & 8 \end{array}$$

∴ the equated time = 8 mo.

Note.—This method is but a rough approximation, and can only be taken as equitable when the various times of payment are not widely apart. It will, in short, be applicable only to cases which occur in the ordinary course of trade, and is therefore all that we require in the present work.

It is also to be observed that the error involved in this method is *slightly in favor of the payer*, because interest is calculated on the payments made before they are due, instead of discount, in the algebraical process from which the method is derived. See Chapter XIX.

Examples xcii.

1. What is the equated time of $250 due 4 mo. hence, and $350 due 10 mo. hence?

2. Find the equated time of $300 due in 3 mo. hence, $400 due in 4 mo. hence, and $500 due in 6 mo. hence.

3. On Jan. 15, I bought a bill of goods amounting to $900, $275 of which was on 30 da. credit, $300 on 60 da., and $325 on 90 da. On what date should the debt be discharged by one payment?

4. A debt of $2400 was contracted on March 6, 1896, payable in 8 mo., but $400 was paid in 2 mo., $600 in 5 mo., $800 in 7 mo. What was the equitable time for paying the balance?

5. The sum of $1200 is due in 14 mo. If $\frac{1}{3}$ of the sum be paid in 9 mo., and $\frac{1}{5}$ of it in 13 mo., in what time ought the remainder to be paid?

6. One-half of a debt of $1000 is due in 10 mo., $\frac{1}{4}$ of it in 12 mo., $\frac{1}{5}$ in 16 mo., and the remainder in 20 mo. When might the whole be paid at one payment?

7. A debt is due in 12 mo. hence, but $\frac{1}{9}$ of it is paid in 6 mo., and $\frac{1}{18}$ in 9 mo. When should the remainder be paid?

8. Of a debt of $1400, $100 is due immediately, $600 at the end of 1 mo., $400 at the end of 7 mo., and the remainder at the end of a year. At what time might the whole debt fairly be paid in one sum?

9. A grocer ought to receive from a customer $50 at the end of 2 mo., $30 at the end of 4 mo., and $20 at the end of $6\frac{1}{2}$ mo. What would be the proper time for receiving the whole sum together?

10. A debt is to be paid as follows: One-sixth now, and one-sixth every three months, until the whole is paid. When might the whole debt be paid at once?

11. If $450 be due in 16 mo., and $250 be due in 13½ mo., find the sum which, if paid now, would be equivalent to the whole debt at the equated time, interest @ 4 %.

12. There is due to a merchant $800, one-sixth of which is to be paid in 2 mo., one-third in 3 mo., and the remainder in 6 mo.; but the debtor agrees to pay one-half *down*. How long may he retain the other half so that neither party may sustain loss?

13. *A* sold goods to *B* at sundry times, and on different terms of credit, as follows: Sept. 30, 1896, $80.75, on 4 mo. credit. Nov. 3, 1896, $150, on 5 mo. credit. Jan. 1, 1897, $30.80, on 6 mo. credit. March 10, 1897, $40.50, on 5 mo. credit. April 25, 1897, $60.30, on 4 mo. credit. How much will balance the account June 3, 1897?

14. *A* owes *B* on the 1st of March the following sums: £140 due on 20th of April. £120 due on the 14th of May. £380 due on 15th of June. On what day may *B* pay these debts together?

15. *M* buys goods of *N*, and has 6 mo. credit from the date of invoice. The goods are delivered on 6 different days, to the following amount: £144 2s. 10d., on Sept. 5. £303 18s. 10d., on Sept. 18. £757 0s. 8d., on Nov. 13. £123 11s. 6d., on Nov. 28. £123 11s. 6d., on Dec. 5. On the 13th January, *N*, who desires to receive all the debts in one payment, reckons that this payment should be made in 100 days. Show that this is approximately correct?

Equation of Accounts.

181. Equation of Accounts (also called "Averaging of Accounts" and "Compound Equation of Payments"), is the process of finding at what time the *balance of an account* can be paid without gain or loss to either party.

The **Balance of an Account** is the difference between the two sides of it, and is what one owes the other.

Ex. T. Black's Account in our Ledger.

DR. T. BLACK. CR.

1900.			1900.		
Jan. 1	To Mdse	$500.00	Feb. 10	By Cash	$1000.00
Feb. 4	" "	600.00	Mar. 4	" "	600.00
Mar. 10	" "	800.00			

Take the latest date, Mar. 10, as focal date.

Due Dates.	Days.		When Paid.	Days.	
Jan. 1	500 × 68 =	34000	Feb. 10	1000 × 28 =	28000
Feb. 4	600 × 34 =	20400	Mar. 4	600 × 6 =	3600
Mar. 10	800 × 0 =	0			
	1900	54400		1600	31600
	1600	31600			
Bal. of ac.	300	22800	Bal. of products.		

Number of days = 22800 ÷ 300 = 76.
Count 76 days backward from Mar. 10.
∴ $300 is due Dec. 24, 1899.

The debit side shows the amount Black has received.
The credit side shows the amount Black has given.
Suppose the account to be settled on the the latest date, March 10.

Dr.

Then, Interest on — Interest on
$500 for 68 da. = $1 for 500 × 68 da., or 24000 days.
600 “ 34 “ = 1 “ 600 × 34 “ “ 20400 “
800 “ 0 “ = 1 “ 500 × 0 “ “ 0 “
$1900 = 1 for 54400 “

Cr.

Interest on — Interest on
$1000 for 28 da. = $1 for 1000 × 28 da., or 28000 da.
600 “ 6 da. = 1 “ 600 × 6 “ “ 3600 “
$1600 = $1 for 31600 da.

∴ Black receives $1900 and the int. on $ 1 for 54400 da.
“ gives $1600 “ “ “ “ $ 1 “ 31600 “
“ owes $300 “ “ “ “ $ 1 “ 22800 “
or “ “ 300 “ “ “ “ $300 “ $\frac{22800}{300}$ “, or 76 da.
∴ $300 is due 76 days before March 10, or Dec. 24.

i. *Find the date when each item is due or paid on both sides.*

ii. *Take the latest due date on either side, thus found, for the focal date. Multiply each item on both sides of the* *account by the number of days between the focal date and the date of the item.*

iii. *Add the products on each side, and subtract the sum of those on the one side from the sum of those on the other, and divide the difference by the balance of the account. The quotient is the number of days to be counted forward from the focal date, when the balance of the products and the balance of the account are on opposite sides, and backward from the focal date when they come on the same side.*

Examples xciii.

1. In the following account it is required to find the balance, and when it is due.

Dr. James Adamson. Cr.

1900.		$	1900.		$
March 9	To merchandise..	300	Mar. 20	By cash..........	247
May 12	" " ..	474	April 11	" draft at 30 da....	400
June 19	" " ..	564	July 10	" cash..........	260

2. Find the equitable balance of the following account:—

Dr. John Jones in account with Wm. Smith. Cr.

1900.		$	1900.		$
Oct. 2	To merchandise	180	Nov. 18	By mdse. 2 mo.	150
Nov. 8	" " 3 mo.	120	Dec. 24	" cash.......	200
Dec. 16	" " 4 mo.	240			

3. Find the equated time for paying the balance of the following account:—

Dr. Scott, Hughes & Co. Cr.

1899.			1899.		
Jan. 29	Mdse., 2 m.	$519.00	Jan. 2	Mdse., 3 m.	$357.00
Feb. 5	Mdse., 3 m.	423.00	" 25	Mdse., 2 m.	738.00
" 19	Mdse., 2 m.	969.00	Feb. 4	Mdse., 3 m.	421.00

4. Average the following account:—

Dr. J. Hughes in account with S. Adams. Cr.

1895.			1895.		
July 4	To Balance.	$375.90	Aug. 10	By Cash..	$316.00
Aug. 20	“ Mdse...	815.58	Sept. 1	“ “ ..	675.00
Aug. 29	“ “ ...	178.25	Sept. 25	“ Mdse..	512.25
Sept. 25	“ “ ...	387.20	Nov. 20	“ Cash..	161.75
Dec. 5	“ “ ...	418.70	Dec. 1	“ “ ..	100.00

5. When is the balance of the following account due?

Dr. A. B. Conron. Cr.

1897.			1897.		
Sept. 12	To Mdse. @ 30 da.	$927.30	Oct. 10	By Cash.	$500.00
Oct. 15	“ “ @ 30 “.	342.75	Nov. 20	“ “ .	300.00
Nov. 18	“ “ @ 60 “.	212.13	Nov. 30	“ “ .	250.00
Dec. 1	“ “ @ 30 “.	175.50	......	....	....

6. When did the balance of the following accounts become due, the merchandise items being on 6 months?

Dr. J. Green in account with Adam Miller & Co. Cr.

1896.			1896.		
Mar. 1	To Mdse..	$720.75	April 1	By Cash...	$700.00
“ 20	“ “ ..	815.30	May 30	“ Mdse..	569.89
April 11	“ “ ..	587.80	July 20	“ Cash..	500.00
“ 30	“ “ ..	300.00	Sept. 25	“ “ ..	100.00
June 15	“ “ ..	625.25	“ 30	“ Mdse..	750.20
July 18	“ “ ..	560.00	Oct. 30	“ “ ..	329.06
Aug. 30	“ “ ..	684.90	Nov. 20	“ “ ..	500.00
Sept. 25	“ “ ..	365.30	....	....	....

EXAMINATION PAPERS.

I.

1. What is the difference between the true and the bank discount of $950 for 3 mo. @ 7 %?

2. A bill is drawn for $722.70 on July 17 @ 2 mo., and discounted on Aug. 11 @ 7½ %. How much did the holder receive?

3. Find the discount charged in discounting a bill for $7850 drawn April 9 @ 7 mo., and discounted June 19th @ 10 %.

4. For what sum must a note be drawn on July 3 @ 3 mo., so that discounted immediately it may produce $501.69, money being worth 7 %?

5. Find the difference between the true and bank discounts on $5555 at 6 % for 1 yr.?

II.

1. Explain the difference between Simple and Compound Interest. Find the interest on $25000 for 3 yr. @ 4 %, supposing interest to make capital at the end of each year.

2. The difference between the Compound and Simple Interest of a certain sum of money for 3 yr. @ 4 % is $3.80. Find the sum.

3. Find at what rate Simple Interest in two years a sum of money would amount to the same sum as @ 4 %, Compound Interest.

4. Find the Compound Interest on $1000 @ 3 % per annum for 2 yr. and 195 da.

5. A person puts out to interest $8000 @ 4 %. He spends annually $300, and adds the remainder of his dividend to his stock. What is he worth at the end of 5 yr.?

III.

1. Explain the distinction between true discount and bank discount. Does the creditor or debtor gain by computing the interest instead of the discount?

2. Find the discount on $400, due 1 yr. hence, if money bear interest @ 5 % per annum. Calculate the interest on this discount for the same time, and show that it is equal to the difference between the interest and the discount of $400.

3. If £10 be the interest on £110 for a given time, what should be the discount off £110 for the same time?

4. What must be the rate of interest in order that the discount on $10292 payable at the end of 1 yr. 73 da. may be $372?

5. A tradesman who is ready to allow 5 % per annum Compound Interest for ready money, is asked to give credit for 2 yr. If he charge $110.25 in his bill, what ought the ready money price to have been?

IV.

1. A speculator borrowed $5000, which he immediately invested in land. Six months afterwards he sold the land for $7500, on a credit of 12 mo., with interest. Money being @ 6 %, what is the speculator's profit at the end of the 12 mo. credit, at which time he returns the $5000?

2. A merchant bought 43 cwt. 3 qr. of sugar @ $5.25 per cwt., which he immediately sold at $7 per cwt. on a credit of 90 da., and then had the purchaser's note for the amount discounted in the bank @ 6 %. What profit did the merchant make?

3. Find the Present Worth of $1000 due 2½ yr. hence @ 5 % per annum. Show that the Discount of the given sum is equal to the interest of the Present Worth for the same time and at the same rate of interest?

4. A man having lent $10000 @ 5 % interest, payable half-yearly, wishes to receive his interest in equal parts monthly, and in advance. How much ought he to receive every month?

5. Show that the interest on £266 13s. 4d. for 3 mo. @ 4½ % per annum is equal to the discount off £83 for 15 mo. @ 3 % per annum.

V.

1. How much may be gained by hiring money @ 5 % to pay a debt of $6400, due in 8 mo., allowing the present worth of this debt to be reckoned by deducting 5 % per annum discount?

2. The difference between the Simple and Compound interests of a sum of money for 3 yr. @ 8 % is $985.60. What is the sum?

3. The interest on a certain sum of money for 2 yr. is £71 16s. 7½d., and the discount on the same sum for the same time is £63 17s., simple interest being reckoned in both cases. Find the rate per cent. per annum, and the sum.

4. *A* offers $8000 for a farm. *B* offers $9500, to be paid at the end of 4 yr. Which is now the better offer, and by how much, allowing 5 %, Compound Interest?

5. A person borrows money at 6 % per annum, and pays the interest at the end of the year. He lends it out @ 8 % per annum, payable quarterly, and receives the interest at the end of the year. By this means he gains $269.18592 a year. How much did he borrow?

For additional examples, see page 317.

Stocks and Shares.

182. The Government of a country, the authorities of a city, etc., often find it necessary to borrow money to carry on public works, etc. A loan is then contracted, and the borrower pledges the credit of the country, city, etc., to pay a fixed rate of interest on the sum borrowed until the debt is paid off.

The term **Stock** is applied to any such government loan. It also denotes the capital of a joint-stock company.

Banks, Railway Companies, and others have their capital divided into *shares* of so many dollars each, usually $50 or $100.

The price of stock is always quoted at so many dollars for $100 stock. Thus, when we read that the stock of the Toronto Bank is at 245, it means that $245 of money will purchase $100 stock in that bank.

The price of stock is always fluctuating, owing to a change in the value of money, *i.e.*, at times money is scarce and consequently in large demand, and hence the rate of interest will be high. At other times it is plentiful, and therefore cheap. Thus, if *A* has money to loan, and can get 5 per cent. for it, he will not invest it in the Dominion stock, which pays 3½ per cent., unless the latter is so cheap that he can make 5 per cent., *i.e.*, unless he can buy at 70. Hence, if *B* wished to sell Dominion 3½ per cent. stock he would have to sell it at a *discount*.

Again, if money could only be loaned at 3 per cent., *B* would be able to sell $100 of such stock for more than $100 money. In this case he would sell at a *Premium*. Among the other causes which determine the value of stock, we may mention its desirability of a safe investment, commercial and political changes at home and abroad, etc.

183. Stock is at **Par** when it sells for its *nominal* value, as when $100 stock sells for $100 money.

It is at a **Premium** when it sells for more than its *nominal* value. Thus, when $100 stock sells for $109 money, it is at a *Premium* of 9 per cent.

It is at a **Discount** when it sells at less than its nominal value. Thus, when $100 stock sells for $85 money, it is at a *discount* of 15 per cent.

The purchase and sale of stocks are usually effected by means of a stock-broker, who is paid a certain percentage on all *stock* that passes through his hands. Thus, if stock is at 92½, and the broker charges ½ per cent., the buyer will have to pay $93 ($92½ + $½) for $100 stock, and the seller would receive $92 ($92½ − $½) for it.

184. Stock is often named from the interest which is is paid to the owners of the stock. Thus, the Dominion Government stock, paying interest at the rate of 4 per cent., is spoken of as the Dominion 4 per cents., or Dominion 4's.

Consols are a part of the National Debt of Great Britain, so called from the Consolidation of the stock of various annuities into a joint 3 per cent. stock.

The National Debt of Great Britain, which now amounts to about 640 millions, has been incurred by loans made to the State by individuals. Interest is paid upon the main part of this debt at the rate of 3 per cent. The names of the persons who have a claim on the nation for such interest, are registered in books kept by the Bank of England on behalf of the Government. Such persons are called *Fundholders*. The debt itself is often called *The Funds;* and the interest, which is payable half-yearly, is called *Dividends*.

Suppose *A* to be a Fundholder in that particular part of the National Debt called *The Three per Cent. Consols*, and suppose the amount of the debt, which he is acknowledged by the Register to hold, be £5000, he is then said to hold £5000 stock. *A* cannot demand the payment of 5000 sovereigns, or any smaller sum, from the Government, as a redemption of the debt, but the Government undertakes to pay him (or any one to whom he may assign his claim) 75 sovereigns, every half-year, that being the amount of interest on £5000 for half a year at 3 per cent.

Now suppose *A* to be desirous of selling his claim to *B*. The value of the claim does not vary much from time to time in the case before us, for England is known to be willing and is acknowledged to be able to pay the interest on her debt, and the security of the claim makes the Fundholder satisfied with a low rate of interest, punctually paid and easily obtained. The value of £100 Stock in Consols is at the present time (Aug. 1st, 1900) 97¼, that is, *A* can obtain £97¼ for each £100 Stock that he holds, and *B*, on the payment of 50 × £97¼, or £4862 10s., can have the £5000 Stock, now held by *A*, transferred to him.

A's name is then removed from the Register, and *B*'s name is inserted in it, and the process is called a *Transfer*.. *A* is said to *sell out* of the Funds, and *B* is said to *invest* in them.

185. Currency is a term used in commercial language,

First, to denote the aggregate of Specie, Bills of Exchange, Bank Bills, Treasury notes, and other substitutes for money employed in buying, selling, and carrying on exchange of commodities between various countries.

Second, to denote whatever circulating medium is used in any country as a substitute for the Government standard. It sometimes happens that the paper currency of a country becomes depreciated in value. Thus, when we read in Stock quotations of buying at 94¾ and selling at 95½, it is meant that a broker would give $94¾ gold for $100 of paper currency, and that he would sell $100 of paper currency for $95½ gold. Also, when we read that gold is 105¼, it is meant that the paper currency is taken as the standard for the time being, and $105¼ of such currency would be given for $100 gold.

186. In Canada the liability on all Bank Stocks is limited to double the amount of the subscribed capital. On all other stocks the liability of shareholders is strictly limited to the amount of the subscribed capital.

When all the Capital of a company has been paid up, it is often changed from Shares to Stock, because in the case of Stock, transactions can be carried on with reference to *any portions of it*, whereas in the case of Shares, fractional parts of those Shares cannot be transferred.

Three points must now be clearly marked:

i. We shall know the amount of money received by *A* for any given amount of stock, if we know the price of the stock at the time of sale.

ii. We shall know how much stock can be bought by *B* for any given amount of money, if we know the price of the stock at the time of sale.

iii. We shall know the amount of income received by *A* (and subsequently by *B*) on any given amount of stock, if we know the rate of interest payable on the stock. The income depending in no way on the price of the stock.

These three cases we now proceed to illustrate:

Ex. 1. What is the value of $2500 stock in the Dominion 4's at 98¼?

The value of $100 stock = $98.25;
∴ " " " $1 stock = $\$\frac{98.25}{100}$;
∴ " " " $2500 stock = $\$\frac{2500 \times 98.25}{100}$;
= $2456.25.

Ex. 2. How much stock can be purchased at 92½ for $740?

For $92.50 I can purchase $100 stock;
for $1 " " " $\$\frac{100}{92.50}$;
for $740 " " " $\$\frac{740 \times 100}{92.50}$, or $800 stock.

Ex. 3. What annual income is derived from investing $3920 in the 6 per cents. at $98?

Here, the owner of $100 stock has an income of $6, and to purchase this stock he must pay $98.

∴ $98 gives an income of $6;
∴ $1 " " " " $\$\frac{6}{98}$;
∴ $3920 " " " " $\$\frac{3920 \times 6}{98}$, or $240.

Ex. 4. What sum must be invested in the Dominion $3\frac{1}{2}$'s at 95 so that I may have an annual income of $1400?

Since $3.50 is got from investing $95,
∴ $1 " " " " $\$\frac{95}{3.50}$;
∴ $1400 " " " " $\$\frac{1400\times95}{3.50}$, or $38000.

Ex. 5. What annual income is derived from $3550 stock in the U.S. 4's?

Income on $100 stock = $4;
" " $1 " = $\$\frac{4}{100}$;
" " $3550 " = $\$\frac{3550\times4}{100}$, or $142.00.

This is merely a case of finding the Interest, where the stock is the Principal.

Ex. 6. Bought stock in the Bank of Commerce at 140. The last dividend was at 7 per cent. What per cent. did I make on the investment?

$140 gives an income of $7;
∴ $1 " " " " $\$\frac{7}{140}$;
∴ $100 " " " " $\$\frac{100\times7}{140}$, or $5.
∴ the per cent. required = 5.

Ex. 7. When stock is at 84, how much stock must be sold to raise $462?

Since $84 is got from selling $100 stock;
∴ $1 " " " " $\$\frac{100}{84}$ stock;
∴ $462 " " " " $\$\frac{462\times100}{84}$ stock,
or $550 stock.

Ex. 8. What is the price of Ontario Bank stock when $6000 stock produces $7620?

Since $6000 stock is worth $7620;
∴ $1 " " " $\$\frac{7620}{6000}$;
∴ $100 " " " $\$\frac{100\times7620}{6000}$, or $127;
∴ the stock was selling at 127.

Ex. 9. By investing in the Dominion $3\frac{1}{2}$'s I make 4 per cent. What is the selling price of this stock?

Since \$4 is got from investing \$100,

∴ \$1 “ “ “ “ $\$\frac{100}{4}$;

∴ $\$3\frac{1}{2}$ “ “ “ “ $\$\frac{3\frac{1}{2}\times 100}{4}$, or $\$87\frac{1}{2}$;

∴ the selling price is $87\frac{1}{2}$.

Ex. 10. Which is the more advantageous stock to invest in, 6 per cents. at 95, or 5 per cents. at $87\frac{1}{2}$, and how much per cent. is it better.

Income for \$95 in the 6 % = \$6;
∴ Income for \$1 in the 6 % = $\$\frac{6}{95}$.

Income for \$1 in the 5 % = $\$\frac{5}{87\frac{1}{2}}$, or $\$\frac{10}{175}$.

We have now to compare the fractions $\frac{6}{95}$ and $\frac{10}{175}$.

Reduced to a common denominator these become $\frac{222}{3515}$ and $\frac{190}{3515}$;

∴ Income for \$1 in the 6 % = $(\frac{222}{3515} - \frac{190}{3515})$ of \$1 better than in the 5 %.

∴ Income for \$100 in the 6 % = $100 \times (\frac{222}{3515} - \frac{190}{3515})$ of \$1 better than in the 5 %;

Now $100 \times \frac{222}{3515} - \frac{190}{3515} = .91$ . . . % required.

Ex. 11. A person transfers £5000 stock from a 3 per cent. stock at 72, and invests the proceeds in a 4 per cent. stock at 90. Find the difference in his income.

First, he sells £5000 stock at 72, and gets £(72 × 50) or £3600.
Then he invests £3600 in the 4 per cent. stock at 90, and buys £$\frac{3600\times 100}{90}$ stock, or £4000 stock.

Now his *first* income on the £5000 stock was £$\frac{5000\times 3}{100}$, or £150.

And his *second* income on the £4000 stock = £$\frac{4000\times 4}{100}$, or £160.

∴ he increases his income by £10.

Ex. 12. A person invests £1075 10s. in Consols when they are at $89\frac{1}{2}$, and sells out when they are at $93\frac{3}{8}$. What is his gain, brokerage at $\frac{1}{8}$ per cent. on each transaction?

Here an annuity which costs £$(89\frac{1}{2} + \frac{1}{8})$ is sold for £$(93\frac{3}{8} - \frac{1}{8})$.

∴ the gain on £$89\frac{5}{8}$ = £$3\frac{5}{8}$;
∴ the gain on £1 = £$\dfrac{3\frac{5}{8}}{89\frac{5}{8}}$, or £$\frac{29}{717}$;
∴ the gain on £1075 10s. = £$(1075.5 \times \frac{29}{717})$, or £43 10s.

Ex. 13. A person invested in Bank stock at $89\frac{3}{4}$, and sold out at $103\frac{1}{2}$, and cleared \$397.50. How much did he invest, brokerage being $\frac{1}{4}$ per cent. on each transaction?

Here what cost \$90 is sold for \$$103\frac{1}{4}$;
∴ he gained \$13.25 by investing \$90;
∴ he gained \$1 by investing \$$\frac{90}{13.25}$;
∴ he gained \$397.50 by investing \$$\frac{397.50 \times 90}{13.25}$, or \$2700.

Ex. 14. A person having to pay \$$3606\frac{2}{33}$ two years hence, invested a certain sum in the Toronto 6 per cent. city bonds, to accumulate interest until the debt be paid, and also an equal sum next year. Supposing the investments to be made when the stock was at 99, and the first year's interest also invested in stock, and the price to remain the same, what must be the sum invested on each occasion that there may be just sufficient to pay the debt at the proper time?

Every \$99 invested will give \$6 interest;
∴ every \$1 invested will give \$$\frac{6}{99}$ interest;
∴ \$ sum invested will give \$ (sum × $\frac{6}{99}$) interest.

Now \$(sum × $\frac{6}{99}$) invested will give \$(sum × $\frac{6}{99}$ × $\frac{6}{99}$) interest.

Hence, at the end of the second year there were on hand the two sums invested.

Two years' interest on the first investment = 2 × sum × $\frac{6}{99}$;
One year's interest on the second investment = sum × $\frac{6}{99}$;
And the interest on the first year's interest = sum × $\frac{6}{99}$ × $\frac{6}{99}$.

Or 2 sums + 3 × sum × $\frac{6}{99}$ + sum × $\frac{6}{99}$ × $\frac{6}{99}$ to meet \$$3606\frac{2}{33}$;

∴ $(2 + \frac{18}{99} + \frac{36}{9801})$ sum = \$$3606\frac{2}{33}$;

∴ sum = \$$\dfrac{3606\frac{2}{33}}{\frac{21420}{9810}}$ = \$1650.

Examples xciv.

1. Find the value of

(*a*) \$7645 stock in the 6 per cents. @ 112.
(*b*) \$9800 stock in the 5 per cents. @ 95.
(*c*) \$7650 stock in the 7 per cents. @ $146\frac{1}{2}$.
(*d*) £3850 stock in the 3 per cents. @ 92.
(*e*) £572 10s. stock in the 3 per cents. @ $91\frac{1}{2}$.

2. How much stock will

(*a*) \$8400 buy in the 3 per cents. @ 75?
(*b*) \$5049 buy in the 8 per cents. @ 187?
(*c*) \$994.50 buy in the 7 per cents. @ 117?
(*d*) £2199 buy in the 3 per cents. @ $91\frac{5}{8}$?
(*e*) £5527 10s. buy in the 3 per cents. @ $92\frac{1}{8}$?

3. What income is got from investing

(*a*) \$1127 in 6 % stock @ 115?
(*b*) \$4147 in 3 % stock @ $72\frac{1}{2}$?
(*c*) \$6720 in $5\frac{1}{2}$ % stock @ 96?
(*d*) \$3725 in 3 % stock @ $74\frac{1}{2}$?
(*e*) £8475 10s. in 3 % stock @ $92\frac{1}{8}$?

4. What amount of stock must be sold

(*a*) In the 8 per cents @ 125 to produce \$750?
(*b*) In the Dominion $3\frac{1}{2}$'s @ $92\frac{1}{2}$ to produce \$629?
(*c*) In the 6 per cents @ 118 to produce \$649?
(*d*) In the $7\frac{1}{2}$ per cents @ 128 to produce \$4096?

5. What per cent is made by investing in the

(*a*) 8 per cents. @ 120?
(*b*) 5 per cents. @ 95?
(*c*) 6 per cents. @ 104?
(*d*) $3\frac{1}{2}$ per cents. @ 75?

6. Find the rate of dividend paid when the income from

(*a*) \$24000 stock = \$2000.
(*b*) \$3600 stock = \$261.
(*c*) \$5400 stock = \$396.

7. What is the price of stock when

(*a*) 7 % stock pays 5 % on the investment?
(*b*) $3\frac{1}{2}$ % stock pays 5 % on the investment?
(*c*) 12 % stock pays $4\frac{4}{9}$ % on the investment?

8. What sum must be invested in the

(*a*) 8 per cents. @ 120 so as to produce an income of \$640?
(*b*) 5 per cents. @ 90 so as to produce an income of \$3750?
(*c*) $4\frac{1}{2}$ per cents. @ 67 so as to produce an income of \$2790?

9. What is the selling price of stock when
   (*a*) \$550 stock in the 6 per cents. produce \$558.25 ?
   (*b*) \$7840 stock in the 4 per cents. produce \$6664 ?
   (*c*) £840 stock in the 3 per cents. produce £773 17s.

10. What must I pay for 5 per cents. that my investment may yield 6 % ?

11. Which is the better investment, the buying of 9 % stocks at 25 % advance, or 6 % stocks at 25 % discount, and how much per cent. better ?

12. The difference between the incomes derived from investing a certain sum in 6 % stock at 126, and in 9 % stock @ 210, is £22 10s. What is the amount invested ?

13. A bank declared a dividend of 3 % for the quarter. How much should a stockholder owing 75 shares (\$200) receive.

14. Sold 48 shares (\$40) Western Assurance stock at 259¾. What did I receive for them, brokerage ⅛ % ?

15. Sold 64 shares (\$50) of Consumers' Gas Company stock, receiving for them \$7132. How was the stock quoted, brokerage being ⅛ %.

16. I sell out of the 3 per cents. @ 96, and invest the proceeds in Railway 5 % stock @ par. Find by how much per cent. my income has increased.

17. If a 3½ % stock be @ 91, how much must I invest in it so as to have a yearly income of £952, after paying 7d. in the pound income-tax ?

18. By selling out £4500 in the India 5 % stock @ 112½, and investing the proceeds in Egyptian 7 % stock, a person finds his income increased by £168 15s. What is the price of the latter stock ?

19. Find the alteration in income occasioned by shifting £3200 stock from the 3 per cent. @ 86⅜ to 4 % stock @ 114⅞, the brokerage being ⅛ %.

20. *A* owns a farm which rents for \$411.45 per annum. If he sells the farm for \$8229, and invest the proceeds in 6 % stock @ 105, paying ½ % brokerage, will his yearly income be increased or diminished, and how much ?

21. Through a broker I invested a certain sum of money in 6 % stock @ 107½, and twice as much in 5 % stock @ 98½, brokerage in each case ½ %. My income from both investments was \$1674. How much did I invest in each kind of stock ?

22. How much stock @ 15 % discount, must be bought and sold at 11 % discount to make a clear gain of $300, brokerage on each transaction ⅛ %?

23. I invest in the 3 per cents. @ 92. They fall to 85, and I sell out and obtain a safe investment paying 5 %, but not subject to fluctuation of value. How long must I hold it before I shall make a profit by the change, in case 3 per cents. rose to the former value?

24. I own $4000 Montreal Bank Stock paying an annual dividend of 10 %. I sell @ 250, and invest in Toronto Gas Company stock @ 225, and receive an annual dividend of 9 %. What change is made in my income, brokerage being ⅖ % and ⅗ % on the respective transactions?

25. A person bought stock @ 95¼, and after receiving the half-yearly dividend at the rate of 7 % per annum, sold out @ 92⅜ and made a profit of $37.50. How much stock did he buy?

26. Whether is it better to invest in the 6 per cents. @ 111½ or in the 5 per cents. @ 94½, brokerage being ½ %?

27. What sum must a man invest in 6 % stocks @ 120, in order to have a clear income of $1977.25, after paying an income-tax of 1¾c. on the $ on all over $700?

28. A gentleman has been receiving 12 % on his capital in Canada. He goes to England to reside, and invests in the 3 per cents. @ 94⅜, and his income in England is £2400. What was his income in Canada, £1 being equal to $4.86⅔?

29. By selling out £4500 in the India 5 % stock @ 112¾, and investing the proceeds in Egyptian 7 % stock, *A* finds his income increased by £168 15s. What was the price of the latter stock, brokerage on each transaction being ¼ %?

30. The 6 per cents. are @ 118 and the 7 per cents. @ 130. A person has a sum of money to invest which will give him $600 more of the former stock than of the latter. Find the difference of income he could obtain by investing in the two stocks.

31. One company guarantees to pay 5 % on shares of $100 each. Another guarantees at the rate of 4⅚ % on shares of $30 each. The price of the former is $124½, and of the latter $34 each. Compare the rates of interest which the shares return to the purchasers.

32. The present income of a railway company would justify a dividend of 3¾ % if there were no preference shares. But as $1200000 of the stock consists of such shares, which are

guaranteed 5 % per annum, the ordinary shareholders receive only 3 %. What is the whole amount of stock?

33. *A* invested a certain sum in 6 % stock @ $119\frac{7}{8}$, brokerage $\frac{1}{8}$, and half as much in 4 % stock @ $79\frac{7}{8}$, brokerage $\frac{1}{8}$. His income from both investments was $450. How much did he invest in each kind of stock?

34. The difference between the annual income derived from investing a certain sum in 10 % stock @ $249\frac{7}{8}$ and that from investing the same sum in 12 % stock @ $268\frac{5}{8}$ is $100. What is the amount invested in each kind of stock, brokerage in each case being $\frac{1}{8}$ %?

35. *A* received $1092 as dividend @ $5\frac{1}{4}$ % on his bank stock. He sold 250 shares ($50) at $114\frac{5}{8}$, and the remainder at $117\frac{3}{8}$, brokerage $\frac{1}{8}$ on each sale. How much did he receive from the sale?

EXAMINATION PAPERS.

I.

1. In a sale of goods for $728 there is a loss of 9 %. For what must 3 times the quantity be sold in order to gain 7 %?

2. If 20 % be gained by selling an article for $2.10, what is the gain or loss per cent. when it is sold for $1.60?

3. A grocer had 150 lb. of tea, of which he sold 50 lb. @ $1.80 per lb., and found he was gaining only $7\frac{1}{2}$ %, but he wished to gain 10 % on the whole. At what rate must the remaining 100 lb. be sold that he may attain his wishes?

4. A tradesman adds 35 % to the cost price of his goods, and gives his customers a reduction of 10 % on their bills. What profit does he make?

5. A bill of $2520 due a year hence can be taken up now @ 5 % discount. Supposing a tradesman can employ his capital so as to obtain interest at the end of every quarter at the rate of $4\frac{1}{2}$ % per annum, had he better so employ it or take up the bill? What will be the difference to him?

II.

1. A tradesman marks his goods with two prices, one for ready money and the other for one year's credit, allowing discount @ 5 %. If the credit price be marked $2.45, what ought the cash price to be?

2. Goods are sold on condition to allow 10 % discount, if payment be made at the end of six months; what discount ought

to be allowed, if payment be actually made (1) three months *before*, and (2) three months *after* the stated time, if money bear interest @ 5 % per annum ?

3. A person purchases goods at $1.20 per lb. Troy weight, and sells them again by Avoirdupois weight. At what rate per ounce must he sell so as exactly to reimburse his outlay ?

4. What is meant when it is said that Consols are at $98\frac{1}{4}$? What are they at when £9000 is paid for £10000 Consols ?

5. A person sells $1200 stock in the 3 per cents. @ 86, in order to invest in Bank stock paying 8 %. What price must he pay for it to be neither a gainer nor loser ?

III.

1. I send $3060 to my agent in Montreal to invest in tea @ 75c. per lb. He deducts his commission of 2 % and purchases the tea. How many pounds do I receive, and at what must I sell per lb. so as to make a profit of 40 %, after paying freightage, $30, and insurance at the rate of $\frac{1}{3}$ % ?

2. Bought land @ $50 an acre. How much must I ask an acre that I may take off 25 % from my asking price and still make 20 % profit on the purchase money ?

3. *A* buys silks @ $2.25 per yd., on a credit of 6 mo. *B* buys the same quality of silks for $2.15 per yd., cash. Which makes the best purchase, money being worth 10 %, and what must the goods be marked at to insure a gain of 25 % ? Or, if the silks be sold @ $3 per yd., what profit per cent. does each make ?

4. A person buys an article and sells it so as to gain 5 %. If he had bought it @ 5 % less and sold it for 5c. less, he would have gained 10 %. Find the cost price.

5. A person buys 6 % City of Toronto bonds, the interest on which is paid yearly, and which are to be paid off at par 3 yr. after the time of purchase. If money be worth 5 %, what price should he give for the bonds ?

IV.

1. Bought cloth @ $3 in gold and sold @ $4 in currency. Did I gain or lose by the transaction, and how much per cent. in currency, gold being @ 118 ?

2. A merchant sold 24 cheese @ $30 each. On one-half he gained 30 %, and on the remainder he lost 30 %. Did he gain or lose on the whole, and how much ?

3. A man wishing to sell his farm asked 36 % more than it cost him, but he finally sold it for 20 % less than his asking

price. He gained $528 by the transaction. How much did the farm cost, what was his asking price, and for how much did he sell it?

4. A person having to pay $1085 at the end of 2 yr., invested a certain sum in 3 % stock, allowing the dividends to accumulate until the payment of the debt, and also an equal sum next year, and also the previous years' interest. If the investment is made and the debt paid when stock was @ 73, what must be the sum invested on each occasion that there may be just sufficient to pay the debt at the proper time?

5. A merchants' stock-in-trade is valued on Jan. 1, 1899, @ $40000. He has $1750 in cash and owes $9350. During the year his personal expenses, $1500, are paid out of the proceeds of the business, and on Jan. 1, 1900, his stock is valued @ $39750. He has $2850 in cash, and owes $7650. What is the whole profit of the year's transactions after deducting 5 % interest on the capital with which he began the year?

V.

1. I received an 8 % dividend on railway stock, and invested the money in the same stock @ 80. My stock having increased to $13750, what was the amount of my dividend?

2. How many shares of $50 each must be bought @ 25 % discount, brokerage 1⅔ %, and sold at 16 %, discount, brokerage 1¼ %, to gain $121.66⅔?

3. When the 3 per cents. are at 87½, and shares paying 5 % are at 130¼, which is the more profitable investment; and what sum does *A* invest when the difference of the incomes resulting from the two investments is $561?

4. The charter of a new railroad company limits the stock to $1500000, of which 3 instalments of 10 %, 20 %, and 40 %, respectively, having been paid in, the cost of construction has reached $850,000, and the estimated cost of completion is $850,000. If the company call in the final instalment of its stock, and assess the stockholders for the remaining outlay, what will be the rate per cent.?

5. A person invests $16380 in the 3 per cents. @ 91. He sells out $1200 stock when they have risen to 93½, and the remainder when they have fallen to 85. How much does he gain or lose by the transaction? If he invests the proceeds in 4½ % stock @ 102, what is the difference in his income?

For additional examples see page 322.

CHAPTER XIV.

SHARING.

Division Into Proportional Parts.

187. Suppose 3 persons, *A*, *B*, and *C*, to be in partnership, and an arrangement made that the profits of the business in which they are engaged are to be divided into 6 *equal* parts, of which *A* is to take 3 parts, *B* 2 parts, and *C* 1 part. The shares of *A*, *B*, and *C* are then said to be in the proportion of 3, 2, and 1.

Ex. 1. Divide $1275 among 3 persons, whose shares are to be in the proportion of 3, 5, and 7.

This may be regarded as a case in which one holds 3 shares, one 5, and one 7, and they hold 15 shares in all.

Hence, if we divide $1275 by 15, we get the amount of *one* share. That is, amount of one share $= \$\frac{1275}{15} = \$85$.

Then one of the persons receives 3 × $85, or $255;
the second receives 5 × $85, or $425;
the third receives 7 × $85, or $595.

Ex 2. Divide $837 among three partners, whose shares are to be in proportion of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$.

The common denominator of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$ is 30;
∴ the shares are to be in the proportion of $\frac{15}{30}$, $\frac{10}{30}$, and $\frac{6}{30}$;
that is the proportion of 15, 10, and 6.
Now $15 + 10 + 6 = 31$;
∴ amount of *one* share out of 31 shares $= \$\frac{837}{31} = \$27$.
Then one of the partners receives 15 × $27, or $405;
the second receives 10 × $27, or $270;
the third receives 6 × $27, or $162.

Ex. 3. A rate of $4212 is to be paid by three townships, and the property on which it is levied is $24700 in the first, $37250 in the second, and $43350 in the third. What sum is paid by each?

Amount of property on which the rate is levied is $105300.
Then $105300 has to pay a rate of $4212;

$$\begin{array}{lllll}
\therefore \$1 & `` & `` & `` & \$\frac{4212}{105300}; \\
\therefore \$24700 & `` & `` & `` & \$\frac{24700\times4212}{105300}, \text{ or } \$988; \\
\$37250 & `` & `` & `` & \$\frac{37250\times4212}{105300}, \text{ or } \$1490; \\
\$43350 & `` & `` & `` & \$\frac{43350\times4212}{105300}, \text{ or } \$1734.
\end{array}$$

Ex. 4. Divide $1000 among *A*, *B*, and *C*, so that *A* may have half as much again as *B*, and *B* a third as much again as *C*.

Representing *C*'s part by 1,
B's part will be $1\frac{1}{3}$,
and *A*'s part will be $1\frac{1}{3} + \frac{1}{2}$ of $1\frac{1}{3}$, or 2;
and, therefore, the parts are to be as the numbers 2, $1\frac{1}{3}$, 1;
$\therefore$ all the shares $= 2 + 1\frac{1}{3} + 1 = 4\frac{1}{3}$ times *C*'s share.

$$\begin{array}{rl}
4\frac{1}{3} \text{ times } C\text{'s share} & = \$1000; \\
C\text{'s share} & = \$1000 \div 4\frac{1}{3} = \$230.769; \\
B\text{'s share} & = \frac{4}{3} \text{ of } C\text{'s} = \$307.692. \\
A\text{'s share} & = 2 \text{ times } C\text{'s} = \$461.538.
\end{array}$$

Ex. 5. Divide the number 237 into three parts, such that 3 times the first may be equal to 5 times the second, and to 8 times the third.

Take the first part as the *unit;* then by the question the second part will be $\frac{3}{5}$ of the first, and the third will be $\frac{3}{8}$ of the first.

$$\begin{array}{rl}
\text{Sum of the parts} & = 1 + \frac{3}{5} + \frac{3}{8} = \frac{79}{40} \text{ times the first.} \\
\text{Hence } \frac{79}{40} \text{ times the 1st} & = 237, \\
\text{the 1st} & = 237 \div \frac{79}{40} = 120, \\
\text{the 2nd} & = \frac{3}{5} \text{ of 1st} = \frac{3}{5} \text{ of } 120 = 72. \\
\text{the 3rd} & = \frac{3}{8} \text{ of 1st} = \frac{3}{8} \text{ of } 120 = 45.
\end{array}$$

Ex. 6. Divide $3400 among *A*, *B*, and *C*, so that *A* may have $800 more than $\frac{2}{3}$ of *B*'s share, and *B* $600 less than $\frac{3}{4}$ of *C*'s share.

Representing *C*'s share by 1, then

$$\begin{array}{rl}
B\text{'s share} & = \frac{3}{4} \text{ of } C\text{'s share} - \$600 \\
A\text{'s share} & = \frac{2}{3} \text{ of } B\text{'s share} + \$800 \\
& = \frac{2}{3}\left(\frac{3}{4} \text{ of } C\text{'s} - \$600\right) + \$800 \\
& = \frac{1}{2} \text{ of } C\text{'s} + \$400
\end{array}$$

$$\begin{aligned}
\text{Sum of all the shares} &= C\text{'s} + \tfrac{3}{4}\,C\text{'s} - \$600 + \tfrac{1}{2}\,C\text{'s} + \$400 \\
&= \tfrac{9}{4}\,C\text{'s} - \$200. \\
\therefore \tfrac{9}{4}\,C\text{'s} - \$200 &= \$3400 \\
\tfrac{9}{4}\,C\text{'s} &= \$3400 + \$200 \\
&= \$3600 \\
C\text{'s} &= \$1600 \\
B\text{'s} &= \tfrac{3}{4}\text{ of }\$1600 - \$600 \\
&= \$600 \\
A\text{'s} &= \tfrac{1}{2}\text{ of }\$1600 + \$400 \\
&= \$1200.
\end{aligned}$$

Examples xcv.

1. Divide $60 into two parts proportional to 11 and 9.

2. Divide $2500 into parts proportional to 2, 3, 7, 8.

3. Divide $8470 into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$

4. Gunpowder is made of saltpetre, sulphur, and charcoal, in parts proportional to 75, 10 and 15. How many pounds of each are contained in 12 cwt. of gunpowder?

5. The sides of a triangle are as 3, 4, 5, and the sum of the lengths of the sides is 480 yards. Find the sides.

6. Divide $640 among *A*, *B*, and *C*, so that *A* may have three times as much as *B*, and *C* as much as *A* and *B* together.

7. Divide 100 apples among three boys, so that the first may receive 7 as often as the second receives 8, and the third may receive 5 as often as the second receives 4.

8. A bankrupt owes £272 10s. to *A*, £354 5s. to *B*, and £490 10s. to *C*. His assets are £418 19s. 4½d. What will each of the creditors receive?

9. A legacy of $36421 was left to four heirs in the proportion of $\frac{1}{6}$, $\frac{2}{5}$, $\frac{4}{9}$, and $\frac{1}{3}$, respectively. How much was the share of each?

10. *A*, *B*, and *C* are employed to do a piece of work for $26.45. *A* and *B* together are supposed to do $\frac{3}{4}$ of the work, *A* and *C* $\frac{9}{10}$, and *B* and *C* $\frac{13}{20}$, and are paid proportionally. How much must each receive?

11. The British silver coin consists of 37 parts of silver, and 3 of copper. How much does the half-crown (2s. 6d.) contain, each pound, troy weight, being coined into 66 shillings?

12. A force of police, 1921 strong, is to be distributed among 4 towns in proportion to the number of inhabitants in each, the population being 4150, 12450, 24900, and 29050, respectively. Determine the number of men sent to each.

13. Divide £29 into an equal number of half-sovereigns, crowns, half-crowns, shillings, sixpences, and fourpences.

14. A piece of land of 200 acres is to be divided among four persons, in proportion to their rentals from surrounding property. Supposing these rentals to be £500, £350, £800, and £90, how many acres must be allotted to each ?

15. Divide £2 5s. among *A*, *B*, and *C*, so that for each threepenny piece received by *A*, *B* may receive a fourpenny piece, and that there are as many shillings in the sum received by *C* as there are sixpences in the sum received by *B*.

16. Divide $10.40 among 5 men, 7 women, and 14 boys, so that each woman may have $\frac{3}{7}$ of each man's share, and each boy $\frac{2}{5}$ of each woman's share.

17. A number of men, women, and children are in the proportions 2, 3, 5. Divide $517.65 among them, so that the shares of a man, a woman, and a child may be proportional to 3, 2, 1, there being 9 women.

18. A man left his property to be divided among his 3 sons in proportion to their ages, which are 20, 18, and 12 yr. The share of the youngest is $1440. What was the value of the property ?

19. Divide $5000 among *A*, *B*, and *C*, so that *A* may get $300 less than $\frac{5}{9}$ of *C*'s share, and *C* $800 more than $\frac{2}{5}$ of *B*'s share. What are the shares of each ?

20. Divide $5000 among *A*, *B*, *C*, and *D*, so that *A* may get $\frac{2}{3}$ of *B*'s share, and $250 ; *B*, $200 more than $\frac{4}{5}$ of *C*'s share ; *C*, $100 less than $\frac{9}{10}$ of *D*'s share. What are the shares of each ?

21. The sum of three fractions is $\frac{183}{242}$; and 22 times the first, 23 times the second, and 24 times the third give equal products Find the fractions.

22. Divide the simple interest on $1171 for 13 yr. @ 6 % into parts which shall have the same relation as $\frac{5}{8}$, $\frac{7}{9}$, $\frac{9}{10}$, $\frac{5}{12}$, $\frac{8}{15}$.

23. Of the boys in a school one-third are over 15 yr. of age, one-third between 10 and 15. A legacy of $400 can be exactly divided amongst them by giving $$\frac{1}{2}$ to each boy over 15, $$\frac{1}{3}$ to each between 10 and 15, and $\frac{1}{6}$ to each of the rest. How many boys are there in the school ?

24. A mixture of gold and silver weighs $10\frac{1}{2}$ oz., and is worth $49. If the proportions of gold and silver were reversed it would be worth $91 more. Gold being worth $16 per ounce, find the price of silver.

25. If 20 men, 40 women, and 50 children receive among them $4000 for 5 weeks' work, and 2 men receive as much as 3 women or 5 children, what sum does a man, a woman, and a child earn per week?

Partnership.

188. When persons unite to carry on any particular branch of business the connection so formed is called a **Partnership.** The method of working questions in Partnership is the same as that explained in the preceding article.

Ex. 1. *A*, *B*, and *C* entered into partnership to carry on a mercantile business for two years. *A* puts in $9000, *B* $6000, and *C* $3000. They gained $4500. What is each one's share of the gain?

The whole capital invested is $18000.
Then $18000 gains $4500;
$\therefore \$\ \ 1$ gains $\$\frac{4500}{18000}$, or $\$\ \frac{1}{4}$.
$\$\ 9000$ gains $\$\frac{9000}{4} = \$2250$.
$\$\ 6000$ gains $\$\frac{6000}{4} = \$1500$.
$\$\ 3000$ gains $\$\frac{3000}{4} = \$\ 750$.

Hence *A*'s share of the gain is $2550; *B*'s, $1500; and *C*'s, $750.

Ex. 2. *A*, *B*, and *C* entered into partnership for trading. *A* puts in $600 for 4 months, *B* $400 for 5 months, and *C* $200 for 6 months. They gained $980. What was each man's share of the gain?

$600 for 4 mo. = $2400 for 1 mo.
$400 " 5 " = $2000 " "
$200 " 6 " = $1200 " "

The whole capital is equivalent to $5600 for 1 mo.
Then $5600 gains $980;
$\therefore \$\ \ 1$ " $\$\frac{980}{5600} = \$\frac{7}{40}$.
$\$2400$ " $\$\frac{2400\times7}{40} = \$420$.
$\$2000$ " $\$\frac{2000\times7}{40} = \$350$.
$\$1200$ " $\$\frac{1200\times7}{40} = \$210$.
$\therefore$ *A*'s share is $420, *B*'s $350, and *C*'s $210.

Examples xcvi.

1. Two men jointly purchased a house for \$2592. The first contributed \$864 towards the purchase, and the second \$1728. They afterwards rented the house for \$132.75 annually. What share of the rent ought each to have?

2. Four persons rent a farm of 115 ac. 32 po. @ \$3.75 an acre. *A* puts on 144, *B* 160, *C* 192, and *D* 324 sheep. How much rent ought each to pay?

3. *A*, *B*, and *C* jointly rented a pasture for 3 mo., agreeing to pay \$22.50 for the use of the same. *A* put in 6 horses, *B* put in 18 cows, and *C* 90 sheep. Considering each horse as equivalent to 2 cows, and each cow as equal to 3 sheep, what part of the rent ought each to pay?

4. *A*, *B*, and *C* entered into partnership for speculating in cotton, their joint capital being \$25780, of which *A* furnished $\frac{2}{5}$, *B* contributed $\frac{3}{4}$ of the remainder, and *C* the balance. Their clear profit was 20 % of the original investment. How should it be divided?

5. Six persons are to share among them \$9450. *A* is to have $\frac{1}{7}$ of it, *B* $\frac{1}{5}$, *C* $\frac{2}{9}$, *D* is to have as much as *A* and *C* together, and the remainder is to be divided between *E* and *F* in the ratio of 3 to 5. How much does each receive?

6. Smith & Brown failed for \$80000. Their assets amounted to \$29,000. What would be the shares of *A* and *B*, if their claims amounted to \$57,000, and *A*'s is 28 % more than *B*'s?

7. *A* starts a business with a capital of \$2400 on the 19th of March, and on the 17th of July admits a partner, *B*, with a capital of \$1800. The profits amount to \$943 by the 31st of December. What is each person's share?

8. *D* and *E* enter into partnership. *D* puts in \$40 for 3 mo., and *E* \$75 for 4 mo. They gain \$70. What is each man's share in the gain?

9. *A*, *B*, and *C* are partners. *A* puts in \$500 for 7 mo.; *B*, \$600 for 8 mo.; and *C*, \$900 for 9 mo. The profit is \$410. What is the share of each?

10. Three graziers hire a pasture for their common use, for which they pay \$106. One puts in 10 oxen for 3 mo., another 12 oxen for 4 mo., and the third 14 oxen for 2 mo. How much of the rent should each pay?

11. Two men complete in a fortnight a piece of work for which they are paid $29.55. One of them works alternately 9 hr. and 8 hr. a day. The other works $9\frac{1}{2}$ hr. for 5 da. in the week, and does nothing on the remaining day. What part of the sum should each receive?

12. *A* and *B* begin to trade in partnership. *A* puts in $400 at first, and $500 at the end of 2 mo. *B* puts in $300 at first, and $600 at the end of 3 mo. The profit at the end of the year is $470. How should this be divided?

13. Johnston and Wilson formed a co-partnership in business for 2 yr. Johnston at first contributed $3000 to joint capital, and at the end of 12 mo. put in $1500 more. Wilson at first put in $3500, but at the end of 15 mo. from the beginning withdrew $1000. At the end of the first year they admitted Miller into the firm, he contributing $2250. Their profits were $1248. How ought this to be apportioned?

14. *A*, *B*, and *C* entered into partnership, and invested, and drew out as follows: *A* invested, July 1st, 1898, $1000; on Nov. 1st, $1500, and on April 1st, 1899, drew out $300. *B* invested, July 1st, 1898, $2500, and on January 1st, 1899, $600, and on June 1st, 1899, $1000. *C* invested, on July 1st, 1898, $3000, and drew out, Nov. 1st, 1898, $2000, and on March 1st, 1899, $500. Their whole gain was $3785, and each partner shares in proportion to the amount of capital invested, and for the time it was employed. If the business is closed on July 1st, 1899, what does each partner withdraw?

15 *A* and *B* rent a field for $88.20. *A* puts in 10 horses for $1\frac{1}{2}$ mo., 30 oxen for 2 mo., and 100 sheep for $3\frac{1}{4}$ mo. *B*, 40 horses for $2\frac{1}{2}$ mo., 50 oxen for $1\frac{1}{4}$ mo., and 115 sheep for 3 mo. If the food consumed in the same time by a horse, an ox, and a sheep be as the numbers 3, 2, 1, what proportion of the rent must each pay?

16. A person in his will directed that $\frac{1}{2}$ his property should be given to *A*, $\frac{1}{3}$ to *B*, $\frac{1}{4}$ to *C*, and $\frac{1}{5}$ to *D*. Shew that this disposition cannot be fulfilled. If his property amount to $1886.50, dispose of it so that their shares may have to one another the relation he intended.

17. *A*, *B*, and *C* had each a cask of rum containing, respectively, 36, 54, and 78 gal. They blended their rum, and then refilled their casks from the mixture. How much of the rums of *A* and *B* are contained in *C*'s cask?

18. *A* rents a house for $187.20. At the end of 4 mo. he takes in *B*, as a co-tenant, and they admit *C*, in like manner, for the last $2\frac{1}{2}$ mo. What portion of the rent must each of them pay?

19. Three men take an interest in a coal mine. *B* invests his capital for 4 mo., and claims $\frac{1}{10}$ of the profits. *C*'s capital is in 8 mo., and *D* invests $6000 for 6 mo., and claims $\frac{2}{5}$ of the profits. How much did *B* and *C* put in?

Partnership Settlements.

189. When a partnership is dissolved, either by mutual consent or by limitation of contract, the adjustment of the proceeds between the members is called a Partnership Settlement. If the **Resources** are found to exceed the **Liabilities,** the difference is termed **Net Capital.** If the Liabilities exceed the **Resources,** the difference is **Net Insolvency.** The investment of the partners is the Net Capital at commencement. If the net capital at closing exceeds the net capital at commencement, the difference is the **Net Gain**; if the opposite, **Net Loss.** This net gain, or net loss, is then shared between the partners in accordance with the original agreement between them. This division is frequently not made in exact proportion to the amount invested. Sometimes the skill of one partner is considered equal to the capital of another. Sometimes a stated salary is allowed each partner, according to his ability or reputation. Sometimes, where unequal amounts are invested, interest is allowed each partner on his investment. But whatever allowance is made *such allowance must be classed as a liability and go to reduce the gain.*

Ex. 1. *A* and *B* are partners. The following is a statement of their property and debts: They have Cash, $3240; Merchandise, $2575; Bills Receivable, $860; J. Brown owes on account, $375. They owe on Bills Payable, $1250, and J. Jones on account, $370. *A* invested at commencing $2500, and drew out, during business, $560. *B* invested $2500, and drew out, during business, $280. They agreed to share equally in gains and losses. What was the net gain? What was the net capital of each at closing?

RESOURCES AND LIABILITIES.

Dr.		Cr.
$3240		$1250
2575		370
860		
375		$1620
7050	Resources at Closing.	
1620	Liabilities " "	
5430	Present Worth of Firm.	
4160	Credit excess of Ownership.	
1270	Net Gain.	
635	*A*'s share of net gain.	
635	*B*'s " " " "	

OWNERSHIP.

Dr.		Cr.
$560	*A* withdrew.	$2500
280	*B* "	2500
$840	Total investment	$5000
	" withdrawn	840
	Firm's net investment	4160

Hence, *A*'s present net capital = $2500 − $560 + $635 = $2575, and *B*'s present net capital = $2500 − $280 + $635 = $2855.

Examples xcvii.

1. *A* and *B*, having conducted business 1 yr. as partners, close with the following resources and liabilities: They have Cash, $3456; Mdse., $2120; Bills Receivable, $1874; E. Corby owes $630. They owe on Bills Payable, $3250; W. Smith on account, $346. *A* invested $1500, and withdrew $175. *B* invested $1500, and drew out $315. What is the net gain, and net capital of each at closing?

2. *A* and *B* close business as follows: They have Cash, $1424; Mdse., $1562; Fixtures, $383; Mortgages Receivable, $3485; Bills Receivable, $826. They owe on Bills Payable, $2450; on accounts, $1240. *A* invested $6000, and a debt for $1000 was assumed by the firm, and paid during business. He drew out $685; and is allowed interest on capital invested, $420. *B* invested $4000, and drew out $1860, and is allowed interest on capital, $280. *A* is to share $\frac{3}{5}$ and *B* $\frac{2}{5}$ of gains and losses. What is the net loss? What is the net capital of each?

3. *A* and *B* close business, and wish to know the financial standing of each. They have cash, $2263, and Real Estate worth $5000. They owe on Mortgages, $3846; on Notes, $4462; on Personal Accounts, $675. *A* invested $6000, and drew out $2860. *B* invested $4000, drew out $5560, and is allowed for extra services $250. *A* shares $\frac{3}{5}$ and *B* $\frac{2}{5}$ of the gains and losses. What is the net loss? What is the financial standing of each?

For additional examples see page 324.

CHAPTER XV.

ALLIGATION.

190. Alligation is the process by which we find the mean or average price of a compound when we mix or unite two or more articles of different values.

Ex. 1. A merchant has brown sugar worth 8 cents per pound, New Orleans worth 9 cents, and refined sugar worth 14 cents. How many pounds of each kind must he use in order to form a mixture worth 12 cents per pound?

By selling the mixture at 12c. per lb. we see that 8c. (brown) gains 4c. on 1 lb. $\therefore$ 1c. is gained on $\frac{1}{4}$ lb.

9c. (New Orleans) gains 3c. on 1 lb.; $\therefore$ 1c. is gained on $\frac{1}{3}$ lb.
14c. (refined) loses 2c. on 1 lb.; $\therefore$ 1c. is lost on $\frac{1}{2}$ lb.

Now with every cent *gain* he must combine a cent *loss*, hence he must have

$$\left.\begin{array}{l} \frac{1}{4} \text{ lb. @ } 8\text{c.} \\ \frac{1}{2} \text{ lb. @ } 14\text{c.} \\ \frac{1}{3} \text{ lb. @ } 9\text{c.} \\ \frac{1}{2} \text{ lb. @ } 14\text{c.} \end{array}\right\} = \left\{\begin{array}{l} 3 \text{ lb. @ } 8\text{c.} \\ 6 \text{ lb. @ } 14\text{c.} \\ 4 \text{ lb. @ } 9\text{c.} \\ 6 \text{ lb. @ } 14\text{c.} \end{array}\right.$$

He must, therefore, have 3 lb. of brown sugar, 4 lb. New Orleans, and 12 lb. refined.

We may show that these quantities will make the mixture required, as follows:—

$$\begin{array}{lcl} 3 \text{ lb. @ } 8\text{c. per lb.} & = & 24\text{c.} \\ 4 \text{ lb. @ } 9\text{c.} \quad \text{“} & = & 36\text{c.} \\ 12 \text{ lb. @ } 14\text{c.} \quad \text{“} & = & 168\text{c.} \\ \hline 19 \text{ lb.} = \text{whole mixture.} & & 228\text{c.} = \text{value of mixture.} \end{array}$$

Hence, if 19 lb. be worth 228c.
1 lb. is worth $\frac{228}{19} = 12$c.

Or we may reason thus: The 1c. *gained* on the $\frac{1}{4}$ lb. of brown exactly balances the 1c. *lost* on the $\frac{1}{2}$ lb. of the refined. Hence he must take $\frac{1}{4}$ lb. of the brown and $\frac{1}{2}$ lb. of the refined, or 2 lb. of one and 4 lb. of the other.

Similarly, for every 2 lb. of New Orleans, there must be 3 lb. of refined. As 4 lb. of refined were required to balance the brown, and 3 lb. of the refined to balance the New Orleans, there must be 7 lb. of the refined in the compound. Therefore, the respective quantities are 2 lb. brown, 2 lb. New Orleans, and 7 lb. refined.

From the above we see that in examples of this kind a variety of answers may frequently be obtained, and all of them may be correct. To ascertain their correctness we resort to the method of proof given in this example.

191. From the above analysis we derive an easy practical method of solving such questions.

Ex. 2. How much sugar at 10, 13, 15, 17 and 18 cents per pound must be taken to make a mixture worth 16 cents per pound.

We proceed as follows:—

Differences.	16	
	—	
6	10	1
3	13	1
1	15	1
..	..	..
1	17	2, 4, 6, 8
2	18	4, 3, 2, 1

Write down the prices in a vertical column, and place the *differences* between these prices and the *mean* in a second vertical column to the left. Now take 1 @ 10, 1 @ 13, and 1 @ 15, (the lowest that could be taken); this would represent a *loss* of 10 as compared with the *mean;* and this loss must be balanced by taking the necessary multiples of the differences, 1 and 2, which represent *gain*, as compared with the mean.

It is seen that this loss of 10 can be made up in *four* ways. By 2 @ 17, 4 @ 18; 4 @ 17, 3 @ 18; 6 @ 17, 2 @ 18; 8 @ 17, and 1 @ 18.

Other combinations may be made, as *e.g.*,

6	10	1
3	13	1
1	15	2
..	..	..
1	17	1, 3, 5, 7, 9
2	18	5, 4, 3, 2, 1

Here 1 @ 10, 1 @ 13, and 2 @ 15, give *loss* of 11, which can be made up by multiples of the differences 1 and 2 (opposite 17 and 18) in *five* ways, as indicated.

Also,

6	10	1
3	13	2
1	15	1
..	..	..
1	17	1, 3, 5, 7, 9, 11
2	18	6, 5, 4, 3, 2, 1

When 1 @ 10, 2 @ 13, and 1 @ 15 give 13 loss, which may be made up in *six* different ways.

Again,

6	10	2
3	13	1
1	15	1
..	..	..
1	17	2, 4, 6, 8, 10, 12, 14
2	18	7, 6, 5, 4, 3, 2, 1

Where 2 @ 10, 1 @ 13, and 1 @ 15 give *loss* of 16, which may be made up in *seven* ways.

Also,

6	10	1
3	13	1
1	15	3
..	..	..
1	17	2, 4, 6, 8, 10
2	18	5, 4, 3, 2, 1

When 1 @ 10, 1@ 13, and 3 @ 15 give *loss* of 12, which may be made up in *five* ways; and thus an indefinite number of the combinations may be formed.

It should be observed that if the differences opposite the prices *less* than the *mean* are together greater than the sum of the other differences (as in the example), we assign numbers (the *lowest* possible) to the prices less than the mean FIRST, and *vice versa*; *e.g.* of the latter case.

Ex. 3. How much coffee at 25, 24, 23, 22, 21, 19, 18, and 17 cents per pound must be taken to make a mixture worth 20 cents per pound?

Diff's.	20	
	—	
3	17	4, 3, 2, 1, 1, 2
2	18	1, 2, 3, 5, 4, 2, etc.
1	19	1, 2, 3, 2, 4, 5
..	..	..
1	21	1
2	22	1
3	23	1
4	24	1
5	25	1

Here the sum of the differences in excess of the mean is greater than that of the differences below the mean. We therefore assign *first* numbers to the prices which are greater than the mean, viz., 1 @ 21, 1 @ 22, 1 @ 23, 1 @ 24, 1 @ 25. This gives a *gain* of 15, which may be balanced as above by 1 @ 19, 1 @ 18, and 4 @ 17; or, by 2 @ 19, 2 @ 18, and 3 @ 17, etc., etc.

Ex. 4. A grocer has 12 lb. of brown sugar, worth 10 cents per pound, which he wishes to mix with clarified sugar worth 16 cents per pound, so that the mixture may be worth 14 cents per pound. How many pounds of clarified sugar must he take?

Proceeding as in the previous examples, without reference to the *quantity* of the brown sugar, we find that there must be 1 lb. brown sugar to 2 lb. clarified sugar. But as 12 lb. of brown sugar are required, we must multiply each of these quantities by 12 in order that the gain and loss may be equal. We shall therefore have 12 × 2 lb. = 24 lb. of clarified sugar.

Ex. 5. A grocer wishes to mix 20 lbs. of sugar worth 9 cents per pound, and 10 pound worth 12 cents per pound, with clarified sugar worth 15 cents, so that the compound may sell at 13 cents. How much of the clarified must he take?

20 lb. @ 9c. = \$1.80
10 lb. @ 12c. = \$1.20
30 \$3.00

Then, if 30 lb. is worth \$3,
1 lb. " $\$\frac{3}{30}$ = 10c.

The value of 1 lb. of the mixture is, therefore, worth 10c.

The question may then be read as follows:—

How many pounds of clarified sugar, worth 15 cents per pound, must be mixed with 30 pound of another kind of sugar, worth 10 cents per pound, so that the mixture may be sold for 13 cents per pound?

The question in this form has already been fully explained.

Ex. 6. A merchant has West India sugar worth 8 cents per pound, and New Orleans sugar worth 13 cents. He wishes to combine these so as to make a barrel containing 175 lb., which he may sell at 11 cents per pound. How many pounds of each kind must he take?

Solving the question without reference to the 175 lb., we find that 2 lb. of West India sugar and 3 lb. of New Orleans sugar will form a mixture worth 11c. per pound. Adding these quantities we find that they form a mixture of 5 lb. But the required mixture is to contain 175 lb., or 35 times 5. We shall therefore have

35 × 2 lb. = 70 lb. West India sugar.
35 × 3 lb. = 105 lb. New Orleans sugar.

Examples xcviii.

1. A jeweler melted together 9 oz. of gold 22 carats fine, 12 oz. 18 carats fine, 9 oz. 21 carats fine, and 10 oz. 15 carats fine. What is the fineness of the mixture?

2. If a merchant mixes 7 lb. of sugar worth 8c. a pound, with 6 lb. worth 9c., 9 lb. worth 10c., and 10 lb. worth 12c., for how much must he sell the mixture to gain 25 %?

3. On a certain day the thermometer ranged at 64° from 6 o'clock to 9, at 76° from 9 to 12, at 85° from 12 to 3, and at 68° from 3 to 6. What was the average temperature?

4. A person mixed 15 gal. of alcohol 80 % strong, 12 gal. 90 % strong, 23 gal. 60 % strong, and 20 gal. 70 % strong. What is the strength of the mixture?

5. What quantities of coffee, worth 23c. and 35c., respectively, per pound, must be mixed together so that the compound may be sold for 30c. per pound?

6. What quantity of oats @ 35c. per bushel, rye @ 60c. per bushel, and barley @ 80c., must be taken to form a mixture worth 55c. per bushel?

7. How much tea, worth, respectively, 55c. and 75c. per pound, must be mixed with 30 lb. worth 90c. per pound, in order that the compound may be sold for 70c. per pound?

8. How much water will it require to dilute 60 gal. of alcohol, worth $1.50 per gallon, so that the mixture may be worth only $1.20 per gallon?

9. How many gallons of kerosene oil, worth 60c. per gallon, must be mixed with 12 gal. of coal oil, worth 36c., and 8 gal. of Aurora oil, worth 56c., so that the compound may be sold for 50c. per gallon?

10. A farmer has 16 bu. of corn, worth 48c. per bushel, and 12 bu. of oats @ 34c. per bushel, which he wishes to mix with rye @ 60c., and barley @ 80c., in order to sell the compound @ 56c. per bushel. How many bushels of rye and barley will be required?

11. A confectioner mixes three different qualities of candy worth, respectively, 14c., 18c., and 30c. per pound, so as to make a box of 84 lb. How many pounds of each sort must he take so as to sell the compound at an average price of 24c. per pound?

12. A farmer has three different qualities of wool, worth, respectively, 33c., 37c., and 45c. per pound. He wishes to make up a package amounting to 120 lb., which he can afford to sell @ 39c. per pound. How many pounds of each kind must he take?

13. How many sheep worth, respectively, $1.50, $2, $2.75, $3, and $4 apiece, can be taken to make a flock of 300 worth $2.50 apiece?

14. How much tea @ 40c. and 50c. per pound must be mixed with 36 lb. @ 60c., so that the mixture may be sold @ 77c., at a gain of 40 %?

15. How much tea @ 30c., 35c., 40c., 45c., and 50c. must be taken to form a mixture of 100 lb. @ 56c., so as to gain 33⅓ %?

16. A farmer wishes to mix corn worth 70c. per bushel with rye worth 75c., barley worth 60c., and oats worth 45c., to make a mixture of 60 bu., which he may sell @ 78c. per bushel, at a gain of 20 %. How many bushels of each kind must he take?

17. If 16 gal of spirits @ $1.25 per gallon are mixed with 9 gal. at a different price, and 25 % is gained by selling a gallon of the mixture @ $1.67½, what is the price of the second kind of spirits per gallon?

18. A man paid $165 to 55 laborers, consisting of men, women, and boys. To the men he paid $5 per week, to the women $1 per week, and to the boys $½ per week. How many were there of each?

19. A man bought calves, sheep, and lambs, 154 in all, for $154. He paid $3½ for each calf, $1⅓ for each sheep, and $½ for each lamb. How many did he buy of each kind?

20. A farmer bought 100 animals for $100. Geese @ $½ each, pigs @ $3½, and calves @ $10. How many animals were there of each kind?

21. A grocer has three kinds of tea; the second being ½ as dear again as the first, and the third ¼ as dear again as the second. If he mixes a certain quantity of the first with twice as much of the second and 112 lb. of the third, and finds the mixture to be ⅕ as dear again as the second sort, of how many pounds does the mixture consist?

For additional examples, see page 327.

CHAPTER XVI.

EXCHANGE.

192. The term *Exchange* is here used for giving or receiving in the money of one country a sum equal in value to a sum of money of another country. For example, if a Canadian merchant pays to a French merchant $487 and receives in return 2600 francs, it is a case of Exchange.

In countries which carry on considerable trade with each other, the debts reciprocally due from the one to the other are generally nearly equal. In England there is always a large number of persons indebted to others in America, and likewise a large number in America owing money in England. Now if coin, or specie, as it is called, were sent from England to pay the debts in America, and from America to England, the specie would have to be transmitted twice, and would necessarily involve risk, loss of interest, and expense of transportation. To avoid this risk, etc., **Bills of Exchange** are used to liquidate debts reciprocally due between two places without any actual transmission of money.

193. A Bill of Exchange is a written order, addressed to a person in a distant place, directing him to pay a certain sum of money, at a specified time, to another, or to his order. The person who signs the bill is called the **Drawer** or **Maker.** The person to whom it is addressed is the **Drawee,** and after the Drawee agrees to pay it, and writes "accepted" with his signature and the date, across the face of it, he becomes the **Acceptor.** The person to whom the money is to be paid is the **Payee.** If he transfers payment to another, he **Endorses** it, *i.e.*, he writes his name across the back of it and becomes responsible for its payment in case the Drawee fails to make payment.

Bills of Exchange are of two kinds, viz., (i) Inland Bills of Exchange, or Drafts, and (ii) Foreign Bills of Exchange.

An Inland Bill of Exchange, or Draft, is one in which the drawer and drawee reside in the same country. (This has been explained in Art. 175 and 176).

A Foreign Bill of Exchange is one in which the drawer and drawee reside in different countries.

194. The *Par of Exchange* between two countries denotes the nominal value of a unit of coinage in one country, as estimated in terms of a unit of coinage in the other country.

As we supposed the exports from England and America to be equal, creditors in England will be as anxious to sell bills on America as debtors to buy them, and the exchange will deviate but slightly from the *par* of Exchange. But if the exports from America are in excess of those from England, or the *Balance of Trade* is in favor of America, the claims of America in England will exceed its liabilities, and the English will give more than the par value of such bills to avoid the cost of transmitting specie. And on the other hand, the exporters in America, not finding sufficient purchasers for all their bills on England, will sell them at less than their par value. Now the real rate of exchange, depending on the balance of trade, is called the **Course of Exchange**, and it is at a *premium* or *discount*, according as it is above or below the par of exchange. Of course no one would give a premium greater than the cost of transmitting specie. But if the balance of trade is against England as regards America, but in favor of England as against France, the English merchant may find it advantageous to remit to France, and then for France to remit to America, and this mode is adopted when the course of exchange by this circuitous route is less than the direct course of exchange. The finding the course of exchange between two places, by comparing the courses of exchange between them and one or more intervening places, is called

Arbitration of Exchange. The arbitration is *Simple* when only *one* place intervenes, and *Compound* when more than one.

Bills of Exchange are usually drawn in sets, three bills constituting a set. These are distinguished from one another by being called the *first*, *second*, and *third* of exchange. These are forwarded by different routes so as to guard against delay or their being lost. The first that arrives is paid, and the other two become void.

195. FORM OF A FOREIGN BILL OF EXCHANGE.

TORONTO, July 12, 1900.

Exchange for £200.

Three days after sight of this first of exchange (second and third of same date and tenor unpaid), pay to W. J. Gage & Co., Limited, or order, Two Hundred Pounds Sterling, value received. and charge the same to the account of

W. B. TAYLOR.

To GEO. H. SIMPSON,
Banker, London.

196. By Act of Parliament the value of the pound sterling was fixed at $\$4\frac{4}{9}$. This was much below its intrinsic value, which is now fixed at $\$4.86\frac{2}{3}$. The rates of exchange which are quoted in commercial papers are still calculated at a certain per cent. *on the old par of exchange.* Exchange is at par between Great Britain and Canada when it is at a premium of $9\frac{1}{2}$ per cent., for $\$4\frac{4}{9}$ increased by $9\frac{1}{2}$ per cent. equals $\$4.86\frac{2}{3}$.

Ex. 1. Jones & Co. of New York owes Wm. Smith of Toronto $1000. Smith draws on Jones & Co. at 60 days, and discounts his draft on the same day at 6 per cent. What does he get for it, exchange being at par and commission at $\frac{1}{4}$ per cent.?

Discount on \$1000 for 63 da. = \$10.36.
Commission on \$1000 @ $\frac{1}{4}$ % = \$2.50.
∴ present worth of draft = \$1000 − \$(10.36 + 2.50),
or \$987.14.

Ex. 2. A broker in Toronto sold a bill of exchange on London, the face of which was for £750 8s. What did he receive for the bill, exchange being quoted at $110\frac{1}{4}$?

Since £1 = $\$(4\frac{4}{9} \times 1.10\frac{1}{4})$, *i.e*, $\$4\frac{4}{9}$ increased by $10\frac{1}{4}$ %,
∴ £750.4 = $\$(750.4 \times 4\frac{4}{9} \times 1.10\frac{1}{4})$
= $3676.96;
∴ he got $3676.96 for the bill.

Ex. 3. What is the value in English money of 4528.7 francs, when the course of exchange between Paris and London is at 25.3 francs per pound sterling?

Since 25.3 francs = £1,
1 franc = £$\frac{1}{25.3}$;
∴ 4528.7 francs = £$\frac{4528.7}{25.3}$, or £179.

Ex. 4. A merchant pays a debt of 4379 milrees in Portugal with £971 11s. $9\frac{3}{4}$d. What is the course of exchange in pence per milree?

£971 11s. $9\frac{3}{4}$d. = 932727 farthings;
Then since 4379 milrees = 932727 farthings,
1 milree = $\frac{932727}{4379}$ farthings, or 213 farthings;
∴ the course of exchange is $53\frac{1}{4}$d. per milree.

Ex. 5. If 11.65 Dutch florins are given for 24.42 francs, 352 florins for 407 marks of Hamburg, and $58\frac{1}{4}$ marks for 32 silver roubles of St. Petersburg, how many francs should be given for 932 silver roubles?

Here 1 silver rouble = $\frac{58.25}{32}$ marks,
1 mark = $\frac{352}{407}$ florins,
1 florin = $\frac{24.42}{11.65}$ francs;
∴ 1 silver rouble = $\frac{58.25}{32} \times \frac{352}{407} \times \frac{24.42}{11.65}$ francs, or 3.3 francs;
∴ 923 silver roubles = 932 × 3.3 francs, or 3075.6 francs.

Ex. 6. A New York merchant remits 27940 florins to Amsterdam by way of London and Paris, at a time when the exchange of New York on London is $4.885 for £1, of London on Paris is 25.4 francs for £1, and of Paris on Amsterdam is 212 francs for 100 florins. $\frac{1}{8}$ per cent. brokerage being paid in London and Paris, how many dollars will purchase the bill of exchange?

Since 100 florins = 212 francs,
∴ 1 florin = $\frac{212}{100}$ francs.
But to buy a bill of 100 franc requires a bill of $100\frac{1}{8}$ francs;
∴ to buy a bill of 1 franc requires a bill of $\frac{801}{800}$ franc.
Again, 25.40 franc = £1,
∴ 1 franc = £$\frac{1}{25.40}$;
but to buy a bill of £100 requires £$100\frac{1}{8}$;
∴ " " £1 " £$\frac{801}{800}$.
Again, £1 = $4.885,

$$\therefore 1 \text{ florin} = \$\left(\frac{212}{100} \times \frac{801}{800} \times \frac{1}{25.40} \times \frac{801}{800} \times 4.885\right);$$

$$\therefore 27940 \text{ florins} = \$\frac{27940 \times 212 \times 801 \times 801 \times 4.885}{100 \times 800 \times 25.40 \times 800}$$

$$= \$11420.317, \text{ sum required.}$$

Ex. 7. A merchant of Toronto wishes to transmit 2400 marcs banco to Hamburg. He finds exchange between Toronto and Hamburg to be 35 cents for 1 marc. The exchange between Toronto and London is $4.83 for £1; that between London and Paris is 26 francs for £1; and that of Paris on Hamburg is 47 francs for 25 marcs. By what way should the Toronto merchant remit?

By direct exchange 1 marc = $0.35;
∴ 2400 marcs = 2400 × $0.35
= $840
By circuitous exchange 25 marcs = 47 francs;
∴ 1 marc = $\frac{47}{25}$ francs;
but 26 francs = £1;
∴ 1 franc = £$\frac{1}{26}$,
and £1 = $4.83;

$$\therefore 1 \text{ marc} = \$\left(4.83 \times \frac{1}{26} \times \frac{47}{25}\right);$$

$$\therefore 2400 \text{ marcs} = \$\frac{2400 \times 4.83 \times 47}{26 \times 25}$$

$$= \$838.19.$$

By direct exchange the merchant pays $840 for his bill of exchange, and only $838.19 by the circuitous mode;
∴ the circuitous mode is better by $1.81.

Examples xcix.

1. John Adams bought a sight draft for $250 on Boston, brokerage $\frac{1}{4}$ %. What did the draft cost him?

2. A merchant sold goods on commission to the amount of \$2375. After deducting 3 % as his commission he purchased a sight draft @ $\frac{1}{4}$ % brokerage. What was the face of the draft?

3. What will a merchant in Toronto pay for a sight draft to settle a bill of goods in Manchester for £558, exchange @ $9\frac{5}{8}$ % and brokerage $\frac{1}{4}$ %?

4. When \$7300 are paid in Toronto for a bill of exchange on Liverpool for £1500, how was sterling exchange quoted?

5. What will be the cost of a bill on Paris for 236874 francs, exchange being 5.3 francs to the dollar?

6. If £1 be worth 12 florins, and also be worth 25 francs 56 centimes, how many francs and centimes is one florin worth?

7. If £1 be worth $25\frac{1}{2}$ francs, and be also worth 2244 copeks in Russian money, what is the value of the napoleon in Russian copeks? (N.B.—20 francs = 1 napoleon).

8. The French franc is divided into 100 centimes, and the Frankfort florin into 60 kreutzers. When the pound sterling is worth 25.50 francs in Paris, and 11 florins 54 kreutzers at Frankfort, what is the worth of the napoleon in florins and kreutzers?

9. A merchant in Toronto wishes to remit \$2767.80 to Manchester, England, exchange being @ 108. What will be the face of his bill in pounds, shillings, and pence?

10. Find the par of exchange between the U. S. gold eagle, weighing 258 grains $\frac{9}{10}$ fine, and the sovereign, of which 1869 weigh 40 lb. of gold $\frac{11}{12}$ fine.

11. Find the arbitrated rate of exchange between London and Paris when the course of exchange between London and Amsterdam is $12.16\frac{1}{4}$ florins for £1, and between Amsterdam and Paris $209\frac{1}{4}$ francs for 100 florins.

12. If a merchant buys a bill in London, drawn in Paris, at the rate of 25.5 francs per pound sterling, and if this bill is sold in Amsterdam at the rate of 30 francs for 14 florins, and the money received be invested in a bill on Hamburg at the rate of 18 florins for 20 marcs banco, what is the rate of exchange between London and Hamburg, or what is a pound sterling in London worth in Hamburg?

13. If the exchange of London on Hamburg is 14 marcs banco per pound sterling; that of Hamburg on Amsterdam is 20 marks banco for 18 florins; that of Amsterdam on Paris is 28 florins for 60 francs; and that of Paris on Toronto is 4 francs for

72 cents, what is the rate of exchange between London and Toronto, or how many dollars are equal to £1 sterling?

14. The exchange at Paris upon London is at the rate of 25 francs 70 centimes for £1 sterling, and the exchange at Vienna upon Paris is at the rate of 40½ Austrian florins for 20 francs. Find how many Austrian florins should be paid at Vienna for a £50 note.

15. What is the arbitrated rate of exchange between London and Lisbon, when bills on Paris, bought in London at 25.65 francs per pound, are sold in Lisbon at 525 rees per 3 francs?

16. Given that 1 ounce Troy equals 31.1 grammes; that 10 grammes of French standard gold are worth 31 francs; and that the worth of a given weight of English standard gold is to that of the same weight of French standard gold as 3151 to 3100, find what number of Troy ounces of English standard gold the franc is equal to, and what is the fixed number of francs equivalent to £1—the English mint price of standard gold being 77s. 10½d. per ounce?

EXAMINATION PAPERS.

I.

1. If three fluids, whose volumes are as 3, 7, and 12, and their specific gravities .95, 1.15, and 1.36, be mixed together, what will be the specific gravity of the compound?

2. A Toronto merchant wishes to pay a debt of £1200 in London. How many dollars must he pay to procure remittances through France and Hamburg if we allow that 21 francs = $4, 19 marcs banco at Hamburg = 35 francs at Paris, and £7 at London = 96 marcs banco at Hamburg?

3. A merchant in Cincinnati wishes to remit $14331.60 to New York. Exchange on New York is ¾ % premium, but in St. Louis ½ % premium, from St. Louis to New Orleans ½ % discount, and from New Orleans to New York 1 % discount. What will be the value in New York by each method, and how much better is the circular?

4. A merchant in Toronto purchased a draft on New York for $2660, drawn at 60 da., paying $2570.89. What was the course of exchange?

II.

1. A merchant mixes 11 lb. of tea with 5 lb. of an inferior quality, and gains 16 % by selling the mixture at 87c. per pound. Allowing that a pound of the one cost 12c. more than a pound of the other, what was the cost of each kind per pound?

2. *A* and *B* are in partnership in a concern in which *A* has $20000 engaged, and *B* $30000. The gross receipts for a year are $12800. Of this one-eighth part is expended in salaries of clerks, and $120 in insurance. By an arrangement between the partners *A* is to receive 8 % upon his capital, and *B*, 4 % upon his, and then the remainder of the profits is to be divided in proportion to the capital employed. Find the net receipts of *A* and *B*.

3. Bills on Amsterdam, bought in London @ 12 florins 15 cents per £1 sterling, are sold in Paris @ 57½ florins for 120 francs. What is the course of exchange between London and Paris?

4. A cask contains 12 gal. of wine and 18 gal. of water. Another cask contains 9 gal. of wine and 3 gal. of water. How many gallons must be drawn from each cask so as to produce by their mixture 7 gal. of wine and 7 gal. of water?

III.

1. A merchant has sugar @ 8, 10, 12, and 20c. a pound. With these he wishes to fill a cask that holds 200 lb. How much of each kind must he take so that the mixture may be worth 15c. a pound?

2. A 15 days' draft on Montreal yielded $1190.234 when sold @ 1½ % discount, and interest off at 6 %. What was the face of the draft?

3. If *A* gain $120 in 6 mo., *B* $150 in 5 mo., and *C* $210 in 9 mo., what was the whole stock, *C*'s part of it being $400?

4. From a cask of wine one-fourth is drawn off, and the cask is filled up with water. One-fourth of the mixture is then drawn off, and the cask again filled up with water. After this has been done four times altogether, what fraction of the original quantity of wine will be left in the cask?

5. A person in London owes another in St. Petersburg 920 roubles, which must be remitted through Paris. He pays the requisite sum to his broker at a time when the exchange between London and Paris is 25.15 francs for £1, and between Paris and St. Petersburg 1.2 francs for 1 rouble. The remittance is delayed until the rates are 25.35 francs for £1, and 1.15 francs for 1 rouble. What does the broker gain or lose by the delay?

IV.

1. If, when the course of exchange between England and Spain is $38\frac{1}{2}$d. per dollar of 20 reals, a merchant in Liverpool draws a bill of £354 16s. 3d. on Madrid, how many dollars and reals will pay the draft?

2. I wish to pay a bill in Naples of 7500 lire. The direct exchange is \$0.22 = 1 lira; the exchange on London is \$4.95; of London on Paris is £1 = 26 francs; of Paris on Naples is $1\frac{1}{8}$ francs = 1 lira. What is the difference between the direct and circuitous exchange?

3. A merchant in New York wishes to pay £3000 in London. Exchange on London is at par. On Paris 5 francs 25 centimes per \$1, and on Amsterdam 40 cents to a guilder. The exchange between France and England at the same time is 25 francs to £1, and that of Amsterdam on England $12\frac{1}{5}$ guilders to £1. Which is the most advantageous, the direct exchange, or through Paris, or through Amsterdam?

4. How many pounds of sugar @ 8c., 13c., and 14c. per pound may be mixed with 3 lb. @ $9\frac{1}{4}$c., and 4 lb. @ 14c. per pound, so as to gain 16 % by selling the mixture at $14\frac{1}{2}$c. per pound?

V.

1. A person mixes 4 gal. of gin @ 15s. per gallon, with 4 gal. of water and a gallon of base spirit worth 10s. What is his gain per cent. on his outlay by selling the mixture @ $2\frac{5}{6}$s. per bottle of 6 to the gallon?

2. The stocks of three partners, *A*, *B*, and *C*, are \$3500, \$2200, and \$2500, respectively. Their gains are \$1120, \$880, and \$1200, respectively. If *B*'s stock is in trade 2 mo. longer than *A*'s, what time was each stock in trade?

3. A merchant every year gains 50 % on his capital, of which he spends £1200 per annum in house and other expenses. At the end of 4 yr. he finds himself in possession of 4 times as much as what he had at commencing business. What was his original capital?

4. There are two mixtures of wine and water, the quantities of wine in which are, respectively, .34 and .46 of the whole. If a gallon of the first is mixed with two gallons of the second, what decimal part will the wine be in the compound, and how much per cent. will the first mixture be strengthened?

For additional examples see page 329.

CHAPTER XVII.

RATIO AND PROPORTION.

Ratio.

197. If A and B be quantities of the same kind, the relative greatness of A with respect to B is called the **Ratio** of A to B.

198. The ratio of one quantity to another quantity is represented in Arithmetic by the fraction which expresses the measure of the first when the second is taken as the unit of measurement.

Thus, if 5s. be the unit, the measure if 3s. is $\frac{3}{5}$, and the ratio of 3s. to 5s. is represented by the fraction $\frac{3}{5}$.

The words, "the ratio of 3 shillings to 5 shillings," are abbreviated thus,

3 shillings : 5 shillings.

199. Ratios may be compared with each other by comparing the fractions by which they are represented.

Thus, 2 pence : 5 pence is represented by $\frac{2}{5}$,
and 3 pence : 7 pence is represented by $\frac{3}{7}$.
Now $\frac{2}{5} = \frac{14}{35}$, and $\frac{3}{7} = \frac{15}{35}$;
$\therefore$ $\frac{3}{7}$ is greater than $\frac{2}{5}$;
and $\therefore$ 3 pence : 7 pence is greater than 2 pence : 5 pence.

When we thus compare the ratios existing between two pairs of quantities, it is not necessary that all *four* quantities should be of the same kind; it is only necessary that *each pair* should be of the same kind.

For example, we can compare the ratio of 4 shillings to 7 shillings with the ratio of 7 days to 12 days, and finding that $\frac{4}{7}$ is less than $\frac{7}{12}$, we may say that the ratio of 4 shillings to 7 shillings is less than the ratio of 7 days to 12 days.

200. When the ratio symbol (:) is placed between two *numbers*, we may substitute for it the fraction symbol.

Thus, if we have to compare the ratios 2 : 3 and 5 : 7, we effect it by comparing the fractions $\frac{2}{3}$ and $\frac{5}{7}$.

201. Ratios are *compounded* by multiplying together the fractions by which they are represented, and expressing the resulting fraction as a ratio.

Thus, the ratio compounded of 2 : 3 and 5 : 7 is 10 : 21.

2 and 3 are called the **Terms** of the ratio 2 : 3.

2 is called the **Antecedent** and 3 the **Consequent** of the ratio.

202. Ratios are either *direct* or *inverse*.

A *direct* ratio is the quotient of the antecedent divided by the consequent.

An *inverse* ratio, or reciprocal ratio, is the quotient of the consequent divided by the antecedent.

Examples c.

1. Compare the ratios 2 : 5 and 4 : 9.

2. Compare the ratios 17 : 39 and 19 : 41.

3. Compare the ratios 4 : 7, 8 : 15, and 13 : 24.

4. Compound the ratios 5 : 7, 13 : 15, 21 : 91, and 45 : 52.

5. Compound the ratios $3\frac{1}{2} : 4$, $3\frac{1}{5} : 7$, $1\frac{1}{3} : 3\frac{1}{4}$, $2\frac{1}{6} : 1\frac{2}{7}$.

6. If the ratio be 25 and the consequent $1.25, what is the antecedent?

7. How much does the ratio $36 \times 4 \times 3 : 12 \times 16 \times 2$ exceed that of $60 \div (3 \times 5) : 20 \times 2 \div 8$?

8. What is the reciprocal ratio of $\frac{1}{3} : \frac{1}{6}$; of $2\frac{1}{3} : 7.\dot{7}$?

9. *A* owns a farm of 180 ac. There are 36 sq. mi. in the township in which it is situated. What is the relation of the latter to the former?

10. The ratio 63 : 52 results from compounding four ratios together; three of these are 7 : 8, 12 : 15, and $\frac{1}{2} : \frac{1}{5}$. Express the fourth ratio in its simplest form.

11. What effect has adding the same quantity to both terms of a ratio?

12. *A* and *B* run a mile race. *A* wins. If *B* had run $\frac{1}{3}$ faster he would have won by 11 yd. Compare their rates.

13. If the time past 9 a.m. is to the time past 1 p.m., as 7 to 2, find the time.

Proportion.

203. Proportion consists in the equality of two ratios.

The Arithmetical test of Proportion is therefore *that the two fractions representing the ratios must be equal.*

Thus the ratio 6 : 12 is equal to the ratio 4 : 8, because the fraction $\frac{6}{12}$ = the fraction $\frac{4}{8}$.

The four numbers 6, 12, 4, 8, written in the order in which they stand in the ratios, are said to be *in proportion*, or *proportionals*, and this relation is thus expressed:—

$$6 : 12 = 4 : 8.$$

The two terms 6 and 8 are called the **Extremes.**
" " " 12 and 4 " " " **Means.**

The sign of equality is usually expressed thus, :: and then the ratios read 6 is to 12 as 4 is to 8.

204. When four numbers are in proportion,
the product of the extremes = the product of the means.

For example, if 6 : 12 :: 4 : 8,

$$6 \times 8 = 12 \times 4.$$

For, since $\frac{6}{12} = \frac{4}{8}$, by hypothesis,

and $\frac{6\times8}{12\times8} = \frac{6}{12}$,

and $\frac{4\times12}{8\times12} = \frac{4}{8}$;

$$\therefore \frac{6\times8}{12\times8} = \frac{4\times12}{8\times12}.$$

Now the *denominators* of these fractions are equal, and therefore the numerators must also be equal, that is,

$$6 \times 8 = 4 \times 12.$$

From this it is evident that if three out of the four numbers that form a proportion are given, we can find the fourth.

Ex. 1. Find a fourth proportional to 3, 15, 7.

3 : 15 = 7 : number required;
∴ 3 × number required = 15 × 7;
∴ number required = $\frac{15\times7}{3}$ = 35.

Ex. 2. What number has the same ratio to 9 that 3 has to 5?

3 : 5 = number required : 9;
∴ 5 × number required = 3 × 9;
∴ number required = $\frac{27}{5} = 5\frac{2}{5}$.

205. Three numbers are said to be in **Continued Proportion** when the ratio of the first to the second is equal to the ratio of the second to the third.

Thus, 3, 6, 12 are in continued proportion,

for $\frac{3}{6} = \frac{6}{12}$.

The second number is called a **Mean Proportional** between the first and the third.

Ex. Find a mean proportional between 6 and 24.

$6 : \text{required number} = \text{required number} : 24.$
$\therefore \text{required number} \times \text{required number} = 6 \times 24;$
$\therefore \text{square of required number} = 144;$
$\therefore \text{required number} = 12.$

206. When two quantities are connected in such a way that when one is increased 2, 3, ... times, the other is also increased 2, 3, ... times, they are in *direct* proportion.

For example, if 1 lb. of sugar cost 9c.,
2 lb. will cost 2 × 9c.;
3 lb. " " 3 × 9c.;
hence, 7 lb. " " 7 × 9c.,
and 25 lb. " " 25 × 9c.;
∴ 7 lb. : 25 lb. :: 7 × 9c. : 25 × 9c.

That is, the *cost* of sugar is *directly* proportional to its *weight.*

207. When two quantities are connected in such a way, that when one is *increased* 2, 3, ... times, the other is *diminished* 2, 3, ... times, they are *inversely* proportional. Thus, if one man can mow a field in 12 days, 2 men can mow it in half the time, or in $\frac{12}{2}$ days; 3 men in a third of the time, or in $\frac{12}{3}$ days, etc.;

hence, 4 men can mow it in $\frac{12}{4}$ da.;
and 12 " " " " $\frac{12}{12}$ da.;
∴ 4 men : 12 men :: $\frac{12}{12}$ da. : $\frac{12}{4}$ da.;

that is, the *number of men* required to do a certain work is *inversely* proportional to the number of days, or *vice versa.*

Examples ci.

1. Arrange 4, 3, 9, and 12 so that they may be in proportion.

2. Find the second term when 18, $2.\dot{6}$, and $1.\dot{8}$ are the other three terms of a proportion.

3. Find a mean proportional to .038 and .00152.

4. If $A = 3\frac{1}{3}$ of B, and $C = 5\frac{1}{5}$ of B, find the ratio of A to C.

5. Find a fourth proportional to 5, 7, and 15.

6. Find a fourth proportional to $\frac{2}{3}$, $\frac{4}{7}$, and $\frac{5}{9}$.

7. Find a fourth proportional to .3, .16, and .09.

8. Find a mean proportional to 14 and 56.

9. Find a mean proportional to $\frac{5}{63}$ and $\frac{35}{81}$.

10. The first term of a proportion is 6.8; the third and fourth terms are .5, and 1.3, respectively. Find the second term.

11. The first and second terms of a proportion are 30 ft. and $12\frac{1}{2}$ ft., respectively, and the fourth term is $650. Find the third term.

12. What number bears the same ratio to $\frac{3}{5}$ that $\frac{2}{7}$ does to $\frac{5}{8}$?

13. Divide $1587 among A, B, C and D, so that A's share $:$ B's share $= 6 : 5$, B's share $:$ C's share $= 4 : 3$, and C's share $:$ D's share $= 3 : 2$.

14. The estate of a bankrupt worth $19687.50 is to be divided among four creditors. The debts due to A and B, are as $2 : 3$; to B and C, as $4 : 5$; and to C and D, as $6 : 7$. What must each receive?

15. Divide $166.50 among A, B, and C, in proportion to $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$, respectively.

16. A debt of $1254 is paid in $10 bills, $5 bills, $2 bills, and $1 bills. The number of each denomination is in proportion to 5, 7, 9, and 11, respectively. How many of each kind were there?

Simple Proportion, or Rule of Three.

208. When *three* terms of a proportion are given to find the *fourth*, it is **Simple Proportion.** In a simple proportion we have two ratios given. One of these has both terms, the other is incomplete, having only one term. Two of the given terms must be of one kind, and the third and the answer of another kind.

Ex. 1. If 5 horses eat 20 bushels of oats in a given time, how many bushels will 8 horses eat in the same time?

Here the number of bushels consumed is *directly* proportional to the number of horses.

Hence 5 : 8 :: 20 bu. : bu. required;

$\therefore$ bu. required $= \frac{8 \times 20}{5} = 32.$

Ex. 2. If 6 men can do a piece of work in 5 days, in what time can 9 men do the same work?

Here the time is *inversely* proportioned to the number of men.

Hence 9 : 6 :: 5 da. : days required;

$\therefore$ days required $= \frac{6 \times 5}{9} = 3\frac{1}{3}.$

Ex. 3. If 3 cwt. 25 lb. of hay cost $2.21, what should 3 tons 5 cwt. cost?

Here the cost is *directly* proportional to the quantity.

Hence, 3 cwt. 25 lb. : 3 t. 5 cwt. :: $2.21 : dollars required.

Here we reduce the 1st and 2nd terms to the common denomination, pounds, and the proportion becomes.

325 : 6500 :: $2.21 : dollars required;

$\therefore$ dollars required $= \frac{6500 \times 2.21}{325} = \$44.20.$

From these examples we deduce the following rule:

Write the given number that is of the same kind as the required fourth term, for the third term of the proportion. Then consider from the nature of the question whether the answer is to be greater or less than the third term. If greater, place the larger of the two remaining numbers in the second place; if less, in the first. Then having reduced the first and second terms to the same denomination, multiply the measures of the second and third terms together, and divide the product by the measure of the first term. The quotient will be the measure of the answer required.

NOTE.—After the third term has been written down, the order of the other two may be ascertained by a question. Thus, in Ex. 1: "If 5 horses eat 20 bu., will 8 horses eat more or less than 20 bu.?" More; hence

5 : 8. In Ex. 2: "If 6 men do a piece of work in 5 days, will it take 9 men a longer or shorter period than 5 days?" Shorter; hence 9 : 6.

Examples cii.

1. A person after paying an income tax of 7d. in the £ has a net income of £1247 10s. 5d. What was the gross income?

2. A watch which is 10 min. too fast at 12 o'clock noon on Monday, gains 3 min. 10 sec. a day. What will be the time by the watch at a quarter past 10 a.m. on the following Saturday?

3. In running a 3 mile race on a course $\frac{1}{3}$ of a mile round, *A* overlaps *B* at the middle of the 7th round. By what distance will *A* win at the same rate of running?

4. A watch was $6\frac{7}{11}$ min. slow at noon; it loses 12 min. in $20\frac{1}{2}$ hours. Find the true time when its hands are together for the fourth time after noon.

5. If 4 men or 6 women or 9 boys can perform a piece of work in $27\frac{1}{2}$ da., in what time can (*a*) 5 men and 9 women perform it? and (*b*) 5 men and 8 boys perform it?

6. If $14\frac{3}{8}$ shares of a property are worth $116.15, what are $5\frac{5}{6}$ shares worth?

7. A floor can be covered by $32\frac{1}{2}$ yd. of carpet 7 quarters wide. How many yards of Brussels carpet 26-in. width will cover the same room?

8. Two clocks, of which one gains 4 min. 15 sec. and the other loses 3 min. 15 sec. in 24 hr., were both within $2\frac{1}{2}$ min. of the true time, the former fast and the latter slow, at noon on Monday; they now differ from one another by half an hour. Find the day of the week and the hour of the day.

9. If 6336 stones $3\frac{1}{4}$ ft. long complete a certain quantity of wall, how many similar stones of $2\frac{3}{8}$ ft. long will raise a like quantity?

10. A beseiged town, containing 22400 inhabitants, has provisions to last 3 weeks. How many must be sent away that they may be able to hold out 7 weeks?

Compound Proportion.

209. Where *five*, *seven*, *nine*, etc., terms of a proportion are given, to find a *sixth*, *eighth*, *tenth*, etc., term, it is called **Compound Proportion** or the **Double Rule of Three.**

In Compound Proportion there are three or more ratios given, all being complete but one.

A Compound Proportion is produced by multiplying together the corresponding terms of two or more simple proportions.

Thus, 12 : 6 :: 4 : 2
9 : 3 :: 6 : 2
5 : 4 :: 10 : 8 multiplied together produce the proportion 540 : 72 :: 240 : 32.

Ex. If 6 men in 8 days, working 10 hours a day, can reap 24 acres of wheat, how many acres could 10 men reap in 15 days of 12 hours each?

$$\begin{array}{r}6 : 10 :: 24 : \text{acres required.}\\ 8 : 15 \phantom{:: 24 : \text{acres required.}}\\ 10 : 12 \phantom{:: 24 : \text{acres required.}}\\ \hline 480 : 1800 :: 24 : \text{acres required;}\end{array}$$

$$\therefore \text{acres required} = \frac{1800\times24}{480} = 90.$$

24, the term of the imperfect ratio, is put in the 3rd place. The other ratios are then considered separately and treated as in Simple Proportion. After all the ratios have been stated, all the first terms are multiplied together for a new first term and similarly with the second terms. The answer is then got as in Simple Proportion.

NOTE I.—Before compounding the complete ratios it is convenient to cancel all the factors common to the 1st term, and to the 2nd or 3rd terms. When any of the 1st and 2nd terms are not of the same denomination, they must be reduced to a common denomination before proceeding with the solution.

NOTE II.—Before stating the question it is convenient to write down the terms of the supposition under one another and opposite these to place the corresponding terms of the demand with an x opposite the term of the same name as the answer required.

Thus, in the above example, 6 men 10,
8 days 15,
10 hours 12,
24 acres x.

Examples ciii.

1. If 18 men in 12 da. build a wall 40 ft. long, 3 ft. thick, and 16 ft. high, how many men must be employed to build a wall 120 yd. long, 8 ft. thick, and 10 ft. high, in 60 da. ?

2. An engineer engages to complete a tunnel $3\frac{3}{4}$ miles long in 2 yr. and 10 mo. For a year and a half he employs 1200 men, and then finds he has completed only three-eights of his work. How many additional men must he employ to complete it in the required time ?

3. Two sets of men perform the same amount of work. Each man in the first set is stronger than each man in the second, in the ratio of 7 to 6. The first set works 6 da. a week for 10 weeks; and the second set 5 da. a week for 7 weeks. If there are 9 men in the first set, how many are there in the second ?

4. If 20 men can excavate 185 cubic yd. of earth in 9 hr., how many men could do half the work in a fifth of the time ?

5. At the siege of Sebastopol it was found that a certain length of trench could be dug by the soldiers and navvies in 4 da., but that when only half the navvies were present it required 7 da. to dig the same length of trench. Compare the amount of work done by the navvies with that done by the soldiers.

6. Two elephants which are 10 in length, 9 in breadth, 36 in girth, and 7 in height, consume one *drona* of grain. How much will be the rations of 10 other elephants, which are a quarter more in length and other dimensions ?

7. How many revolutions will be made by a wheel which revolves at the rate of 360 revolutions in 7 min., while another wheel, which revolves at the rate of 470 in 8 min., makes 658 revolutions ?

8. A piece of work is to be done in 36 da. 15 men work at it 15 hr. a day, but after 24 da. only $\frac{3}{5}$ of it is done. If three more men are put on, how many hr. a day must all work to finish it in the given time ?

9. If 248 men, in $5\frac{1}{2}$ da. of 12 hr. each, dig a ditch of 7 degrees of hardness, $232\frac{1}{2}$ yd. long, $3\frac{2}{3}$ yd. wide, and $2\frac{1}{3}$ yd. deep, in how many days of 9 hr. each will 24 men dig a ditch of 4 degrees of hardness, $387\frac{1}{2}$ yd. long, $5\frac{1}{4}$ yd. wide, and $3\frac{1}{2}$ yd. deep ?

10. If 5 compositors in 16 da. of 11 hr. each can compose 25 sheets, of 24 pages in a sheet, 44 lines in a page, and 40 letters in a line, in how many days, of 10 hr. each, can 9 compositors compose a volume (to be printed in the same kind of type), consisting of 36 sheets, 16 pages to a sheet, 50 lines to a page, and 45 letters to a line?

11. If 12 men, working 9 hr. a day for $15\frac{5}{9}$ da., were able to execute $\frac{2}{3}$ of a job, how many men may be withdrawn and the job be finished in 15 da. more, if the laborers are employed only 7 hr. a day?

12. If a pane of glass 18 in. long and $12\frac{1}{2}$ in. wide cost 20c., what will be the cost, at the same rate, of a pane $22\frac{1}{2}$ in. long and 15 in. wide?

13. A miller has a bin 8 ft. long, $4\frac{1}{5}$ ft. wide, and $2\frac{1}{2}$ ft. deep, and its capacity is 75 bu. How deep must he make another bin which is to be 18 ft. long and $3\frac{5}{6}$ feet wide, that its capacity may be 575 bu.?

14. A ship's crew of 300 men were so supplied with provisions for 12 mo. that each man was allowed 30 oz. per day. But after having been 6 mo. on their voyage they found it would take 9 mo. longer to finish it, and 50 of their number had been lost. It is required to find the daily allowance of each man during the last 9 mo.

15. A trench 920 ft. long, 17 ft. wide, and 10 ft. deep, has been dug by 7 men and 2 boys. The work could have been done in the same time by 6 men and 5 boys. What length of a trench, 15 ft. wide and 12 ft. deep, could be dug by 5 men and 3 boys in half the time?

16. Five men are employed 7 hr. a day on a certain work. After 4 da. they have done $\frac{1}{12}$ of it. If 6 additional men are put on to assist them, when will the work be finished, if all work $7\frac{1}{2}$ hr. a day?

17. If 15 men, 12 women, and 9 boys can complete a piece of work in 50 da., what time would it take 9 men, 15 women, and 18 boys to do $2\frac{1}{4}$ times as much, the parts done by a man, a woman, and a boy being as 3, 2, 1?

For additional examples see page 330.

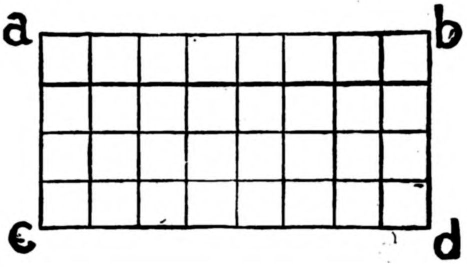
CHAPTER XVIII.

MENSURATION.

The Rectangle.

210. The unit of measurement, by which we measure Area or Surface, is derived from the unit of Length. Thus, if we take an inch as the unit of length, and construct a square whose side is an inch, this Square Inch may be taken as the unit of Area, and the *measure* of of any given area will be the number of times it contains this unit, in accordance with the remarks in Art. 33.

Let **a b d c** be a rectangle, and let the side **a b** be 8 inches in length, and the side **a c** 4 inches in length.



Then, if the Unit of Length be an inch, the *measure* of **a b** is 8, and the *measure* of **a c** is 4.

Divide **a b, a c** into eight and four equal parts, respectively, and draw lines through the points of division parallel to **a c, a b,** respectively. Then the rectangle **a b d c** is divided into a number of equal *squares*, each of which is a square inch.

If one of these squares be taken as the Unit of Area, the *measure* of the area of **a b d c** will be the number of these squares.

Now this number is the same as that obtained by multiplying the measure of **a b** by the measure of **a c**;

that is, measure of **a b d c** = 4 × 8 = 32;

∴ area of **a b d c** is 32 sq. in.

Hence, to find the area of a rectangle, *multiply the measure of the length by the measure of the breadth, and the product will be the measure of the area.*

Ex. 1. A rectangular garden is 48 feet long, and 25 feet broad. What is its area?

Taking a foot as the unit of length, and therefore a square foot as the unit of area,

$$\text{measure of the area} = 48 \times 25 = 1200;$$
$$\therefore \text{ the area is } 1200 \text{ sq. ft.}$$

Ex. 2. A rectangular board is 2 feet 7 inches long, and 1 foot 4 inches broad. What is its area?

Taking 1 in. as the unit of length, and therefore 1 sq. in. as the unit of area,

$$\text{measure of the area} = 31 \times 16 = 496;$$
$$\therefore \text{ the area is } 496 \text{ sq. in.}$$

Or, we might take 1 ft. as the unit of length, and then

$$\text{measure of area} = 2\tfrac{7}{12} \times 1\tfrac{1}{3} = \frac{31 \times 4}{12 \times 3} = \tfrac{31}{9} = 3\tfrac{4}{9};$$
$$\therefore \text{ the area is } 3\tfrac{4}{9} \text{ sq. ft.}$$

Ex. 3. The length of the side of a square croquet-ground is 49 yards. What is its area?

Taking 1 yd. as the unit of length,

$$\text{area} = (49 \times 49) \text{ sq. yd.} = 2401 \text{ sq. yd.}$$

NOTE.—Observe the difference between the expressions 49 *yards square* and 49 *square yards.* The former refers to a square whose side is 49 yards, and whose area is 2401 square yards. The latter, to a surface whose area is 49 square yards.

Examples civ.

1. Find the area of the rectangles having the following dimensions :—

(*a*) 7 ft. by 5 ft.
(*b*) $13\frac{1}{2}$ ft. by 10 ft.
(*c*) $22\frac{1}{4}$ ft. by $13\frac{1}{2}$ ft.
(*d*) 5 ft. 4 in. by 2 ft. 3 in.
(*e*) 17 ft. 5 in. by 8 yd. 2 ft.
(*f*) 5 yd. 1 ft. by 4 yd. 2 ft.
(*g*) 12 yd. 2 ft. by 5 yd. 1 ft.
(*h*) 7 yd. 2 ft. by 5 yd. 6 in.

2. Find the area of the squares whose sides have the following lengths :—

(*a*) $5\frac{1}{2}$ yd.
(*b*) $37\frac{1}{2}$ yd.
(*c*) $17\frac{3}{4}$ ft.
(*d*) $29\frac{1}{2}$ ft.
(*e*) 9 ft. 7 in.
(*f*) 3 ft. 4 in.
(*g*) 7 yd. 1 ft. 5 in.
(*h*) 15 yd. 2 ft. 3 in.

3. Find the breadth of the following rectangles, having given the area and length :—

(*a*) Area 176 sq. ft., length 11 ft.
(*b*) Area 71 sq. ft. 100 sq. in., length 9 ft. 8 in.
(*c*) Area 854 sq. ft. 84 sq. in., length 97 ft. 8 in.
(*d*) Area 1 ac., length 440 yd.
(*e*) Area 5 ac., length 275 yd.
(*f*) Area 5 ac. 1 ro. 36 po., length 267 yd. 2 ft.

4. What are the sides of the squares whose areas are

(*a*) 1178 sq. yd. 7 sq. ft.
(*b*) 33 ac. 4305 sq. yd.

5. The perimeter of a square and a rectangle are each 160 in. Find the difference in their areas, the sides of the rectangle being in the ratio of 2 to 3.

Carpeting Rooms.

211. If we know the area of the floor of a room, we know how many square inches of carpet will be required to cover it. Carpets are sold in strips, and when the *width* of a strip is known, we shall know how much *length* of carpet will be required to cover a given surface.

For instance, if the surface be 162 square feet, and the carpet selected be 27 inches wide, we reason thus,

$$162 \text{ sq. ft.} = 162 \times 144 \text{ sq. in.};$$

$$\therefore \text{ length of carpet required } = \frac{162 \times 144}{27} \text{ in.} = 864 \text{ in.} = 24 \text{ yd.}$$

Examples cv.

1. How many yards of carpet, 27 in. wide, will be required for rooms whose dimensions are

(*a*) 15 ft. by 13 ft.
(*b*) 22 ft. 4 in. by 20 ft. 3 in.
(*c*) 35 ft. 4 in. by 27 ft. 3 in.
(*d*) 25 ft. by 12 ft. 6 in.
(*e*) 27 ft. by $14\frac{1}{2}$ ft.

2. Find the expense of carpeting rooms whose dimensions are

(*a*) 18 ft. by 14 ft., with carpet 30 in. wide @ $1 a yard.

(*b*) 22 ft. by 15½ ft., with carpet 27 in. wide @ $1.80 a yard.

(*c*) 29 ft. 9 in. by 23 ft. 6 in., with carpet a yard wide @ $1.08 a yard.

(*d*) 34 ft. 8 in. by 13 ft. 3 in., with carpet ¾ yd. wide @ 3s. 4½d. a yard.

Papering the Walls of a Room.

212. To find the quantity of paper required to cover *one* wall of a room, we find the area of the surface of the wall by taking the product of the measures of the length and breadth of that wall, the latter being the same as the height of the room. Hence, we shall find the area of the *four* walls of the room *if we take the measure of the compasss of the room and multiply it by the measure of the height.*

By the compass of a room, we mean the length of a string stretched tight on the floor, and going all round the room.

Deductions for doors, windows, and fire-place must be made in practice.

Suppose, then, we have to find how much paper is required for the walls of a room whose length is 22 feet 3 inches, breadth 17 feet 4 inches, and height 9 feet 6 inches.

We first find the compass of the room, thus,

$$\begin{array}{rr l}
\text{ft.} & \text{in.} & \\
22 & 3 & \multirow{4}{*}{} \\
17 & 4 & \left.\begin{array}{c}\\ \\ \\ \\ \end{array}\right\} \text{dimensions of the four sides.} \\
22 & 3 & \\
17 & 4 & \\
\hline
79 & 2 &
\end{array}$$

To get the area of paper required, we multiply the measure of the compass of the room by the measure of the height, thus,

$$\text{area} = (9\tfrac{1}{2} \times 79\tfrac{1}{6}) \text{ sq. ft.} = \frac{19 \times 475}{12} \text{ sq. ft.} = 752\tfrac{1}{12} \text{ sq. ft.}$$

NOTE.—Papers, like carpets, are sold in strips, and if we know the width of a strip, we shall know how many feet in length will be required to cover a given surface.

Thus, in the room under consideration, if the paper be 20 in. wide,
length of paper required $= (752\frac{1}{12} \div \frac{20}{12})$ ft. $= \frac{9025}{20}$ ft. $= 451\frac{1}{4}$ ft.

Examples cvi.

1. How many square feet of paper will be required for rooms whose dimensions are :—

(*a*) Length, 19 ft.; breadth, 16 ft.; height, 9 ft. ?
(*b*) Length, $24\frac{1}{2}$ ft.; breadth, $18\frac{1}{4}$ ft.; height, 10 ft.?
(*c*) Length, 25 ft. 7 in.; breadth, 19 ft. 4 in.; height, 9 ft. 9 in.?
(*d*) Length, 23 ft. 5 in.; breadth, 18 ft. 7 in.; height, 9 ft. 6 in. ?

2. Find the expense of papering rooms whose dimensions are :—

(*a*) Length, 18 ft.; breadth, 14 ft.; height, 8 ft.; with paper 16 in. wide @ 20c. a yard.

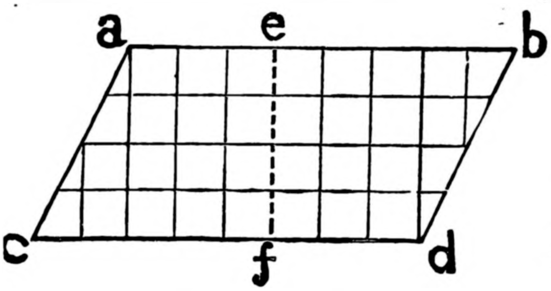
(*b*) Length, 20 ft. 6 in.; breadth, 17 ft. 4 in.; height, 9 ft.; with paper 20 in. wide @ 10c. a yard.

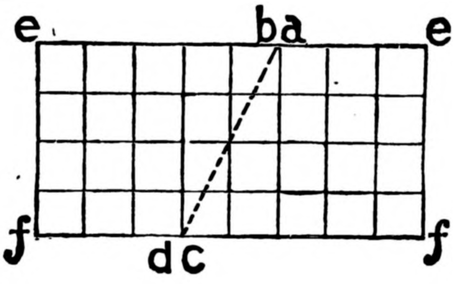
(*c*) Length, 30 ft. 8 in.; breadth, 26 ft. 5 in.; height, 10 ft. 6 in.; with paper 2 ft. wide @ 8d. a yard.

(*d*) Length, 26 ft.; breadth, 21 ft.; height, 10 ft.; with paper 20 in. wide @ 9d. a yard, allowing for a fireplace which is 5 ft. 3 in. by 4 ft., a door which is 7 ft. by $4\frac{1}{2}$ ft., and two windows, each 6 ft. by $3\frac{1}{2}$ ft.

The Parallelogram.

213. A Parallelogram is a quadrilateral figure whose opposite sides are parallel.





From the above figures it is easily seen that a parallelogram can be changed into a rectangle whose length is the length of the parallelogram and whose breadth is the perpendicular width, or altitude, of the parallelogram.

Examples cvii.

1. Find the area of the following parallelograms :—

(*a*) 7 po. in length and 22 yd. in width.

(*b*) 7 yd. 2 ft. in length and $2\frac{3}{4}$ yd. in width.

(*c*) 17 ft. 9 in. in length and 14 ft. 3 in. in width.

(*d*) 3 po. 3 yd. in length and $17\frac{1}{2}$ yd. in width.

2. A parallelogram is 352 yd. long and contains 10 ac. Find its width.

3. Find the area of a field half as long and half as wide as that in example 2.

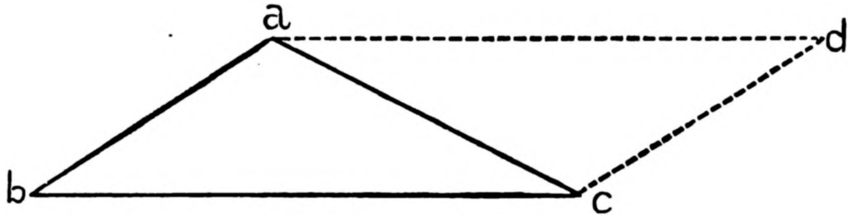
4. If the parallel sides of a garden are 198 ft. long and the perpendicular distance between them is 55 ft., what is the garden worth @ $400 per acre ?

5. There are two fields in the shape of parallelograms of equal areas. Their lengths are 1344 yd. and 1134 yd., respectively. The width of the former is 945 yd. Find the width of the latter.

6. The length of a parallelogram is 88 ft. If its width is increased by 8 ft., the area will be 616 sq. yd. Find the original width.

The Triangle.

214. To find the area of a Triangle.



By drawing a straight line through **a** parallel to **b c** and another through **c** parallel to **b a,** it becomes evident that the area of the triangle **a b c** is half the area of a parallelogram on the same base and of the same altitude as the triangle. Hence, *to find the area of a triangle, take half the area of a parallelogram whose base is equal to that of the triangle and whose breadth is equal to the altitude of the triangle.*

Examples cviii.

1. Find the area of each of the following triangles, the base and perpendicular upon the base from the opposite angle being respectively:

(*a*) 22 ft. 6 in. and 9 ft. 4 in.

(*b*) 12 ft. 9 in. and 9 in.

(*c*) 45 chains 16 links and 24 chains.

2. The area of a triangle is 63 sq. yd. The length of one side is 42 ft. Find the length of the perpendicular upon this side from the opposite angle.

3. The area of a triangle is 5 ac. 36 po. and its base is $3\frac{3}{4}$ chains long. Find the length of the perpendicular from the opposite angle upon the base.

4. The area of a triangle is 134 sq. yd. 64 sq. in. The perpendicular from an angle to the opposite side is 10 ft. 2 in. long. Find the length of this side.

Irregular Quadrilaterals.

215. A Trapezoid is a quadrilateral having two of its sides parallel.

216. To find the area of a Trapezoid.

When the lengths of the parallel sides and the perpendicular distance between them are known, by drawing a diagonal, it is obvious that the Trapezoid is divided into two triangles of which the bases and perpendicular heights are known, and hence their areas may be determined and thus the area of the Trapeziod is determined.

Ex. Find the area of a trapezoid, the parallel sides of which are 20 inches and 14 inches long, respectively, and the perpendicular distance between them 9 inches.

$$\begin{aligned} \text{Area of a triangle with base 20 in.} &= (\tfrac{20}{2} \times 9)\text{ sq. in.}\\ \text{“ \ “ \ “ \ \ “ \ \ \ “ \ \ “ \ 14 in.} &= (\tfrac{14}{2} \times 9)\text{ sq. in.}\\ \therefore \text{ “ \ \ “ \ “ trapezoid} &= \{(\tfrac{20}{2} + \tfrac{14}{2}) \times 9\}\text{ sq. in.}\\ &= \{(\tfrac{20+14}{2}) \times 9\}\text{ sq. in.}\\ &= 153\text{ sq. in.} \end{aligned}$$

Hence, to find the area of a Trapezoid,

Multiply the measure of half the sum of the two parallel sides by the measure of the perpendicular distance between them, and the result will be the measure of the area.

217. To find the area of any quadrilateral, the diagonal and perpendiculars on it from the opposite angles being given.

It is obvious that the diagonal divides the quadrilateral into two triangles, the area of each of which can be found from the measurements given.

Ex. Find the Area of an irregular quadrilateral, the diagonal of which is 50 inches long and the perpendiculars upon it from the opposite corners are 25 inches and 31 inches respectively.

$$\begin{aligned} \text{Area of triangle with altitude 25 in.} &= (\tfrac{25}{2} \times 50)\text{ sq. in.}\\ \text{“ \ \ “ \ \ \ “ \ \ \ \ “ \ \ \ \ “ \ \ \ 31 in.} &= (\tfrac{31}{2} \times 50)\text{ sq. in.}\\ \therefore \text{ “ \ \ “ quadrilateral} &= \{(\tfrac{25}{2} + \tfrac{31}{2}) \times 50\}\text{ sq. in.}\\ &= \{(\tfrac{25+31}{2}) \times 50\}\text{ sq. in.}\\ &= (28 \times 50)\text{ sq. in.}\\ &= 1400\text{ sq. in.} \end{aligned}$$

Hence, to find the Area of an irregular quadrilateral,

Multiply the measure of half the sum of the two perpendiculars by the measure of the diagonal, and the result will be the measure of the area.

Examples cix.

1. Find the area of a trapezoid whose parallel sides are 85 ft. and 110 ft., and the distance between them is 200 ft.

2. How many acres are there in a field in the form of a trapezoid, the parallel sides being 650 links and 850 links, and the distance between them $2\frac{1}{2}$ chains?

3. The area of a trapezoid is 306 sq. yd. and the parallel sides are 81 ft. and 72 ft. in length. Find the distance between them.

4. Find the surface of a board 18 in. wide at one end, 25 in. at the other, and 16 ft. long.

5. A B C D is an irregular quadrilateral. The diagonal A C is 760 links long, and the perpendiculars upon A C from B and D are 1 chain and 1 chain 18 links, respectively. Find the area of A B C D.

6. The longest diagonal of an irregular quadrilateral figure is 63 yd. and the perpendiculars let fall upon it from the remaining angles are 9 ft. 6 in. and 13 ft. 10 in. Find the area.

The Right-Angled Triangle.

Ex. 1. A rectangular bowling-green is 56 yards long and 42 yards broad. Find the distance from corner to corner.

218. By Euclid I. 47, we know that in a right-angled triangle the square on the side *opposite* the right angle is equal to the sum of the squares on the sides *containing* the right angle.

Hence, the square of the measure of the side opposite the right angle is equal to the sum of the squares of the measures of the sides containing the right angle.

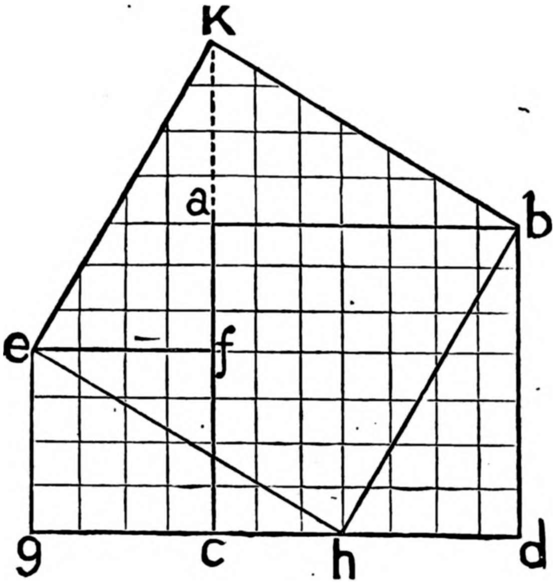
Thus, in our present example,

$$\text{square of measure of distance from corner to corner}$$
$$= (56 \times 56) + (42 \times 42) = 4900;$$
$$\therefore \text{distance is 70 yd.}$$

For those who have not studied geometry, the following concrete proof is given:—

In figure, page 248, take **h d** = **g c**. Then the square on **g c** is equal to the square on **h d.** **b h d** is a right-angled triangle, and the squares **a b d c**, **e f c g** are the squares on its sides. Move each of the triangles **h e g**, **b h d** along the hypothenuse of the other, without rotation into positions **a b k**, **e f k**. The figure formed is the square on **b h,** the hypothenuse of the right-angled triangle

b h d, and is evidently equal to the sum of the squares on its sides.



Examples cx.

1. Find the hypothenuse of each of the following right-angled triangles whose base and perpendicular are respectively :—

(*a*) 40 ft. and 42 ft.
(*b*) 119 in. and 120 in.
(*c*) 153 ft. and 104 ft.
(*d*) 210 yd. and 176 yd.

2. Find the base of each of the following right-angled triangles, the hypothenuse and perpendicular being respectively :—

(*a*) 410 in. and 168 in.
(*b*) 617 ft. and 105 ft.
(*c*) 1013 yd. and 45 yd.
(*d*) 557 in. and 165 in.

3. Find the perpendicular of the following right-angled triangles, the base and hypothenuse being respectively :—

(*a*) 510 yd. and 514 yd.
(*b*) 2380 ft. and 2381 ft.
(*c*) 624 in. and 820 in.
(*d*) 1950 ft. and 2146 ft.

4. A ladder 41 ft. long stands erect close to the wall of a building. How many inches will its top fall if the foot is drawn out 9 ft. from the wall ?

5. A rectangular field is 330 yd. long and 104 yd. wide. Find the distance from corner to corner along the diagonal.

6. One end of a rope 145 ft. long is tied to the top of a pole 144 ft. high, and the other is fastened to a peg in the ground.

If the pole is vertical and the rope tight, find how far the peg is from the centre of the pole at the ground.

7. Find the cost of fencing in a piece of ground in the form of a right-angled triangle whose base is 792 ft. and perpendicular 1175 ft. @ 10c. per yd.?

8. A rectangular plantation, whose width is 88 yd., contains $2\frac{1}{2}$ ac. Find the distance from corner to corner on the diagonal.

9. The area of a square is 390625 sq. ft. What is the length of the diagonal?

10. A man carrying a ladder 50 ft. long, places it upon the street in such a position that it will exactly reach a window 28 ft. high on one side, or another window 36 ft. high on the other side. Find the width of the street.

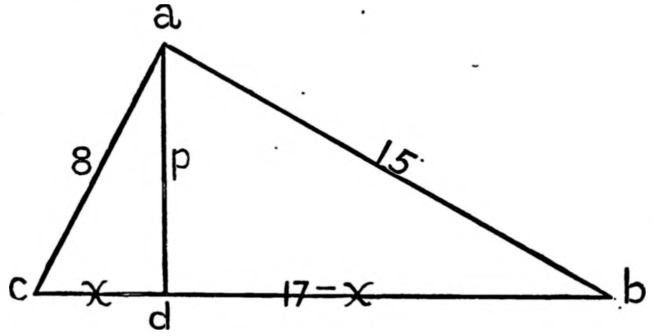
11. A tree 98 ft. high breaks off and the top strikes the ground 84 ft. from the centre of the tree at the ground. Where did the tree break?

12. A rope 106.6 ft. long will just reach from one side of a street to the top of a house 87 ft. high, exactly opposite. How wide is the street?

13. Each equal side of an isosceles triangle is twice as long as the base, which is 48 ft. long. Find the altitude of the triangle?

219. To find the length of a perpendicular, let fall upon the longest side of a triangle from the opposite angle.

Ex. 1. The sides of a triangle are 8, 15, and 17 units in length. Find the length of the perpendicular upon the longest side from the opposite angle.



In the diagram let A B contain 15 units; A C, 8 units; C B, 17 units; the perpendicular, p units; and C D, x units.

$$\text{Thus } p^2 + (17-x)^2 = 225. \ (1);$$
$$\text{also } p^2 + x^2 = 64 \ (2);$$

Subtracting (2) from (1)

$$\therefore 289 - 34x = 161;$$
$$\therefore x = \tfrac{64}{17}.$$

Substitute this value for x in (2)

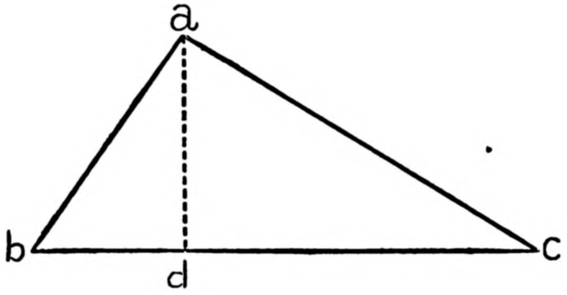
$$\text{and } p^2 + (\tfrac{64}{17})^2 = 64;$$
$$\therefore p = \tfrac{120}{17} = 7\tfrac{1}{17}.$$

Examples cxi.

1. Find the length of the perpendicular dropped upon the longest side of each of the following triangles from the opposite angle.—

(*a*) Sides are 3, 4, and 5 in. long, respectively.
(*b*) " " 5, 12, and 13 in. " "
(*c*) " " 7, 24, and 25 in. " "
(*d*) " " 9, 40, and 41 in. " "
(*e*) " " 11, 60, and 61 in. " "

220. To find the area of a triangle, the lengths of the three sides being given.



Let ABC be a triangle having its three sides AB $= c$, BC $= a$, and AC $= b$.

Let the perpendicular AD be drawn on BC.

Let BD $= x$.

Then DC will equal $a - x$.

Now, $\text{AD}^2 = c^2 - x^2$, and $\text{AD}^2 = b^2 - (a-x)^2$.

$$\therefore c^2 - x^2 = b^2 - (a-x)^2 = b^2 - a^2 + 2ax - x^2$$
$$\therefore 2ax = a^2 + c^2 - b^2$$
$$\therefore x = \frac{a^2 + c^2 - b^2}{2a} = \text{BD}$$

$$\begin{aligned}
\text{But } AD^2 &= c^2 - x^2 \\
&= c^2 - \left(\frac{a^2 + c^2 - b^2}{2a}\right)^2 \\
&= \left(c + \frac{a^2 + c^2 - b^2}{2a}\right)\left(c - \frac{a^2 + c^2 - b^2}{2a}\right) \\
&= \frac{(a + b + c)(a + c - b)(b + a - c)(b + c - a)}{4a^2}
\end{aligned}$$

$$\therefore AD = \frac{1}{2a}\sqrt{(a + b + c)\cdot(a + c - b)(b + a - c)(b + c - a)}$$

$\therefore$ area of triangle

$$\begin{aligned}
&= \frac{a}{2} \times \frac{1}{2a}\sqrt{(a + b + c)(a + c - b)(b + a - c)(b + c - a)} \\
&= \tfrac{1}{4}\sqrt{(a + b + c)(a + c - b)(b + a - c)(b + c - a)}
\end{aligned}$$

Now, let $a + b + c = 2s$ (1)

Subtract $2b$ and

$$a + b + c - 2b = 2s - 2b$$

or $a + c - b = 2(s - b)$

Similarly $b + a - c = 2(s - c)$

$b + c - a = 2(s - a)$

$\therefore$ area of triangle

$$\begin{aligned}
&= \tfrac{1}{4}\sqrt{2s \times 2(s - b) \times 2(s - c) \times 2(s - a)} \\
&= \frac{4}{4}\sqrt{s(s - b)(s - c)(s - a)} \\
&= \sqrt{s(s - a)(s - b)(s - c)}.
\end{aligned}$$

RULE.—*From half the sum of the three sides subtract each side separately. Multiply the measures of the half sum and of the three remainders together, and extract the square root of the product. The result will be the measure of the area.*

Examples cxii.

1. Find the area of the triangles whose sides are, respectively, as follows:—

(*a*) 57 yd., 60 yd., and 111 yd.
(*b*) 50 ft., 40 ft., and 30 ft.
(*c*) 125 yd., 85 yd., and 60 yd.
(*d*) 13 ch., 14 ch., and 15 ch.
(*e*) 29 ft., 52 ft., and 69 ft.
(*f*) 375 ch., 143 ch., and 296 ch.

2. The sides of a quadrilateral figure are 123 ft., 208 ft., 116 ft., and 231 ft., respectively, and the diagonal, from the first to the third corners, is 325 ft. Find the area.

3. The three sides of a triangle are 13 ft., 14 ft., and 15 ft. Find the length of the three perpendiculars from the angles on the opposite sides.

4. A B C D is a four-sided figure. B C is parallel to A D; A B, B C, and C D are each 325 ft. long, and A D is 733 ft. Find the area.

5. The area of a triangle is 690 sq. ft., and the lengths of the perpendiculars from the angles on the opposite sides are $47\frac{17}{29}$ ft., $26\frac{7}{13}$ ft., and 20 ft., respectively. Find the triangle.

The Circle.

221. It is found that if the length of the circumference of a circle be divided by the length of the diameter, the quotient is 3.1415..., or about $3\frac{1}{7}$. This is usually denoted by the Greek letter π.

In the following examples regard π as $3\frac{1}{7}$.

Examples cxiii.

1. Find the circumference of each of the following circles:—

(*a*) Diameter is 14 in. long.
(*b*) Diameter is 6.3 ft. long.
(*c*) Diameter is 7912 mi. long.
(*d*) Diameter is 483 ft. long.

2. Find the diameter of each of the following circles whose circumferences are, respectively:—

(*a*) 187 ft.
(*b*) 68.2 ft.
(*c*) 11 ft. 11 in.
(*d*) 25.3 mi.

3. Find the circumference of each of the following circles whose radii are, respectively:—

(*a*) 1 ft. 9 in.
(*b*) 5 yd. 1 ft. 4 in.
(*c*) 4 ch. 76 l.
(*d*) 7 yd. 2 ft. 4 in.

4. The radius of a circle is 10 ft. Find the perimeter of the semicircle.

5. The radius of a circle is 5 ft. 3 in. Find the length of an arc of 10°; of 12°; of 16°; of 75°.

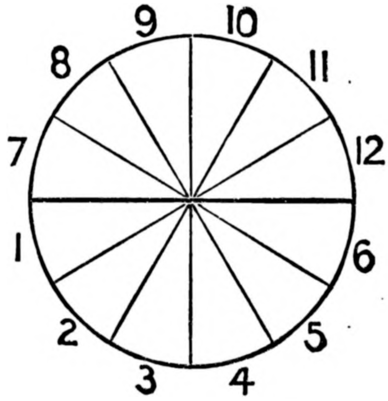
6. A farmer's roller is 6 ft. 3 in. long, and $2\frac{1}{4}$ ft. in diameter. Find the area of the surface passed over in making 140 complete revolutions.

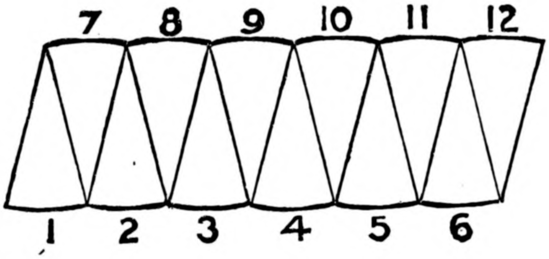
7. The inner circumference of a circular road is 3872 ft. long. The road is 42 ft. wide. Find the length of the outer circumference.

8. The end of a string is fastened by a tack to the edge of a rectangular block 4 in. by 6 in., and is wound round the block. If it is unwound in the plane in which it is now situated, and always kept stretched, find the length of the curve, which is traced by the extremity of the string in one complete revolution.

9. The minute-hand of a tower clock is $10\frac{1}{2}$ ft. long. How many miles does its extremity travel during the month of September?

10. A locomotive running at the rate of 45 mi. per hour has a driving-wheel which makes 3 revolutions per second. Find the diameter of the wheel?





222. If a circle be divided, as in the figure on the left, and the parts rearranged, as in the other figure, the area of the circle will equal the area of the parallelogram. The length of such a parallelogram is half the circumference of the circle and its width is half the diameter. The product of the measure of half of the circumference and the measure of half the diameter will be the measure of the area of the parallelogram or the circle.

Hence, to find the area of a circle we have the following rule:—

Multiply the measure of half the length of the circumference by the measure of the radius, and the product is the measure of the area.

$$\text{Since } \frac{c}{d} = \pi, \therefore c = \pi d = 2\pi r.$$

$$\text{And, area} = \tfrac{1}{2} c \times r = \tfrac{1}{2} \times 2\pi r \times r = \pi r^2.$$

Examples cxiv.

1. Find the area of each of the following circles:—

(*a*) Diameter 42 ft.	(*d*) Circumference 77 ch.
(*b*) Diameter 70 in.	(*e*) Radius 1760 yd.
(*c*) Circumference 308 in.	(*f*) Radius 110 ft.

2. Find the radius of the circle whose area is

(*a*) 12474 sq. ft.	(*c*) 98.56 sq. yd.
(*b*) $38\frac{1}{2}$ sq. yd.	(*d*) 1386 sq. ch.

3. Out of a circular piece of paper 18 in. in radius, a circle 17 in. in radius is cut. Find the area of the part left.

4. A circular field contains one acre. Find the length of the fence enclosing it.

5. The radius of a circle is 84 in. Find the radius of another circle sixteen times as large.

6. The side of a square is 4 ft. 8 in. Find the area of the inscribed circle.

7. The circumference of a circle is 88 in. long. Find the side of a square inscribed in this circle.

8. A road runs round a circular fair ground. The outer circumference of the road is 880 yd. long, and the inner one is 792 yd. Find (i.) the width of the road, (ii.) the area of the road, and (iii.) the area of the grounds within the road.

9. The difference between the diameter and the circumference of a circle is 75 in. Find its area.

10. In the grounds of a gentleman there is a circular pond with a gravel walk round its margin. The area of the pond is 2464 sq. yd., and that of the walk is $1886\frac{1}{2}$ sq. ft. Find the width of the walk.

11. The radius of the outer boundary of a ring is 52 in. long. The area of the ring is 2134 sq. in. Find the circumference of the inner boundary.

12. The radius of a circle is 84 ft. long. Find the radius of another circle of $\frac{1}{4}$ the area.

13. A circle and a square are of equal areas. The side of the square is 198 in. long. Find the circumference of the circle.

14. A circle and a square have the same perimeter, viz., 7744 in. Find the area of each.

223. A **Sector** of a circle is a figure bounded by two radii and the arc between them.

224. To find the area of a Sector.

It is evident that the area of the sector must bear the same ratio to the area of the circle as the length of its arc bears to the circumference of the circle.

$$\text{Hence, measure of area of sector} : \text{measure of area of circle} :: l : 2\pi r,$$

where l is the measure of length of the arc of the sector.

$$\therefore \text{ measure of area of sector} = \frac{\text{measure of area of circle} \times l}{2\pi r}$$

$$= \frac{\pi r^2 \times l}{2\pi r}$$

$$= \frac{lr}{2}.$$

Hence, *Multiply the measure of the length of the arc of the sector by the measure of the radius and divide the product by two, the result is the measure of the area of the sector.*

Examples cxv.

1. The radius of a circle is 12 ft., and the length of an arc of a sector is 6 ft. Find the area of the sector.

2. The length of an arc of a sector is 2 ft. 6 in., and the radius of the circle is 4 ft. 4 in. Find the area of the sector.

3. Find the area of a sector whose radius is 14 ft., and which subtends an arc of 18°.

4. The area of a sector is 56 sq. ft. The area of the circle is 616 sq. ft. Find the length of arc of the sector.

5. The radius of a circle is 21 in. Find the area of the sector and the length of the arc which subtends an angle of 115°.

6. The area of a circle is 3850 sq. ft., and the area of a sector of this circle is 1540 sq. ft. Find the arc of the sector (i) in degrees, (ii) in feet.

Similar Surfaces.

225. Rectilineal Figures are similar when

(i) They are equiangular; and

(ii) They have their sides about the equal angles proportional.

Thus, all regular polygons are similar, as equilateral triangles, squares, regular pentagons, circles, etc.

The Areas of Similar Surfaces are to one another as the squares of the measures of corresponding lines of the surfaces.

Ex. 1. A map of a county is drawn on the scale of 1 inch to 30 feet. Find what space on the map will be occupied by a farm of 144000 square yards.

$$\text{Area on map} : 144000 \text{ sq. yd.} :: 1^2 : 360^2$$

$$\therefore \text{measure of area on map} = \frac{144000 \times 1}{360 \times 360}$$

$$= 1\tfrac{1}{9}.$$

$$\therefore \text{area on map} = 1\tfrac{1}{9} \text{ sq. yd.}$$

Examples cxvi.

1. The sides of a rectangle are in proportion of 3 to 4, and its area is 768 sq. in. Find the rectangle.

2. Determine the scale used in the construction of a plan upon which a square foot of surface represents an area of 10 ac.

3. A pipe $\frac{1}{2}$ in. in diameter will fill a cistern in 20 min. How long will be required to fill it when there is a discharge pipe of $\frac{1}{3}$ in. in diameter opened at the same time.

4. One of the sides of a field containing 10 ac. is 42 ch. long. What is the area of a similar field whose corresponding side is 28 ch. long?

5. The area of a quadrilateral is 1323 sq. ft., and one of its diagonals is 63 ft. Find the area of a similar quadrilateral in which the corresponding diagonal is 61 ft.

6. A map is constructed on the scale of 20 mi. to an inch. What is the area of a county represented on the map by $3\frac{1}{5}$ sq. in.?

7. Of two circles the area of the first is 120 sq. ft., and the diameter of the second is $\frac{3}{4}$ of that of the first. Find the area of the second circle.

8. The side of one square is $\frac{4}{5}$ of that of a second, and the area of the smaller square is 1 ac. Find the area of the larger.

9. The parallel sides of a trapezoid are respectively $10\frac{1}{2}$ ft. and 18 ft. in length, and the non-parallel sides are, respectively, 16 ft. and 12 ft. long. These are produced to meet. Find the respective lengths of the produced sides between the point of meeting, and the longer of the parallel sides of the trapezoid.

10. Find the difference between the perimeter of a square field containing 10 ac. and the perimeter of a rectangular one of equal area, the length of the latter being 4 times its width.

11. If it costs $360 to fence a square field @ $3 per rod, what would it cost to fence a rectangular field of the same area, the sides being in the ratio of 4 to 9?

12. A pipe $\frac{1}{2}$ in. in diameter will fill a cistern in 20 min. How long a time will be required to fill it where there is (i) a second supply pipe $\frac{1}{4}$ in. in diameter opened at the same time, (ii) a discharge pipe $\frac{1}{4}$ in. in diameter opened at the same time?

Miscellaneous Examples.

1. The area of a square garden is 4 ro. 1 po. 29 sq. yd. $6\frac{3}{4}$ sq. ft. Find the length of its side.

2. Find the expense of turfing a plot of ground which is 40 yd. long, and 100 ft. wide, with turfs each a yard in length and 1 ft. in breadth, the turfs, when laid, costing 6s. 9d. per hundred.

3. A square room, whose floor measures 32 sq. yd. 1 sq. ft., is 11 ft 6 in. in height. Find the expense of whitewashing its ceiling and walls @ 5c. per square yard.

4. It costs $99 to cover the floor of a room $8\frac{1}{4}$ yd. long by $6\frac{2}{3}$ yd. wide with carpet 2 ft. wide. Find the price of the carpet per yard.

5. If the cost of papering a room $8\frac{1}{4}$ yd. long, and $6\frac{2}{3}$ yd. wide, with paper 2 ft. wide @ 4d. per yard, be £2 19s. 8d., find the height of the room.

6. The length of a room is 21 ft. and its height 10 ft. 6 in., and the area of the floor is $\frac{5}{11}$ of the area of the four walls. Find the breadth of the room.

7. What length must be cut off a board which is $6\frac{3}{4}$ in. broad, that the area may contain a square foot?

8. What is the expense of papering a room 4 yd. $6\frac{3}{4}$ in. long, 3 yd. $11\frac{1}{4}$ in. wide, and 3 yd. 1 ft. high, with paper half a yard wide @ 12c. a yard.

9. How many stones, each 2 ft. long and $15\frac{1}{2}$ in. wide, would be required to pave a square courtyard whose side is 124 ft.?

10. Find the cost of papering a room 21 ft. long, 15 ft. wide, and 12 ft. high, with paper $2\frac{1}{2}$ ft. wide @ 15c. per yard, allowing for a door 7 ft. high and 3 ft. wide, two windows each 5 ft. high and 3 ft. wide, and a pannelling 2 ft. high round the floor.

11. The length of one side of a rectangular field is 572 yd., and the area of the field is 50 ac. 2 ro. 32 po. Find the length of the other side and of the diagonal.

12. A rectangular field, 300 yd. long, and 150 broad, is separated into 4 equal parts by 2 bands of trees, 20 ft. wide, parallel to the sides. How large will each part be, and what will be the area covered by the trees?

13. A room, whose height is 11 ft., and length twice its breadth, takes 143 yd. of paper 2 ft. wide for its four walls. How many yards of moulding will be required?

14. What will be the cost of painting the walls and ceiling of a room whose height, length, and breadth are 12 ft. 6 in., 27 ft. 4 in., and 20 ft., respectively, @ 36c. per sq. yd.?

15. If the cost of carpeting a room 11 yd. long and 8 yd. wide, with carpet @ 3s. a yard, be £19 16s., find the width of the carpet.

16. How many flagstones, each 5.76 ft. long, and 4.15 ft. wide, are requisite for paving a cloister which incloses a rectangular court 45.77 yd. long, and 41.93 yd. wide, the cloister being 12.45 ft. wide?

17. The four sides of a field are 75 ch., 100 ch., 125 ch., and 200 ch. long, respectively. The first two sides form a right angle. Find the area in acres.

18. The shadow of a man standing upright and 5 ft. 8 in. high, was found to measure 8 ft. 4 in. The shadow of a steeple, measured at the same time, was found to be 325 ft. How high is the steeple?

19. The sides of a rectangle are 16 and 12. Find the distance between the feet of the perpendiculars drawn from opposite vertices to a diagonal.

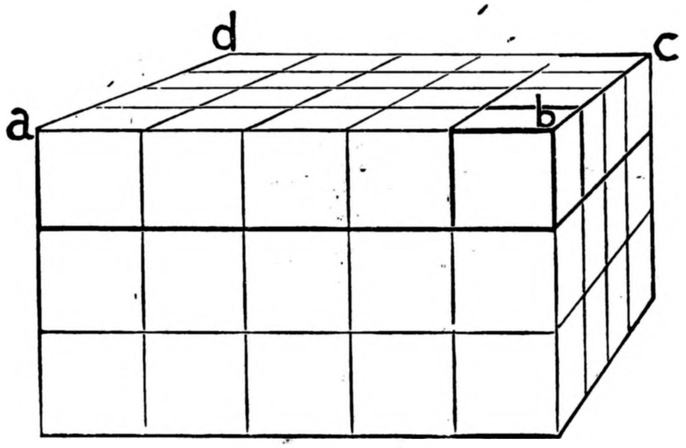
20. The sides of a triangle are proportional to the numbers 13, 20, 21. Its area being 1134 sq. ft., find the sides in feet.

21. The length of a sector is 36°, and its area is 385 sq. ft. Find the length of its arc.

Rectangular Solids.

226. The Unit of Measurement, by which we measure the Volume of a Solid body, or the Capacity of a vessel, is derived from the Unit of Length. Thus, if we take an inch as the unit of length, and construct a *cube*, each of whose edges is an inch in length, this Cubic Inch may be taken as the Unit of Volume; and the measure of any given volume will be the number of times it contains this unit.

227. Let **a b c d** be a rectangle, and let the side **a b** be 5 inches in length, and the side **a d** 4 inches in length.



Then **a b c d** will contain 20 square inches.

Now, suppose we construct a number of blocks of wood, perfect cubes, whose volume is a cubic inch, and place one of these on each of the squares in **a b c d**, and then place another of the blocks on the top of each of the first set, and so on, till we have piled up 3 layers. Then we shall have constructed a rectangular solid, whose length is 5 inches, breadth 4 inches, and depth or thickness, 3 inches.

Now, the number of cubic inches in this solid we estimate in the following way: for each of the squares in **a b c d** we shall have a pile of 3 cubic inches. Therefore, the number of cubic inches in the solid will be 3×20, or 60.

Hence, we obtain the following Rule:—

To find the cubic content of a rectangular solid, find the continued product of the measures of the length, breadth, and thickness, and the result is the measure of the cubic content.

Ex. 1. Find the cubic content of a rectangular piece of timber whose length is 47 feet, breadth 4 feet, and thickness 3 feet.

Taking a foot as the unit of length, and therefore a cubic foot as the unit of cubic content,

$$\text{measure of cubic content} = 47 \times 4 \times 3 = 564;$$
$$\therefore \text{ the cubic content} = 564 \text{ cu. ft.}$$

Ex. 2. What is the cubic content of a room whose length is 22 feet 6 inches, breadth 18 feet 3 inches, and height 10 feet?

$$\begin{aligned}\text{Cubic content} &= (22\tfrac{1}{2} \times 18\tfrac{1}{4} \times 10) \text{ cu. ft.}\\ &= \frac{45 \times 73 \times 10}{2 \times 4} \text{ cu. ft.} = 4106\tfrac{1}{4} \text{ cu. ft.}\end{aligned}$$

Ex. 3. A rectangular sheet of water, of uniform depth, is 430 yards long, 270 yards broad, and contains 7314300 cubic feet of water. What is its depth?

Reducing the length and breadth to feet,

$$\text{area of surface} = (430 \times 3 \times 270 \times 3) \text{ sq. ft.};$$
$$\therefore \text{ depth} = \frac{7314300}{430 \times 3 \times 270 \times 3} \text{ ft.} = 7 \text{ ft.}$$

Examples cxvii.

1. Find the cubic content of the rectangular solids whose dimensions are,

(*a*) 8 ft., 7 ft., 6 ft.
(*b*) $10\frac{1}{2}$ ft., $8\frac{1}{4}$ ft., $6\frac{1}{3}$ ft.
(*c*) 5 ft. 6 in., 4 ft. 3 in., 3 ft. 7 in.
(*d*) 11 ft. 8 in., 9 ft. 10 in., 7 ft. 5 in.
(*e*) 6 yd. 2 ft. 4 in., 3 yd. 1 ft. 7 in., 4 ft. 11.

2. Find the volume of a cube, the length of whose edge is $7\frac{1}{2}$ in.

3. A rectangular block of marble is 4 ft. 3 in. long, 2 ft. 6 in. wide, and 3 ft. deep. Find (i) its surface, (ii) its volume.

4. The surface of a cube contains 384 sq. in. Find (i) the length of its side, and (ii) its volume.

5. A rectangular solid contains 147 cu. ft. It is 8 ft. long by $3\frac{1}{2}$ ft. deep. Find its width.

6. How many bricks will be required to build a wall 75 ft. long, 6 ft. high, and 18 in. thick, each brick being 9 in. long, $4\frac{1}{2}$ in. wide, and 3 in. deep?

7. A lake, whose area is 45 ac., is covered with ice 3 in. thick. Find the weight of the ice in tons, if a cubic foot of ice weigh 920 oz. avoirdupois.

8. If 500 men excavate a basin 800 yd. long, 500 yd. wide, and 40 yd. deep, in 4 mo., how many men will be required to excavate a basin 1000 yd. long, 400 yd. wide, and 50 yd. deep, in 5 mo.?

9. A square block of stone, 2 ft. in thickness, is in cubic content 5 cu. ft. 24 in. What is the length of its edge?

10. What weight of water will a rectangular cistern contain, the length being 4 ft., the breadth 2 ft. 6 in., and the depth 3 ft. 3 in., when a cubic foot of water weighs 1000 oz.?

11. A block of stone is 4 ft. long, $2\frac{1}{2}$ ft. broad, and $1\frac{1}{4}$ ft. thick. It weighs 27 cwt. Find the weight of 100 cu. in. of the stone.

12. A cubic foot of water weighs 1000 oz. Find the length of the side of a cubic vessel whose contents (water) weigh 4 t. 12 cwt. 3 qr. 10 lb. 7 oz. (112 lb. = 1 cwt.)

13. If 120 men can make an embankment $\frac{3}{4}$ of a mile long, 30 yd. wide, and 7 yd. high, in 42 da., how many men would it take to make an embankment 1000 yd. long, 36 yd. wide, and 22 ft. high, in 30 da.?

14. A rectangular cistern, 9 ft. long, 5 ft. 4 in. wide, and 2 ft. 3 in. deep, is filled with liquid which weighs 2520 lb. How deep must a rectangular cistern be which will hold 3850 lb. of the same liquid, its length being 8 ft., and its width 5 ft. 6 in.?

15. Find the cost of making a road 110 yd. in length, and 18 ft. wide; the soil being first excavated to the depth of 1 ft., at a cost of 1s. per cubic yard; rubble being then laid 8 in. deep, @ 1s. per cubic yard, and gravel placed on the top, 9 in. thick, @ 2s. 6d. per cubic yard.

The Cylinder.

228. A Cylinder is a solid, bounded by two circular faces and a curved one, every part of which is the same distance from a straight line joining the centres of the circular faces.

229. If a cylinder be taken and the curved surface be exactly covered with paper, it will be found that the paper is in the form of a rectangle, whose length is equal to the circumference of the cylinder, and whose width is the length of the cylinder. Hence, to find the **surface** of a cylinder, we have the following rule:—

Multiply the measure of the circumference by the measure of the length of the cylinder. The product will be the measure of the area of the curved surface. To this product add the measure of the area of the two ends, and the sum will be the measure of the area of the entire surface of the cylinder.

230. To find the cubic content of a Cylinder.

Multiply the measure of the area of one end by the measure of the length of the cylinder, and the product will be the measure of the cubic content.

Ex. 1. Find the number of cubic feet of iron in a water pipe 3 feet in diameter, 12 feet long, the iron being 1 inch thick.

$$\begin{aligned}
\text{Radius of water pipe} &= 18 \text{ in.}\\
\text{Radius of opening} &= 17 \text{ in.}\\
\text{Area of end of pipe} &= (3\tfrac{1}{7} \times 18^2) \text{ sq. in.}\\
\text{Area of opening} &= (3\tfrac{1}{7} \times 17^2) \text{ sq. in.}\\
\therefore \text{ area of iron surface in end} &= 3\tfrac{1}{7}\,(18^2 - 17^2) \text{ sq. in.}\\
&= (3\tfrac{1}{7} \times 35 \times 1) \text{ sq. in.}\\
&= \tfrac{110}{144} \text{ sq. ft.}\\
\therefore \text{ the volume of pipe} &= (12 \times \tfrac{110}{144}) \text{ cu. ft.}\\
&= 9\tfrac{1}{6} \text{ cu. ft.}
\end{aligned}$$

Examples cxviii.

1. Find the area of the curved surface of the cylinder whose length and diameter are, respectively,

(*a*) 8 ft. and 7 in,
(*b*) 17 ft. and 1 ft. 9 in.
(*c*) 5 ft. and $3\frac{1}{2}$ in.
(*d*) 24 ft. and $5\frac{1}{4}$ in.

2. Find the total surface of a cylinder whose diameter and height are, respectively,

(*a*) 14 in. and 10 ft.
(*b*) $8\frac{3}{4}$ in. and $2\frac{1}{2}$ ft.
(*c*) $5\frac{3}{5}$ in. and 3 ft.
(*d*) $9\frac{1}{3}$ in. and $7\frac{1}{2}$ in.

3. The curved surface of a cylinder is $3\frac{1}{3}$ sq. ft. and its height is 6 in. Find the area of the ends.

4. A pillar 21 ft. high and 15 in. in diameter is to be decorated @ 35c. per square foot. Find the cost.

5. Find the volume of a cylinder whose diameter and height are, respectively,

(*a*) 4 ft. and 14 ft.
(*b*) 21 in. and 24 ft.
(*c*) $3\frac{1}{2}$ in. and 28 in.
(*d*) $5\frac{1}{4}$ in. and 12 ft.

6. A well is 24 ft. deep and $5\frac{1}{4}$ ft. in diameter. Find the number of cubic yards of earth taken out in digging it.

7. A circular shaft 120 ft. deep and $4\frac{1}{2}$ ft. in diameter is sunk at a cost of $3.50 per cubic yard of earth removed. Find the cost.

8. A flat ring 2 in. high has an outer diameter of 5 ft. 6 in., and the thickness of the metal is 3 in. Find the volume of the ring.

9. Find the number of cubic feet of iron in a water pipe 12 ft. long and 15 in. in radius, the iron being 2 in. thick.

10. How much iron will be required to cast a water pipe 12 ft. long and 19 in. in radius, the iron being 3 in. thick?

11. A hollow cylinder is 6 ft. high, its outer diameter is 6 ft. 2 in., and its inner one 5 ft. 6 in. Find its solidity.

12. How fast must the water rise in a well whose diameter is $3\frac{1}{2}$ ft., so that it may remain at the same depth when a pump is emptying it at the rate of 33 cu. ft. per hour?

13. A bucket is to hold 8 gal. It is 14 in. in diameter with vertical sides. How deep is it, a gallon containing 277.274 cu. in?

14. A vessel in the form of a right cylinder is to hold 4 gal. The depth of the vessel is to equal the length of the diameter of the end. Find the depth.

15 How many coins 1 in. in diameter and $\frac{1}{8}$ in. in thickness can be coined from material in the form of a cube, the edge of which is $5\frac{1}{2}$ in.?

16. A cask of an average internal diameter of 2 ft. 4 in. when full of water weighs $846\frac{1}{2}$ lb. When empty it weighs $44\frac{5}{12}$ lb. Find the depth of water in the cask.

17. An iron roller is in the shape of a hollow cylinder whose length is 4 ft., external diameter 2 ft. 8 in., and thickness 4 in. Find its weight, if a cubic foot of iron weigh 486 lb.

The Pyramid.

231. A Right Pyramid is a solid, bounded by a plane face, enclosed by three or more straight lines, called the base, and as many triangular plane faces as the base has sides.

232. To find the area of the slant surface of a Pyramid.

Multiply the measure of the perimeter of the base by the measure of half the slant height of the pyramid, and the product will be the measure of the area of the slant surface.

Ex. 1. Find the total surface of a square-based pyramid, whose edge measures 3 feet 6 inches, and slant height 5 feet 6 inches.

Measure of perimeter of base = 14.
" " half slant height = $2\frac{3}{4}$.
∴ " " area of " " = $14 \times 2\frac{3}{4} = 38\frac{1}{2}$,
and " " " " base = $5\frac{1}{2} \times 5\frac{1}{2} = 30\frac{1}{4}$,
∴ " " total area = $68\frac{3}{4}$;
∴ area = $68\frac{3}{4}$ sq. ft.

Examples cxix.

1. Find the whole surface of a square pyramid, each side of the base being 15 ft., and the slant height 30 ft.

2. A triangular pyramid is $43\frac{1}{5}$ ft. in slant height, and the sides of its base are $16\frac{1}{2}$ ft., 18 ft., and $14\frac{1}{4}$ ft., respectively. Find the area of the slant surface.

3. Find the lateral surface of a regular hexagonal pyramid, whose side at the base is $4\frac{1}{2}$ ft., and whose slant height is 32 ft.

4. Find the lateral surface of a pyramid whose slant height, measured from the apex to the centre of one of its pentagonal sides of the base, is 10 ft., and each side of the base 18 in.

233. To find the cubic content of a pyramid. Procure a hollow rectangular vessel, and also a hollow pyramid with a base equal to the base of the rectangular vessel and of the same altitude. Fill the pyramid with dry sand, or with water, and when exactly full empty it into the rectangular vessel, and continue to do this until the vessel is full. It is found that the rectangular vessel holds just **three** times as much as the pyramid.

Hence, we have the following rule for finding the cubic content of a pyramid:—

Multiply the measure of the area of the base of the pyramid by the measure of its altitude and divide the result by three, the quotient will be the measure of the cubic content.

Examples cxx.

1. A triangular pyramid covers an area of 63 sq. ft., and the perpendicular height is 30 ft. Find its volume.

2. Find the cubic content of a pyramidal tent which covers a rectangular piece of ground 15 ft. by 18 ft., and is 25 ft. in altitude.

3. Find the solidity of a triangular pyramid whose sides at the base are 4 ft. 7 in., 15 ft. 3 in., and 11 ft. 4 in., respectively, and whose altitude is 66 ft.

4. A pyramid 24 ft. high stands upon a base of 5 sq. ft., and is of uniform density and weighs 335 lb. per cubic foot. Find its weight.

5. The great pyramid of Egypt was 481 ft. high, and its base was 764 ft. sq. Find its volume in cubic yards.

6. Find the volume of a pyramid on a square base whose side is 22 ft., and whose slant height is 61 ft.

The Cone.

234. A Right Cone is a solid bounded by a circular plane face, called the base, and a curved face tapering from the circumference of the base to a point. It may be supposed to be formed by the revolution of a right-angled triangle round one of the sides containing the right angle.

235. If a piece of paper be cut to fit the lateral surface of the cone and then spread out, it will form a sector of a circle, and the area may be found as in Art 224.

Hence, to find the area of the lateral surface of a cone,

Multiply the measure of the circumference of the base of the cone by the measure of the slant height, and divide the product by two. The result will be the measure of the area of the curved surface.

236. To find the total surface of a cone.

It is evident that *the total surface is the Sum of the area of the base and the lateral surface.*

Examples cxxi.

1. How many square yards of canvas are there in a tent 21 ft. in diameter and 18 ft. in slant height?

2. A cone is 7 ft. in diameter; its slant height is 50$\frac{2}{5}$ ft. Find (i) the area of the lateral surface, (ii) its total surface.

3. Find the lateral surface of a cone, the radius of whose base is 3$\frac{1}{2}$ ft., and whose slant height is equal to its circumference.

4. The slant height of a conical spire is 45 ft., and the circumference of the base is 27 ft. The cost of painting it is $40.50. At what rate per square foot is this charged?

5. Find the lateral surface of a cone 38 ft. 6 in. in circumference and 21 ft. in altitude.

6. The area of the lateral surface of a cone is 363 sq. ft.; its slant height is 54 ft. Find the radius of its base.

7. How many square yards of canvas will be required for a conical tent 10 ft. in diameter and 12 ft. high, no allowance being made for seams?

8. A conical galvanized iron vessel has a lid. The diameter of the lid is 4 ft., and the vessel is 5 ft. 10 in. deep. How many square feet of iron are there in the surface of the vessel?

237. To find the cubic content of a cone.

Procure a hollow cylinder and a hollow cone of the same area of base as the end of the cylinder, and of equal altitude, and proceed as in the case of the pyramid. It is found that the cubic content of the cylinder is just three times that of the cone.

Hence, to find the cubic content of a cone,

Multiply the measure of the area of the base of the cone by the measure of the altitude and divide the product by three. The result will be the measure of the cubic content.

Ex. 1. What is the volume of the largest possible cone cut out of a cubical block of stone whose edge is 14 inches.

Diameter of cone = 14 in.
Area of base of cone = 154 sq. in.
Altitude of cone = 14 in.

$$\therefore \text{ measure of cubic content} = \frac{154 \times 14}{3}$$
$$= 718\tfrac{2}{3}$$
$$\therefore \text{ cubic content} = 718\tfrac{2}{3} \text{ cu. in.}$$

Examples cxxii.

1. Find the volume of a cone whose base is 3 ft. 6 in. in radius, and whose altitude is equal to the circumference of the base.

2. Find the volume of a cone, the diameter of whose base is $3\frac{1}{2}$ ft., and whose altitude is 6 ft.

3. A cone 60 ft. high has a cubic content of 3080 cu. ft. Find the diameter of the base.

4. How often can a wine glass in the form of a cone $2\frac{1}{2}$ in. in diameter and 2 in. deep, be filled out of a cylindrical bottle 4 in. in internal diameter, and 7 in. deep?

5. A cone is 8 yd. 1 ft. high, and the diameter of its base is 35 in. Find its volume.

The Sphere.

238. **A Sphere** is a solid, bounded by a curved surface, every part of which is equally distant from a point within it, called the centre.

239. To find the area of the surface of a Sphere.

Bore a hole through the centre of a circular piece of board. Tie over one of the circular faces a thin rubber membrane by means of cord passing round the circumference of the board, and mark on the rubber any small area. Insert a cork and tube into the hole in the board, and introduce water until the rubber is bulged into the form of a hemisphere. Measure the area which was marked on the rubber, and it will be found to be now just twice as large as before the water was introduced. From this it is evident that the area of the curved face of a hemisphere is just twice that of its plane face.

Now, the measure of the area of the plane face $= \pi r^2$;
∴ " " " " " " " curved face $= 2\pi r^2$;
∴ " " " " " surface of sphere $= 4\pi r^2$,
or πd^2.

Hence, *the measure of the area of the surface of a sphere is found by multiplying the square of the measure of the radius by four times* π.

Ex. 1. A globe, whose radius is 28 inches, has to be gilded at a cost of 36 cents per square foot. Find the cost.

$$\text{Measure of area} = 4 \times \tfrac{22}{7} \times 2\tfrac{1}{3} \times 2\tfrac{1}{3};$$
$$\therefore \text{area} = \tfrac{616}{9} \text{ sq. ft.}$$
$$\therefore \text{cost} = \tfrac{616}{9} \times 36\text{c.}$$
$$= \$24.64.$$

Examples cxxiii.

1. Find the surface of each sphere of the following dimensions:—

(*a*) Radius $4\frac{2}{3}$ in.
(*b*) Diameter 35 in.
(*c*) Circumference 66 in.
(*d*) Circumference 132 in.

2. The surface of a sphere contains $17\frac{1}{9}$ sq. ft. Find its radius.

3. The cost of gilding a ball on the top of a spire is $34.65 @ $2\frac{1}{2}$c. per sq. in. What is the diameter of the ball?

4. Find the cost of painting a globe whose diameter is 56 in. @ 8c. per square foot.

5. Find the diameter of a sphere whose surface is 6732 sq. ft.

6. If the diameter of the earth is 7912 mi., find the area of its surface.

7. The surface of a globe is $86\frac{5}{8}$ sq. ft. Find its circumference.

8. The earth being considered a sphere 4,000 mi. in radius, find the scale in miles per inch by which its surface must be represented upon a 12 in. globe.

240. To find the cubic content of a Sphere.

A Sphere may be regarded as composed of a large number of pyramids, the apex of each meeting at the centre of the sphere, and the bases forming its curved face.

Thus, the area of all the bases of the pyramids would be the surface of the sphere and their height would be its radius.

$$\begin{aligned}\text{Hence, measure of area of bases} &= 4\pi r^2\\ \text{And measure of height} &= r\\ \therefore\ \text{measure of cubic content} &= 4\pi r^2 \times \frac{r}{3}\\ &= \frac{4\pi r^3}{3}.\end{aligned}$$

Hence, to find the cubic content of a sphere,

Multiply the cube of the measure of the radius of the sphere by four-thirds of π.

Ex. 1. A solid sphere of iron 6 inches in radius is melted and cast into a hollow cylinder 24 inches long and $6\frac{1}{2}$ inches in radius. Find the radius of the opening in the cylinder.

$$\begin{aligned}\text{Measure of cubic content of sphere} &= \tfrac{4}{3} \times 3\tfrac{1}{7} \times 6^3\\ \text{“ \quad “ \quad “ \quad “ \quad of cylinder} &= 24 \times 3\tfrac{1}{7}\,\{(6\tfrac{1}{2})^2 - r^2\}\\ \therefore\ 24 \times 3\tfrac{1}{7}\,\{(6\tfrac{1}{2})^2 - r^3\} &= \tfrac{4}{3} \times 3\tfrac{1}{7} \times 6^3\\ \therefore\ (6\tfrac{1}{2})^2 - r^2 &= 12;\\ \therefore\ r^2 &= 30\tfrac{1}{4};\\ \therefore\ r &= 5\tfrac{1}{2};\\ \therefore\ \text{radius of opening} &= 5\tfrac{1}{2}\text{ in.}\end{aligned}$$

Examples cxxiv.

1. Find the volume of each sphere of the following dimensions:—

(*a*) 3 in. in radius.
(*b*) 12 in. in diameter.
(*c*) 15 in. in diameter.
(*d*) 44 in. in circumference.

2. A cubical block of wood 1 ft. long is turned into the largest globe possible. How many cubic inches of wood are cut away?

3. The surface of a sphere contains 154 sq. ft. Find its cubic content.

4. A solid metal sphere 8 in. in diameter is melted and formed without loss into shot $\frac{1}{5}$ in. in diameter. How many shot are there?

5. A cubic foot of copper is drawn into a wire one-fourth of an inch in diameter. Find its length.

6. A solid sphere of iron 12 in. in diameter is cast without loss into a hollow cylinder $6\frac{1}{2}$ in. in radius and 8 in. long. Find the thickness of the iron.

7. A sphere and a cube have the same volume. Compare their surfaces.

8. A sphere and a cube have the same surface. Compare their volumes.

9. Iron is 7.8 times as heavy as water. Find the weight of a solid cannon ball 6 in. in diameter.

10. The surface of a sphere is $346\frac{1}{2}$ sq. in. Find its volume.

11. A ball of lead 4 in. in diameter is covered with silver. Find the thickness of the silver in order that the surface of the silver may be twice that of the lead.

Frusta of Pyramids and Cones.

241. A Frustum of a pyramid or cone is the portion of the pyramid or cone included between the base and a plane cutting the solid parallel to the base.

242. To find the lateral surface of a Frustum.

Multiply half the sum of the measures of the perimeters of the ends of the frustum by the measure of the slant height, and the product will be the measure of the area of the lateral surface.

Ex. 1. Find the total surface of the frustum of a cone whose ends are 4 feet and 3 feet in diameter, and whose slant height is 12 feet.

Perimeter of ends $= 4 \times 3\frac{1}{7}$ ft. and $3 \times 3\frac{1}{7}$ ft.
$= 12\frac{4}{7}$ ft. and $9\frac{3}{7}$ ft.

$$\therefore \text{ area of lateral surface} = \left(\frac{12\frac{4}{7} + 9\frac{3}{7}}{2} \times 12\right) \text{ sq. ft.}$$

$= 132$ sq. ft.
Area of ends $= 12\frac{4}{7}$ sq. ft. and $7\frac{1}{14}$ sq. ft.
$\therefore$ total area $= (132 + 12\frac{4}{7} + 7\frac{1}{14})$ sq. ft.
$= 151\frac{9}{14}$ sq. ft.

243. To find the cubic content of the frustum of a pyramid or cone.

Find the areas of the two ends of the frustum; also find a mean proportional (Art. 205*) between these areas. Then multiply the sum of the measures of these three quantities by one-third of the measure of the height of the frustum, and the product will be the measure of the solid content of the frustum.*

Ex. 1. The sides of the ends of the frustum of a square pyramid are $2\frac{1}{2}$ feet and 3 feet, and the distance between them is $1\frac{5}{7}$ feet. Find the volume of the frustum.

Area of ends $= 6\frac{1}{4}$ sq. ft. and 9 sq. ft., respectively.
Mean proportional $= \sqrt{(6\frac{1}{4} \times 9)}$ sq. ft.
$= 7\frac{1}{2}$ sq. ft.
Measure of volume $= \frac{1}{3}$ of $1\frac{5}{7}\,(6\frac{1}{4} + 7\frac{1}{2} + 9)$
$= 13.$
$\therefore$ volume $= 13$ cu. ft.

Examples CXXV.

1. A block of marble is in the form of a frustum of a square pyramid. The side of the smaller end is 1 ft., and that of the larger end 2 ft. 6 in., and the slant height is 16 ft. Find the total surface of the block.

2. Find the lateral surface of the frustum of a cone whose base is $38\frac{1}{2}$ sq. ft., and top $9\frac{5}{8}$ sq. ft., and slant height 15 ft.

3. A square reservoir in the form of an inverted frustum is 144 yd. long at the top, and 121 yd. long at the bottom, and 30 ft. deep. How many gallons of water will it hold?

4. Find the value of a stick of square timber 25 ft. 6 in. long, the girth of the larger end being 10 ft., and of the smaller end 5 ft., @ 25c. per cubic foot.

5. The water in a drain with slanting sides flows at the rate of half a mile per hour. How many gallons flow past a point in 10 min., the water being 25 ft. wide at the surface, 16 ft. at the bottom, and 4 ft. deep?

6. Find the volume of a squared piece of timber, its length being 18 ft., each side of the greater end being 18 in., and of the smaller one 12 in.

7. Find the volume, in cubic feet, of the frustum of a cone, the radii of whose ends are 1 ft. and 4 ft., respectively, the slant height of the frustum being 5 ft.

Similar Solids.

244. Solids are similar when

(i) They are of the same form; and

(ii) They have their corresponding dimensions proportional.

The volumes of similar solids are to one another as the cubes of the measures of their corresponding lines of measurement.

Ex. 1. A sphere 5 inches in diameter weighs 75 ounces. Find the weight of a sphere 4 inches in diameter made of material 25 per cent. heavier than the other.

If the first sphere were of the same material as the second it would weigh $\frac{125}{100}$ of 75 ounces, or $93\frac{3}{4}$ ounces.

$$\text{Hence, weight of second} : 93\tfrac{3}{4} :: 4^3 : 5^3$$

$$\therefore \text{ measure of weight of second} = \frac{93\frac{3}{4} \times 4^3}{5^3}$$

$$= 48$$

$$\therefore \text{ weight of second} = 48 \text{ oz.}$$

Examples cxxvi.

1. If a cube of metal, the edge of which is 1 in., weighs 9 oz., find the weight of a cube of the same metal whose edge is 3 in. long?

2. A sphere 2 in. in diameter weighs 16 oz. Find the weight of a sphere made of the same material 3 in. in diameter.

3. If the thickness of a 25c. piece be to that of a 5c. piece as 7 to 5, compare their diameters.

4. The breadth of a rectangular solid is 20 ft. What must be the breadth of a similar solid whose volume is three times as great?

5. Compare the volumes of two similar cones, the circumferences of whose bases are 20 ft. and 25 ft., respectively.

6. If one edge of a prism is 5 in., and its volume is 64 cu. in., what is the edge of another similar prism whose volume is 27 cu. in.?

7. The height of a right cylinder is $4\frac{1}{3}$ ft. Find the height of a similar cylinder 27 times the volume.

8. If two cubes have their contents, the one double of the other, and the edge of the larger is 1 ft., find the edge of the smaller.

9. There are two similar pyramids whose volumes are 162 cu. in. and 3072 cu. in. If the altitude of the smaller is $4\frac{1}{2}$ in., what is the altitude of the greater.

10. If a solid weighing 27 lb. cost $3.60 to gild, what will a similar solid weighing 125 lb. cost to gild?

11. Find the edge of a cube which is 7 times the volume of a cube, the edge of which is 7 in.

12. The edge of a cube is increased by $\frac{1}{7}$ of itself. By what fraction of itself is the volume increased?

13. Two circular plates of lead, each one inch thick, the diameters of which are 7 in. and 14 in., respectively, are melted and formed into a single circular plate 5 in. thick. Find its diameter.

Miscellaneous Examples.

1. Compare the volumes of a cube 1 ft. in length, a cylinder 1 ft. in diameter and 1 ft. in height, and a sphere 1 ft. in diameter.

2. Find the cost of painting the convex surfaces of 5 cylindrical pillars, each 14 ft. high and a foot in diameter, @ 18c. per square yard.

3. The diameter of a sphere is 6 ft. Find the volume of the largest cube that can be cut from it.

4. The three edges of a rectangular solid that meet at an angle are 25 in., 54 in., and 160 in. Find the edge of a cube which has the same volume.

5. The iron of a spherical shell 12 in. in diameter is $1\frac{1}{2}$ in. thick. Find the number of cubic inches of iron in the shell.

6. A spherical stone is found to displace $22458\frac{1}{3}$ cu. in. of water. Find its diameter.

7. A sphere of gold $\frac{1}{4}$ in. in diameter is beaten out into a circle of gold leaf .000006 in thickness. Find the radius of the circle.

8. Find the solidity of a spherical shell whose inner and outer radii are 14 in. and $10\frac{1}{2}$ in., respectively.

9. The circumference of the base of a cone is 44 ft., and its slant height is $8\frac{3}{4}$ ft. Find the volume of the cone.

10. If water is poured into a cylindrical reservoir 84 ft. in diameter at the rate of 3696 gal. a minute, and a gallon measures $277\frac{1}{4}$ cu. in., find how much the water rises in 2 hr. 24 min.

11. Find the cubic content of an iron ring $3\frac{1}{2}$ in. thick, whose internal and external diameters are 5 ft. and 15 ft., respectively.

12. A globe $18\frac{1}{2}$ in. in diameter weighs 73 lb. Find the weight of a globe of the same material whose diameter is 37 in.

13. A circular hole is to be cut in a circular plate whose diameter is 24 in., so that the weight of the plate may be reduced $\frac{1}{4}$. Find the diameter of the hole.

14. What is the volume of a cylindrical gas holder 140 ft. in diameter and 120 ft. high?

15. Find the length of a cylinder, the radius of whose base is 7 in., and whose volume is equal to the volume of a cube whose edge is 22 in.

16. Find the lateral surface of a cone whose height is 15 in., and the diameter of whose base is 40 in.

17. The cubic content of a box is 100 cu. ft. The depth is 5 ft. 4 in.; the length is three times the width. Find the length and width.

18. A sovereign is $\frac{7}{8}$ in. in diameter and $\frac{1}{16}$ in. in thickness. If 2964500 of them are melted and formed into a cube, find the length of the edge of the cube.

19. Iron is 7.8 times as heavy as water. Find the weight of a rectangular iron box, the outer dimensions being 6 ft long, 4 ft. deep, and 3 ft. 6 in. wide, the iron being 1 in. thick, and there being no lid.

For additional examples see page 332.

CHAPTER XIX.

INTEREST, ANNUITIES, Etc.

Interest.

245. To find the amount of a given sum, in any given time, at Simple Interest.

If P be the principal in *dollars*, n the length of time in years, r the interest of \$1 for 1 yr.; then the interest of \$P for 1 yr. will be Pr, and for n yr. will be Prn; wherefore, if I be the interest, then

$$I = Prn$$

If M be the amount, we have

$$\begin{aligned} M &= P + Prn \\ &= P(1 + rn). \end{aligned}$$

246. To find the amount of a given sum, in any given time, at Compound Interest.

Let P = the principal in *dollars;*
" r = the interest of \$1 for one year;
" n = the number of years;
" R = the amount of \$1 for 1 yr. $= 1 + r$
then PR will be the amount of \$P for 1 yr.,
and this becomes the *Principal* for the 2nd year;

$$\therefore PR.R = PR^2,$$

which will be the amount of \$P for 2 yrs.,
and this becomes the *Principal* for 3rd year;

$$\because PR.^2R = PR^3,$$

which will be the amount of \$P for 3 yr., etc.;

$$\begin{aligned} \text{hence, } M &= PR^n \\ &= P(1 + r)^n, \end{aligned}$$

which will be the amount of \$P for n years.

$$\begin{aligned} \text{Interest} &= PR^n - P \\ &= P(R^n - 1). \end{aligned}$$

247. To show that the formula $M = PR^n$ is true when n is fractional.

If n is fractional we can always find a whole number such that na is a whole number $= q.$, suppose. Divide the year into α equal intervals, and let m be the amount of \$1 in one of these intervals, then the amount of $1 in a intervals is m^{α}, and is equal to R; also the amount of $1 in n years, that is $n\alpha$ intervals, is equal to $m^{n\alpha}$, and, therefore, equal to R^n. Hence, the amount of $P = PR^n; therefore, the formula is true for fractional values of n.

Thus, if r' is the nominal yearly rate of interest of $1 payable q times a year, meaning that $\frac{r'}{q}$ is the interest payable at the end of each qth part of a year, then the amount of $1 in a year $= \$\left(1+\frac{r'}{q}\right)^q$, and the *true* yearly rate of interest $= \$\left(1+\frac{r'}{q}\right)^q - \$1$.

Ex. 1. Find the amount of $100 in $2\frac{1}{2}$ years at 8 per cent., Compound Interest.

$$M = 100\left(1+\frac{8}{100}\right)^{\frac{5}{2}}$$
$$= 100\left\{1+\frac{5}{2}\cdot\frac{8}{100}+\frac{5.3}{1.2.2^2}\left(\frac{8}{100}\right)^2+\frac{5.3.1}{1.2.3.2^3}\left(\frac{8}{100}\right)^3+\ldots\right.$$
$$= 100\,(1+.2+.012+.00016+\cdots)$$
$$= \$121.216\cdots$$

Ex. 2. Find the advantage, when Compound Interest is reckoned, of having the interest paid half-yearly, quarterly, etc., instead of yearly.

The advantage per $1 for a year, when the interest is paid half-yearly, and the half-yearly payment is half the yearly one.

$$=\left(1+\frac{r}{2}\right)^2-(1+r)$$
$$=1+r+\frac{r^2}{4}+\cdots-(1+r)$$
$$=\frac{r^2}{4}.$$

Similarly, when the interest is paid quarterly, the advantage

$$=\frac{3r^2}{8}, \text{ nearly, since } r \text{ is a small fraction.}$$

And generally, when the interest is paid p times a year the advantage

$$= \left(1 + \frac{r}{p}\right)^p - (1 + r)$$

$$= \left(\frac{p-1}{2p}\right) r^2 \text{ nearly.}$$

Ex. 3. Find the amount of a given sum, at compound interest, the interest being supposed due every instant.

If the interest were paid q times per annum, then

$$M = P\left(1 + \frac{r}{q}\right)^{nq}$$

$$= P\left\{1 + nq.\frac{r}{q} + \frac{nq(nq-1)}{1.2}.\left(\frac{r}{q}\right)^2 + \dots\right\}$$

$$= P\left\{1 + nr + \frac{n\left(n - \frac{1}{q}\right)}{1.2} r^2 + \dots\right\}$$

Now, if q be indefinitely great, that is, the intervals between the payments indefinitely small, then, neglecting $\frac{1}{q}$ and its powers, we have

$$M = P\left(1 + nr + \frac{n^2r^2}{1.2} + \frac{n^3r^3}{1.2.3} + \dots\right)$$

$$= Pe^{nr}, \text{ where } e = 2.7182818.$$

Todhunter's Algebra, Art. 542.

Ex. 4. If P represents the population of any place at a certain time, and every year the number of deaths is $\frac{1}{p}$th, and the number of births $\frac{1}{q}$th, of the whole population at the beginning of that year. Required, the amount of population at the end of n years from that time.

At the end of one year from the time the population was P,

$$\text{the increase} = \frac{P}{q} - \frac{P}{p} = P\frac{p-q}{pq}$$

$\therefore$ population at end of 1st yr.

$$= P + P\frac{p-q}{pq} = P\left\{1 + \frac{p-q}{vq}\right\} = P_1, \text{ say.}$$

Similarly, population at end of 2nd yr.

$$= P_1\left\{1 + \frac{p-q}{pq}\right\} = P\left\{1 + \frac{p-q}{pq}\right\}^2;$$

and so on, as in Compound Interest.

Hence, population at end of nth year $= P\left\{1 + \frac{p-q}{pq}\right\}^n$.

248. To deduce the formula for Simple Interest from the formula for Compound Interest.

$$\begin{aligned} M &= PR^n \\ &= P(1+r)^n \\ &= P\left\{1 + nr + \frac{n(n-1)}{1.2}r^2 +, \text{etc.}\right\} \end{aligned}$$

Now, Compound Interest may be regarded as consisting of two parts,

(i) Interest on principal, and

(ii) Interest on interest.

If from the value of M, given above, we take away the part that represents *interest on interest*, there remains the interest on the principal, or the *Simple Interest.* Now, the third term contains r^2 or $r \times r$, that is interest on interest. Similarly for succeeding terms.

Therefore, for Simple Interest we have

$$M = P(1 + nr), \text{ as before.}$$

Hence, any formula for Simple Interest may be deduced from the corresponding one for Compound, by neglecting r^2 and all higher powers.

Therefore, in general, we shall find the formula for Compound Interest, and deduce the corresponding formula for Simple Interest. Indeed, this is the only rational method of treating the subject. There is but one kind of interest, viz., Compound Interest. Simple Interest is incorrect in principle, and, of course, may lead to very incorrect results. When any sum of money is due, it matters not whether it is called principal or interest, it is of value to the owner, and should bear interest. The results obtained by the principle of Simple Interest are merely approximations to the correct results obtained by the principle of Compound Interest.

Examples cxxvii.

1. A sum of \$P is put out at Simple Interest for n years. Find an expression for its amount at the end of that time.

2. If R be the amount of \$1 in one year at any rate of interest, the amount of P dollars in n years will be PR^n, whether n be integral or fractional.

3. If \$P, at Compound Interest, amount to \$M in t years, what sum must be paid down to receive \$P at the end of t years?

4. If \$P at Compound Interest, rate r, double itself in n years, and at rate $2r$, in m years, show that $\frac{m}{n} > \frac{1}{2}$.

5. In what time will a sum of money treble itself @ 5 %, Compound Interest?

$$\log. 3 = .4771212, \log. 1.05 = .0211893.$$

6. A sum of money, \$P, is left among A, B, C, in such a manner that at the end of a, b, c years, when they respectively come of age, they are to possess equal sums. Find the share of each, at compound interest.

7. Two men invest sums of \$4410 and \$4400, respectively, at the same rate of interest, the former at simple, the latter at compound interest. At the end of two years their properties amount to equal sums. Find the rate of interest.

8. In a certain county the births in a year amount to an mth of the whole population, and the deaths to an nth. In how many years will the population be doubled?

9. A person spends in the first year m times the interest of his property; in the second year $2m$ times that of the remainder; in the third year $3m$ times that at the end of the second, and so on; and at the end of $2p$ years he has nothing left. Show that in the pth year he spends as much as he has left at the end of that year.

10. If interest be payable at every instant, in how many years would \$1 amount to \$6, reckoning interest @ 5 %.

11. A person starts with a certain capital, which produces him 4 % per annum, compound interest. He spends every year a sum equal to twice the original interest on his capital. Find in how many years he will be ruined, having given log. 2 = .3010300, log. 13 = 1.1139434.

12. The population of a county is 35743. There is no emigration or immigration. The annual deaths are 27 in the 1000, and

the births 62 in 1000. What will be the increase in the population in five years?

13. If the population of a country be P, and every year the number of deaths is one-sixtieth, and the number of births one-forty-fifth, of the whole population at the beginning of the year, find in what time the population will be doubled.

$$\log. 181 = 2.25768,\ \log. 3 = .4771213.$$
$$\log. 2 = .30103.$$

14. On a sum of money borrowed, interest is paid at the rate of 5 %. After a time $600 of the loan is paid off, and the interest on the remainder reduced to 4 %, and the yearly interest is now lessened one-third. What was the sum borrowed?

15. If a debt a, at Compound Interest, is discharged in n years by annual payments of $\frac{a}{m}$, show that

$$(1 + r)^n (1 - mr) = 1.$$

Discount.

249. To find the Present Worth and Discount on any sum for a given time, (i) at Compound Interest, (ii) at Simple Interest.

The principal difference between Amount and Present Worth is that the former is reckoned *forwards* from a given date, while the latter is reckoned *backwards* from the same date. Hence, it is evident that if V represents the Present Worth, then,

$$V = P(1 + r)^{-n}$$
$$= \frac{P}{(1 + r)^n} \text{ for Compound Interest;}$$

expanding and neglecting r^2 and higher powers we have

$$\frac{P}{1 + nr} \text{ for Simple Interest.}$$

If D be the Discount, then

$$D = P - V$$
$$= P - \frac{P}{(1 + r)^n}, \quad \text{Compound Interest.}$$
$$= P - \frac{P}{1 + nr}, \quad \text{approximately.}$$
$$= \frac{Pnr}{1 + nr}, \quad \text{Simple Interest.}$$

250. If we expand $P(1+r)^{-n}$, and neglect r^2 and higher powers, we get

$$P(1-nr)$$

which may be called the *common* present worth.

The *true* present worth is

$$\frac{P}{1+nr}\text{; by division}$$
$$= P(1-nr+n^2r^2-n^3r^3+\text{, etc.})$$

Subtracting the *common* from the *true* present worth, we have

$$Pn^2r^2(1-nr+n^2r^2-\text{, etc.})$$
$$= Pn^2r^2\frac{1}{1+nr};$$

and, therefore, when n is small the error committed in taking *common* for *true* discount is nearly proportional to the square of the time.

In the expression

$$P(1-nr),$$

if $n=\frac{1}{r}$ the *common* present worth is nothing; while if $n>\frac{1}{r}$ it is a negative quantity. That is, the *common* present worth of a bill for \$100 due 20 yr. hence @ 5 % is nothing, and for any period beyond 20 yr. the holder of the bill would require to pay a certain sum to get quit of it, which is absurd. The *true* present worth of \$100 due in 20 yr., as given by the formula $\frac{P}{1+nr}$, is \$50.

251. *The interest is greater than the discount.*

$$\text{Since } D=\frac{Pnr}{1+nr},$$
$$\frac{1}{D}=\frac{1+nr}{Pnr}$$
$$=\frac{1}{Pnr}+\frac{1}{P}$$
$$=\frac{1}{I}+\frac{1}{P};$$
$$\therefore \frac{1}{D}>\frac{1}{I};$$
$$\therefore I>D.$$

252. Since
$$V = \frac{P}{1 + nr},$$
and
$$D = \frac{Pnr}{1 + nr}$$
$$= \frac{I}{1 + nr},$$
we see that *the Discount is the Present Worth of the Interest.*

253. *The Discount is half the harmonic mean between the Principal and the Interest.*

$$D = \frac{I}{1 + nr}$$
$$= \frac{PI}{P + Pnr}$$
$$= \frac{PI}{P + I}$$
$$= \frac{1}{2}\,\frac{2\,PI}{P + I}$$

= half the harmonic mean between the principal and the interest.

Ex. 1. The Simple Interest on a certain sum of money for a certain time is $28, and the discount for the same time at the same rate of simple interest is $24. What is the sum of money? If the time be 3½ years, what is the rate per cent.?

From the above formula we have
$$24 = \frac{28\,P}{P + 28};$$
$$\therefore 24\,P + 24 \times 28 = 28\,P;$$
$$\therefore 4\,P = 24 \times 28;$$
$$\therefore P = 168;$$
$$\therefore \text{the sum required} = \$168.$$

Again,
$$D = \frac{I}{1 + nr},$$
or
$$24 = \frac{28}{1 + 3\tfrac{1}{2}r};$$
$$\therefore 3 = \frac{7}{2 + 7r};$$
$$\therefore r = \frac{1}{21};$$
$$\therefore \text{rate per cent.} = 100r = \frac{100}{21} = 4\tfrac{16}{21}.$$

Ex. 2. If the Simple Interest on a sum of money for a given time and rate is $\frac{1}{n}$th of that sum itself, the True Discount will be $\frac{1}{n+1}$ of the sum.

$$D = \frac{PI}{P+I};$$

but in this case, $I = \frac{1}{n}P$;

$$\therefore D = \frac{P\frac{1}{n}P}{P + \frac{1}{n}P}$$

$$= \frac{P}{n+1}.$$

Similarly, if the interest be $\frac{a}{b}$ of the principal, the discount is $\frac{a}{a+b}$ of the principal.

Ex. 3. Bank Discount at 5 per cent. being $130.90, find the True Discount on the same amount.

$$\frac{D}{I} = \frac{n}{n+1}, \text{ where } n = \tfrac{5}{100} = \tfrac{1}{20}$$

$$= \frac{20}{21};$$

$$\therefore D = \frac{20}{21} \times \$130.90$$

$$= \$124.66\tfrac{2}{3}.$$

254. *Bank Discount exceeds True Discount by the Simple Interest on the True Discount.*

Bank Discount − True Discount = I − D

$$= Pnr - \frac{Pnr}{1+nr}$$

$$= \left(P - \frac{P}{1+nr}\right)nr$$

$$= \frac{Pnr}{1+nr}\,nr$$

$$= Dnr$$

= Simple Interest on the True Discount.

Ex. 4. The True Discount on a bill due in 1 year, and discounted at 8 per cent., being \$500, what would have been the Bank Discount thereon?

$$\begin{aligned}\text{Bank Discount} &= \text{True Discount} + Dnr\\ &= \$500 + \$500 \times \tfrac{8}{100}\\ &= \$540.\end{aligned}$$

Examples cxxviii.

1. Bank discount being 5 %, a person receives \$37.10 less than the nominal value of his bill. What should he receive for his bill if true discount only were deducted?

2. A person possesses a sum of money, the simple interest of which, @ 4 %, is \$536.25. With this sum he purchases an estate, for which he pays, by a note payable in 4 mo. time, and which, being discounted @ 4 %, is worth at present exactly the money he possesses. For how much is the note drawn?

3. True discount, @ 4 %, on a sum of money, being \$15, fiind the simple interest on the same sum, @ 5 %.

4. The interest on a certain sum of money is \$180, and the discount on the same sum for the same time and the same rate of interest is \$150. Find the sum.

5. If the interest on \$A for a year be equal to the discount on \$B for the same time, find the rate of interest.

6. If the three per cents. are at 90 one month before the payment of the half-yearly dividend, what is the rate of interest.

7. *A* gives *B* a bill for \$$a$, due at the end of m yr., in discharge of a bill for \$$b$, due at the end of n yr. For what sum should *B* give *A* a bill due at the end of p yr. to balance the account, at Compound Interest?

8. Given *A*, my income; a, the premium for assuring \$100; r the rate of interest per cent. per annum. Find what sum I must lay out in assuring my life, so that my executors may receive a sum whose interest will equal my reduced income.

9. *A* sells goods to *B*, and allows him 10 % discount, if he pays in six months. What discount ought he to allow if payment be made in two months @ 5 % per annum, simple interest?

10. The discount on a promissory note of \$100 amounted to \$7.50, and the interest made by the banker was 5.405 %. Find the interval at the end of which the note was payable.

Equation of Payments.

255. To find the equated time of payment of two sums due at different times at a given rate of interest.

Let P_1, P_2, be the sums due at the end of the time n_1, n_2; r the rate of interest; n the equated time. Take time N greater than n_2. Then it is manifest that the amounts of P_1, P_2, at the time N, should in equity be together equal to the amount of their sum $(P_1 + P_2)$, in the same time.

Whence,

$$P_1\left(1+r\right)^{N-n_1} + P_2\left(1+r\right)^{N-n_2}$$
$$=\left(P_1 + P_2\right)\left(1+r\right)^{N-n};$$

or, dividing both sides by $\left(1+r\right)^{N}$, we have

$$P_1\left(1+r\right)^{-n_1}+P_2\left(1+r\right)^{-n_2}=\left(P_1+P^2\right)\left(1+r\right)^{-n} \ldots(1),$$

that is, the present values of these sums, due at their respective times, are equal to the present value of their sum, due at the equated time.

If we expand, neglecting r^2 and higher powers, we have

$$P_1(1-n_1r)+P_2(1-n_2r)=(P_1+P_2)\,(1-nr)$$
$$\text{or, } P_1n_1+P_2n_2=(P_1+P_2)n;$$
$$\therefore\ n=\frac{P_1n_1+P_2n_2}{P_1+P_2}$$

which is the rule given in Art. 180.

256. We have seen (Art. 250) that the expansion of $(1+r)^{-n}$, neglecting r^2 and higher powers, gives *common* present worth, instead of *true* present worth. The above process is, therefore, incorrect. It may easily be seen that we have taken the interest instead of the discount of the sum paid before it is due, and thus, since interest is greater than discount (Art. 251), a small advantage has been given to the payer.

257. If we write equation (i) in the form,

$$\frac{P_1}{(1+r)^{n_1}}+\frac{P_2}{(1+r)^{n_2}}=\frac{P_1+P_2}{(1+r)^n},$$

and expand, neglecting r_2 and higher powers, we have

$$\frac{P_1}{1+n_1 r}+\frac{P_2}{1+n_2 r}=\frac{P_1+P_2}{1+nr},$$

which is the form of the equation for Simple Interest.

Solving for n we get

$$n=\frac{P_1 n_1+P_2 n_2+r(P_1+P_2)n_1 n_2}{P_1+P_2+r(P_1 n_1+P_2 n_2)};$$

which is the correct value of the equated time.

If r be a very small quantity, as in practice it usually is, and P_1, P_2, not very large, we shall have

$$n=\frac{P_1 n_1+P_2 n_2}{P_1+P_2}, \text{ as before.}$$

Annuities.

258. The term Annuity is understood to signify any interest of money, rent, or pension, payable from time to time, at particular periods; and these payments may take place *yearly*, *half-yearly*, *quarterly*, *etc.*

259. To find the Amount of an annuity to be paid for a given number of years, at Compound Interest.

Let A be the annuity, n the number of years, R the amount of one dollar in one year, M the required amount.

We have

Amount due at the end of

$$\begin{aligned}
1 \text{ yr.} &= A;\\
2 \text{ ``} &= A + AR;\\
3 \text{ ``} &= A + AR + AR^2;\\
\text{etc. ``} &= \text{etc.}\\
n \text{ ``} &= A + AR + AR^2 + \ldots. + AR^{n-1}\\
&= A\frac{R^n-1}{R-1}
\end{aligned}$$

$$\text{Hence, } M = A\frac{R^n-1}{R-1}.$$

260. For Simple Interest, expanding and neglecting r^2 and higher powers, we get

$$M = \frac{A}{r}\left\{1 + nr + \frac{n(n-1)}{1.2}r^2 + \cdots - 1\right\}$$

$$= A\left(n + \frac{n}{2}(n-1)r\right).$$

261. To find the Present Value of an annuity, to be paid for a given number of years, at Compound Interest.

(i) The amount of the annuity at the end of the first year is A, while the present value is AR^{-1}; similarly, the amount at the end of the nth year is AR^{n-1}, and the present value is AR^{-n}. Hence, in order to obtain the present value from the amount, we must first multiply the formula for the amount by R, and then change the sign of the index of R.

$$M = A\cdot\frac{R^n - 1}{R - 1}.$$

Multiplying by R we get

$$A\cdot\frac{R^{n+1} - R}{R - 1}.$$

Changing sign of index we have

$$P = A\cdot\frac{R^{-(n+1)} - R^{-1}}{R^{-1} - 1}$$

$$= A\cdot\frac{R^{-n} - 1}{1 - R}$$

$$= \frac{A}{r}(1 - R^{-n}).$$

(ii) We may obtain the same result by proceeding on the principle that if the present value P be put out to compound interest for n years, it ought to amount to the same as the annuity for that time.

$$\text{Hence,}\quad PR^n = A.\frac{R^n - 1}{R - 1};$$

$$\therefore P = A.\frac{1 - R^{-n}}{R - 1}$$

$$= \frac{A}{r}\left(1 - R^{-n}\right).$$

(iii) We will now proceed on the principle that the present value P is the sum of the present values of the respective annual payments.

$$\begin{array}{lr}
\text{Present value of A due 1 yr. hence} & = AR^{-1}, \\
\text{“ \quad “ \quad “ \quad 2 \quad “} & = AR^{-2}, \\
\text{etc.} & = \text{etc.}, \\
\text{“ \quad “ \quad “ \quad } n \text{ \quad “ \quad “} & = AR^{-n};
\end{array}$$

$$\begin{aligned}
\therefore\ P &= AR^{-1} + AR^{-2} + AR^{-3} \ldots . + AR^{-n} \\
&= AR^{-1}.\frac{R^{-n} - 1}{R^{-1} - 1} \\
&= \frac{A}{r}\left(1 - R^{-n}\right).
\end{aligned}$$

262. *For Simple Interest, expanding and neglecting* r^2 *and higher powers, we get*

$$\begin{aligned}
P &= \frac{A}{r}\left(1 - \frac{1}{R^n}\right) \\
&= \frac{A}{r}.\frac{(1+r)^n - 1}{(1+r)^n} \\
&= \frac{A}{r}.\frac{1 + nr + n\left(\frac{n-1}{1.2}\right)r^2 + \cdots - 1}{1 + nr} \\
&= A\cdot\frac{n + \frac{n}{2}(n-1)r}{1 + nr} \\
&= \frac{nA}{2}.\frac{2 + (n-1)r}{1 + nr}.
\end{aligned}$$

263. To find the present value of a perpetual annuity.

(i) Reckoning Compound Interest.

$$\begin{aligned}
P &= AR^{-1} + AR^{-1} + \ldots . \textit{ad infinitum}. \\
&= \frac{AR^{-1}}{1 - R^{-1}} \\
&= \frac{A}{R - 1} \\
&= \frac{A}{r}
\end{aligned}$$

(ii) *Reckoning Simple Interest.*

$$P = \frac{nA}{2} \cdot \frac{2 + (n - 1)r}{1 + nr}$$

$$= \frac{nA}{2} \cdot \frac{\frac{2}{n} + (1 - \frac{1}{n})r}{\frac{1}{n} + r}.$$

Now, when $n = \infty$, the limit of

$$\frac{\frac{2}{n} + (1 - \frac{1}{n})r}{\frac{1}{n} + r} = \frac{0 + (1 - 0)r}{0 + r} = 1.$$

Hence, the limit of P, when $n = \infty = \frac{\infty A}{2} = \infty$.

This result shows that an *infinite sum* of money is required to be left, in order to insure an equal annual payment forever, which is absurd. It indicates, therefore, that the only correct method of computing annuities is on the compound interest principle.

264. To find the Present Value of an annuity, to commence at the end of p years, and then to continue q years.

The present values of the first, second, etc., qth payments, due at the end of $p + 1$, etc., $p + q$ years, respectively, will evidently be

$$AR^{-(p+1)}, \qquad AR^{-(p+2)}, \qquad \text{etc.}, \qquad AR^{-(p+q)}.$$

whence the present value

$$P = AR^{-(p+1)} \cdot \{1 + R^{-1} + R^{-2} + \ldots + R^{-(q-1)}\}$$

$$= AR^{-(p+1)} \cdot \left(\frac{1 - R^{-q}}{1 - R^{-1}}\right)$$

$$= \frac{A}{R^{p+q}} \cdot \left(\frac{R^q - 1}{R - 1}\right).$$

If the annuity is payable *forever* after p years have expired, by summing the above series *ad infinitum*, we have

$$P = \frac{A}{R^p (R - 1)}.$$

These formulæ enable us to compute the values of *Reversions*, or *Annuities in Reversion;* and the latter determines the value of the *Fee Simple* of the freehold estate, which is to fall in at the expiration of p years.

Ex. 1. A sum of $\$a$ is borrowed for a period of m years, to be repaid by equal annual instalments, the first payment to be made after one year. Find the amount of the annual instalment.

Let A be the annual instalment.

Then the amount of this annual payment in m years

$$= \frac{A}{r}\left(R^m - 1\right).$$

Again, if the sum a be allowed to accumulate for m years, at compound interest, its amount

$$= aR^m.$$

Now these two amounts ought to be equal.

Hence, we have

$$\frac{A}{r}\left(R^m - 1\right) = aR^m;$$

$$\therefore A = ar \cdot \frac{R^m}{R^m - 1}$$

$$= ar \cdot \frac{1}{1 - R^{-m}}.$$

Ex. 2. The present value of an annuity of \$1, to continue q years, is \$10; and the present value of an annuity of \$1, to continue $2q$ years, is \$16. Find the rate of interest.

$$\text{Here,}\quad 10 = \frac{1}{r}(1 - R^{-q}),\ \text{Art. 261,}$$

$$\text{and}\quad 16 = \frac{1}{r}(1 - R^{-2q});$$

$$\therefore \frac{16}{10} = \frac{1 - R^{-2q}}{1 - R^{-q}}$$

$$= \frac{(1 - R^{-q})(1 + R^{-q})}{1 - R^{-q}}$$

$$= 1 + R^{-q};$$

$$\text{or}\quad 2 - \frac{16}{10} = 1 - R^{-q};$$

$$\therefore 1 - R^{-q} = \frac{2}{5}.$$

Substituting in the first equation, we get

$$\frac{2}{5} \cdot \frac{1}{r} = 10;$$

$$\therefore r = \frac{1}{25},$$

$$\text{or} \quad 100\,r = 4.$$

The rate is, therefore, 4 %.

Ex. 3. A mortgage of $5,000, interest at 6 per cent. per annum, has 7 years and 10 months to run. Find its present value, interest at 10 per cent. per annum, payable half-yearly.

The first payment of interest is $300, and will be due in 10 mo. Its amount for seven years, @ 10 %, payable half-yearly, will be $300(1.05)^{14}$. Similarly, the amount of the second payment of interest at the end of the 7 yr., will be $300(1.05)^{12}$; and so on. The amount of the last payment will be $300.

Hence, the whole amount of the mortgage and interest will be

$$\begin{aligned} & 5000 + 300\,(1.05)^{14} + 300(1.05)^{12} + \ldots\ldots\ldots + 300 \\ &= 5000 + 300\,\{(1.05)^{14} + (1.05)^{12} + \ldots\ldots\ldots\ldots + 1\} \\ &= 5000 + 300\left(\frac{(1.05)^{16} - 1}{(1.05)^2 - 1}\right) \\ &= 8462.06. \end{aligned}$$

Now, if the present value, P, be put out at compound interest, @ 10 % per annum, for 7 yr., and 10 mo., it ought to amount to the same as the mortgage for that time.

$$\therefore P\,(1.05)^{15\frac{2}{3}} = 8462.06;$$

$$\therefore P = 3940.13.$$

The Present Value of the Mortgage is, therefore, $3940.13.

The value of $(1.05)^{15\frac{2}{3}}$ may be found

(i) By means of a table of logarithms.

(ii) By raising 1.05 to the 16th power, dividing this by 1.05 we obtain the 15th power, taking $\frac{2}{3}$ of the difference and adding to the 15th power, we get, approximately, the $15\frac{2}{3}$ power of 1.05.

(iii) By the Binomial Theorem, as follows:—

$(1.05)^{\frac{1}{3}} = (1 + \frac{1}{20})^{\frac{1}{3}} = 1 + \frac{1}{60} - \frac{1}{3600} = 1.0163$; then $(1.05)^{16} \div 1.0163$ will give a close approximation to $(1.05)^{15\frac{2}{3}}$.

For additional information on this subject, consult Loan Tables by Professors Cherriman and Loudon.

Examples cxxix.

1. What sum of money must be paid for a perpetual annuity of $1200 to secure $4\frac{1}{2}$ % per annum on the money paid?

2. What is the present value of a perpetual annuity of $400, the first instalment to be paid at the end of 5 yr., calculated @ $4\frac{1}{2}$ % per annum?

3. Find the present value of an annuity, deferred 2 yr., and to run 5 yr., calculated @ 5 % per annum.

4. What sum invested now @ $4\frac{1}{2}$ %, compound interest, payable yearly, will, at the end of 5 yr., provide for a perpetual annuity of $500?

5. What sum of money, deposited at the end of each year, for the next ten years, will then be sufficient to purchase a perpetual annuity of $100, money being worth 5 % per annum?

6. A township offers debentures for sale, which are to run for 10 yr., and bear interest @ 6 % per annum. At what rate should they sell, money being worth 5 % per annum, payable yearly?

7. What is the present value of the reversion of a perpetuity of $120 per annum, to commence 7 yr. hence, allowing the buyer $4\frac{1}{2}$ % for his money?

8. A person's dividend from his Bank stock is $530 a year. What is the present value of his income for five years to come, computing by simple, and also by compound interest @ 7 %?

9. What annuity, to continue 20 yr., can be purchased for $10000, allowing compound interest @ 5 %?

10. For what sum might the Government of a country undertake to pay an annuity of $1000 a year, forever, on the supposition that money may always be invested @ 6 %?

11. For what sum might an annuity of $400 a year, for ten years, to commence in 5 yr., be purchased, allowing compound interest @ 6 %?

12. A person who enjoyed a perpetuity of $1000 per annum, provided in his will that, after his decease, it should descend to his only son for 10 yr., to his only daughter for the next 20 yr., and to a benevolent institution forever afterwards. What was the value of each bequest at the time of his decease, allowing compound interest @ 6 %?

13. A person at the age of 22 put \$100 at interest @ 6 %, and \$100 each year afterwards, until he was 40 yr. old. He also collected the interest annually, and converted the same into *principal.* What amount was, by these means, accumulated ?

14. A corporation borrows £3769 @ 4 %, to be paid in 30 yr. by equal annual instalments. What will be the annual payment ?

15. A property is let out on lease for a yr. at an annual rental of \$$b$, and after c yr. the lease is renewed on paying a fine of \$$d$. What is the additional rent equivalent to this fine ?

16. A farm is let for n yr. at a fixed rent and a fine of \$$p$. When p yr. of the lease remain, what fine must be paid to extend these p yr. to q yr., at compound interest ?

17. If two joint proprietors have an equal interest in a freehold estate worth \$$a$ per annum, but one of them purchased the whole to himself by allowing the other an equivalent annuity of \$$b$ for n yr., find the relation between a and b.

18. Find the present value of an annuity of \$1, paid n times per annum, and continuing for m yr., allowing compound interest at the rate of r % per annum; and prove that, as n is indefinitely increased, this present value continually approaches the limit $\frac{1 - e^{-mr}}{r}$.

19. A monthly instalment of \$10 has 2 yr. 1 mo. to run. What sum must be paid at once to reduce the period six months, money being worth one-half per cent. per month ?

20. A mortgage of \$4000, interest @ 5 % per annum, payable half-yearly, has 17 yr. and 8 mo. to run. Find its present value, interest 10 % per annum, payable half-yearly.

21. If two sums, s_1, s_2, due at times t_1, t_2, be paid together at an intermediate time t, t being determined from the equation

$$s_1 R^{-t^1} + s_2 R^{-t^2} = (s_1 + s_2) R^{-t}.$$

Show that whichever mode of payment be adopted

(i) At any antecedent period, the present values are the same;

(ii) At any subsequent period, the amounts are the same;

(iii) At the intermediate time of payment, the interest of the sum overdue is the discount of that not due.

CHAPTER XX.

ASSORTED PROBLEMS.

Simple Rules.

1. Find a number such, that if it be added twenty-three times to 37601, the sum will be 40200.

2. If a person spends in four months as much as he earns in three, how much can he lay by annually, supposing that he earns $420 every six months?

3. Two ships get under weigh at the same time for the same port, distant 1200 mi. The faster vessel averages 10 mi. an hour, and arrives at the port a day and a half before the other. What will the latter vessel average an hour?

4. The quotient in a division question equals seven times the divisor, and the divisor equals seven times the remainder; the three amount together to 855. Find the dividend.

5. Multiply 57875 by 729819 with three lines of multiplication, and divide 123456 by 63, using short division.

6. *M* starts from C and travels towards D at a rate of 6 mi. per hour. Two hours afterwards *N* starts from C, and, going 10 mi. an hour, reaches D 4 hr. before *M*. Find the distance from C to D.

7. Two trains start at the same time from London and Edinburgh, and proceed towards each other at the rates of 30 and 50 mi. per hour, respectively. When they meet, it is found that one train has run 100 mi. farther than the other. Find the distance between London and Edinburgh.

8. If a ship containing 150 hhd. of wine pays for toll at the Suez Canal the value of 2 hhd., wanting $30; and another, containing 240 hhd., pays at the same rate, the value of 2 hhd. and $90 besides, what is the value of the wine per hogshead?

9. If 1 lb. of tea is worth 50 oranges, and 70 oranges are worth 84 lemons, what is the value of a pound of tea when a lemon is worth a penny?

10. A man hired a laborer to do a certain amount of work, on the agreement that for every day he worked he should have

$1.50, but that for every day he absented himself he should lose 60c. He worked twice as many days as he absented himself, and received on the whole $72. Find how long he was doing the work.

11. Multiply 32856 and 121711, using 3 lines of multiplication only.

12. In a hundred yards race *A* can give *B* four and *C* five yards' start. If *B* were to race *C*, giving him one yard in a hundred, which would win?

13. The product of the 1st and 2nd of three numbers is 176382, of the 1st and 3rd is 279152, of the 2nd and 3rd is 215496. What are the numbers?

14. A speculator gained $1050 one year, the next year he lost as much again as he had gained, and the next year gained as much again as he had lost, and then had $50000. With what capital did he begin?

15. Two brothers, *A* and *B*, had each $25000. *A* loaned *B* $7500 and then borrowed of him $15450, and lost so much in speculation that *B* had $7050 more than *A*. How much did *A* lose?

16. The first of four numbers is 3125, the second is greater than the first by 5108, the third is equal to the sum of the first and second, and the fourth is equal to the sum of the third and first. What is the sum of the four numbers?

17. A boy rubbed a subtraction example off his slate, and said that the answer was 736858 and the sum of the minuend and subtrahend was 1106450. What was the example?

Factors, Measures and Multiples.

1. Eight bells begin tolling together at the same instant, and they toll at intervals of 1, 2, 3, 4, 5, 6, 7, 8 seconds, respectively. After what time will they be again tolling at the same instant?

2. *A* and *B* run a mile race. At first *A* runs 11 yd. to *B*'s 10, but after *A* has run a half a mile he tires and runs 9 yd. in the time in which he at first ran 11, *B* running at his original rate. Which wins, and by how much?

3. Two cogged wheels are together, there being 32 cogs in one and 40 in the other. The smaller wheel makes 64 revolutions per second. How often are the same two cogs together in one working day of 10 hr.?

4. Three men, whose steps are 2 ft. 6 in., 2 ft. 9 in., and 3 ft., walk a mile together. How often are they in step together?

5. Find the largest and the smallest number that will divide 9027 and 6863, leaving as remainders 27 and 23, respectively. Find the other divisors that would satisfy the question.

6. Find the smallest number of bushels of wheat that would equal in weight an exact number of bushels of Indian corn, or of barley.

7. The front wheel of a carriage is 10′ 6″ in circumference, and the hind one 12′ 6″. Find the distance the carriage has gone, when two spots on the wheel which were touching the ground at starting, have touched the ground at the same instant 704 times.

8. If three bodies move uniformly round a centre in 87, 224. and 365 days, respectively, and if they are now in the same straight line, on a radius of the circle the most distant one is describing, when will they all be on a radius of this circle again?

9. *A* can walk 3 mi. in 48 min., *B* 3 mi. in 72 min., *C* 3 mi. in 84 min., and *D* 3 mi. in 96 min. How far may each one go so that on their returning they may arrive together at the place of starting?

10. If in 2 da. *A* can build 30 rods of a fence, *B* 48 rods, *C* 36 rods, and *D* 40 rods. Find the least number of rods that will furnish exact day's work for any pair of them working together.

11. How many oranges must a boy buy and sell to make a profit of $1.35 if he buys at the rate of 5 for 3c., and sells at the rate of 4 for 3c.?

12. The H. C. F. of two numbers is 1259, their L. C. M. is 72644830039. One of the numbers is 36268013. Find the other.

13. Separate 10290 into three factors that shall be to each other as 2, 3, and 5.

14. In a Long Division sum the dividend is 529565, and the successive remainders from the first to the last are 246, 222, 542. Find the divisor and quotient.

15. What is the least sum of money for which I could purchase a number of hogs at $\$20\frac{4}{7}$, a number of cows at $\$42\frac{2}{3}$, or a number of horses at $72, and what number of each could I purchase for that sum?

16. Four numbers have one factor in common, and there is no other factor which is common to any two of them. Their product is 65975910 and their L. C. M. is 30030. Find the numbers.

17. The product of five consecutive *odd* numbers is 28035315. Find them.

Vulgar Fractions.

1. If $\frac{2}{5}$ of $1\frac{1}{4}$ of an estate is worth \$300, what will be the value of $\dfrac{2\frac{1}{2}}{\frac{5}{14}}$ of the estate?

2. Of an electric cable $\frac{11}{13}$ rests on the bottom of the sea, $\frac{1}{15}$ hangs in the water, and $234\frac{2}{3}$ yd. are employed on land. What is the length of the cable?

3. If $\frac{5}{7}$ of the cargo of a ship is worth \$16000, what will be the value of $\frac{2}{3}$ of $\frac{7}{8}$ of the remainder?

4. Three persons divide the cost of an entertainment amongst them in such a manner that the first pays $\frac{1}{3}$ of the whole, and the second $\frac{6}{5}$ of what the first pays, and the third pays the remainder, which is \$2.50. What is the amount of the bill?

5. $\frac{4}{5}$ of *A*'s stock was destroyed by fire, $\frac{7}{8}$ of the remainder was injured by water and smoke. He sold the uninjured goods at cost price, and the injured goods at a third of cost price. He realized \$1155. What did he lose by the fire?

6. Simplify

$$\frac{1}{7\frac{1}{4} \text{ of } 3\frac{3}{11} + 3\frac{3}{11}} \div \left(\frac{3}{13} - \frac{2}{9}\right) - \left(\frac{13}{3} + \frac{1}{6}\right) \div \frac{2}{3} \text{ of } \frac{3}{8} \text{ of } 63.$$

7. In a dormitory $\frac{11}{23}$ of the boys are in the upper school, $\frac{2}{3}$ of the remainder in the middle, and the rest, 8 in number, in the lower. Find the number in the dormitory.

8. How much ore must one raise, that on losing $\frac{17}{40}$ in roasting, and $\frac{8}{19}$ of the residue in smelting, there may result 506 t. of pure metal?

9. If a population is now ten millions, and the births are 1 in 20, and the deaths 1 in 30, annually, what will the population become in 5 yr.?

10. Show that

$$\cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{5}}}}} + \cfrac{1}{5 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{1}}}}} = \frac{8}{9}.$$

11. A general, after losing a battle, found that he had only two-thirds of his army left fit for action, one-ninth of the army had been wounded, and the remainder, 2000 men, killed or missing. Of how many did the army consist before the battle?

12. At a certain battle two-thirds of the defeated army ran away with their arms, five-sevenths of the remainder left their arms on the field, and of the rest seven-eighths were missing, the remaining 500 being either killed or wounded. Find the whole number of the army.

13. One-tenth of a rod is colored red, one-twentieth orange, one-thirtieth yellow, one-fortieth green, one-fiftieth blue, one-sixtieth indigo, and the remainder, which is 302 in. long, violet. What is the length of the rod?

14. Prove that $\frac{3+4}{4+5}$ is greater than $\frac{3}{4}$ and less than $\frac{4}{5}$.

15. A ship 40 mi. from the shore springs a leak, which admits $3\frac{3}{4}$ t. of water in 12 min.; 60 t. would suffice to sink her; but the ship's pumps can throw out 12 t. of water in an hour. Find the average rate of sailing that she may reach the shore just as she begins to sink.

16. Two persons, walking at the rate of 3 and 4 mi. an hour, respectively, set off from the same place in opposite directions to walk around a park, and meet in 10 min. Find the length of the walk round the park.

17. Simplify

$$\frac{4\frac{5}{6} + 1\frac{11}{45} - 5\frac{11}{18}}{6\frac{1}{7} \times 3\frac{1}{2} - \frac{2\frac{1}{3}}{\frac{2}{7}} \times 1\frac{2}{7} + 1\frac{3}{5}} \text{ and } \frac{5}{6\frac{7}{8}} \times \left(1\frac{4}{7} \times 5\frac{1}{4}\right) + \frac{1}{3} + \frac{1}{1\frac{10}{17}},$$

and find their sum.

18. Alfred owed Robert two-thirds of the amount that Robert owed Charles, and to settle matters Robert gave 10d. to Alfred, who then paid Charles. What did Robert owe Charles?

19. A man walks a certain distance, and rides back in 3 hr. 45 min.; he could ride both ways in $2\frac{1}{2}$ hr. How long would it take him to walk both ways?

20. In a field in which cows and sheep are grazing $\frac{1}{7}$ of the total number are cows; but if 3 cows more are put into the field, the latter will number $\frac{2}{11}$ of the whole. How many sheep are there?

21. Find a fraction equivalent to $\frac{5}{13}$ and having its numerator 44 less than its denominator.

22. If a certain number is divided by 208, the sum of the quotient, dividend, and divisor is 36783. Find the number.

23. A tree 95 ft. high, in falling, broke into two pieces, so that $\frac{4}{5}$ of the longer piece equals $\frac{8}{9}$ of the shorter. How long was each?

24. The outfit of a livery stable is worth $2700. One-seventh of the value of the horses is equal to one-fifth of the value of the vehicles, harness, etc. Find the value of the horses.

25. A boy's age is now one-fifth of his father's age. In 6 yr. it will be one-third of his father's age. How old is the boy?

Decimals.

1. Divide .14 by 7, 140 by .07, and .014 by 7000. Add the results together, and express the decimal as a vulgar fraction.

2. Simplify the expression $7.\dot{5}\dot{7} \times .3\dot{6} - 2.3\dot{4}\dot{5}$.

3. Divide $\frac{41}{162} - \frac{9}{49} - \frac{3}{54}$ by $\frac{4}{9} + \frac{1}{2} - \frac{3}{14}$, and express the result as a decimal.

4. Find the value of

$$\frac{(3\frac{1}{3} - 2\frac{1}{2}) \div \frac{5}{6} \text{ of } \frac{3}{8}}{2\frac{2}{3} \div (\frac{1}{2} + \frac{1}{4})},$$

and express the result as a decimal.

5. Simplify the expression $1.\dot{3} \times (2.\dot{4} + 7.\dot{5}) + 2.3\dot{6}\dot{4} - 1.6\dot{9}\dot{7}$.

6. Express as vulgar fractions in the lowest terms 24.0025 and .0008125; and divide 1.1214 by 5.34 and 1121.4 by .534.

7. Find the vulgar fraction equivalent to $\frac{1.010\dot{1}\dot{5}}{.55}$.

8. A man owns $\frac{3}{16}$ of a mine, and sells $.1\dot{3}5\dot{1}$ of his share. What fraction of the mine has he left?

9. Simplify $\frac{.004 \div .0005}{2.4\dot{2}\dot{3} + 3.\dot{5}7\dot{6} + 2.00\dot{0}191\dot{1}}$.

10. Two lines are 41.06328 in. and .0438 of an inch long, respectively. How many lines as long as the latter can be cut off from the former, and what will be the length of the remaining line?

11. Of two stalactites hanging from the flat roof of a cavern, one is 1.02 in. longer than the other, and the shorter one increases in length at the rate of 3.014 in. in a century. Find the rate of

increase of the other, in order that they may be of the same length at the end of 125 yr.

12. The masters of a school are $.041\dot{6}$ of its whole number, but after 40 new boys have been added the masters became .0375 of the whole. How many boys and masters were there before the new boys came?

13. Simplify $\dfrac{3.\dot{5} - 1.8\dot{3}}{9.\dot{7} - 6.\dot{4}} \times \dfrac{1}{71} \div \dfrac{3.1 \times .\dot{1}0\dot{1}}{2.\dot{1}\dot{5}}$.

14. The following rule has been given to divide by 3.14159:—"Multiply by 7, divide by 11, then by 2, and add $\frac{1}{8}$th of $\frac{1}{1000}$th of the result." Find the error made in obtaining $1 \div 3.14159$ by this process.

15. Simplify (.006 of £2 1s. 8d. + $3.4\dot{5}\dot{4}$ of £3 6s.) × $5\frac{5}{11}$.

16. Find the L. C. M. of 10.5, 7.7, $.08\dot{3}$, 49, and 3.4375.

17. Calculate the value of

$$1 - \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3} - \frac{1}{1 \times 2 \times 3 \times 4} + \frac{1}{1 \times 2 \times 3 \times 4 \times 5}, \text{ etc.,}$$

correctly to 9 places of decimals, the series being unending.

18. Simplify $16\left\{\frac{1}{5} - \frac{1}{3} \cdot \frac{1}{5^3} + \frac{1}{5} \cdot \frac{1}{5^5} - \frac{1}{7} \cdot \frac{1}{5^7} + \cdots\right\} - \frac{4}{239}$.

19. Simplify $\frac{1}{10^3} \times \left\{1 - \frac{3}{10^2} + \frac{3 \times 4}{1 \times 2} \times \frac{1}{10^4} + \frac{3 \times 4 \times 5}{1 \times 2 \times 3} \cdot \frac{1}{10^6}\right\}$.

Involution and Evolution.

1. Extract the square root of 167.9616, and of $\frac{529}{2401}$.

2. Extract the cube root of 16777216.

3. Extract the square root of 30712.5625 of $\frac{625}{2401}$, and of .000000133225.

4. What must be the least number of soldiers in a regiment to admit of its being drawn up 2, 3, 4, 5, or 6 deep, and also of its being formed into a solid square?

5. Find the square root of 10747.4689 and the cube root of 189119224.

6. Find the square roots of 15376.248001 and $\dfrac{31.36}{39.69}$.

7. Find the square roots of 4957.5681 and $\dfrac{129.4947}{60.75}$.

8. Simplify $\dfrac{\sqrt[3]{3.43} + \sqrt[3]{.02744}}{\sqrt[3]{270} - \sqrt[3]{.08}}$.

9. Simplify $\dfrac{(.045)^3 - (.015)^3}{(.045)^2 + (.045)(.015) + (.015)^2}$.

10. (i) In what ways may the numerator and denominator of a fraction be simultaneously altered without any change in the value of the fraction.

(ii) Simplify $\dfrac{(.011)^5 + (.022)^5 + (.033)^5}{(.0022)^5}$.

11. A cubical box, the interior of which measures 2 ft. 9 in. each way, contains 15625000 small cubes. Find the length of an edge of each of the small cubes in decimals of an inch.

12. A wall 5 times as high as it is broad, and 8 times as long as it is high, contains 18225 cu. ft. Find the breadth of the wall.

13. Find the square root of

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3.$$

14. The product of the sum of two numbers by their difference is 12032013. One of the numbers is 2006. Find the other.

15. Prove from the ordinary multiplication table that the product of all the numbers in any block formed with 2, 3, etc., numbers in its side is a number which is itself a perfect square, cube, etc.

16. What is the least number that, being a cube, is exactly divisible by 12, 15, and 21 ?

17. Find the value of $\sqrt[3]{343^2}$, and of $\sqrt[3]{5^4}$ to 3 decimal places.

18. 614656 is the square of a certain number. Find the square (i) of the next number less than this, and (ii) of the next number greater than it, in both cases without involution.

Reduction and Compound Rules.

1. Divide 9366 farthings into an equal number of sovereigns, half-sovereigns, half-crowns, and farthings.

2. If telegraph posts are placed 66 yd. apart, and a train passes one in every three seconds, how many miles an hour is the train running ?

3. Reduce 11 ro. 11 po. 11 yd. to inches, and find what fraction the result is of 3 ac.

4. The distance between two wickets was marked out for 22 yd., but the yard measure was $\frac{5}{12}$ of an inch too short. What was the actual distance?

5. How long would a column of men, extending 3420 ft. in length, take to march through a street a mile long at the rate of 58 paces in a minute, each pace being $2\frac{1}{2}$ ft.?

6. Having given that the weight of a cubic foot of water is 1000 oz., and that the imperial gallon contains 277.274 cu. in., find the weight of a pint of water.

7. Add together .60625 of £1 + $.\dot{1}4285\dot{7}$ of 14s. $10\frac{1}{2}$d., and $\frac{23}{11}$ of $\frac{3}{71}$ of £3 5s. 1d., and express the result as the decimal of 27s.

8. How many times does the 29th day of the month occur in 400 consecutive years?

9. The circumference of the fore-wheel of a carriage is 8 ft., and that of the hind-wheel is 10 ft. In what distance will the fore-wheel make 100 revolutions more than the hind-wheel?

10. Gold of the value of £423267 arrives from Australia. What is its weight in lbs. avoirdupois, the price being £3 18s. per ounce, troy?

11. The whole time occupied by a train 120 yd. long, travelling at the rate of 20 mi. an hour, in crossing a bridge is 18 sec. Find the length of the bridge.

12. Find the value of $.857\dot{1}4285\dot{7}$ of £10 14s. 1d. accurately; and show that the error committed by neglecting all decimals of an order higher than the fifth is less than $\frac{1}{125}$ of a penny.

13. The price of gold in this country is £3 17s. $10\frac{1}{2}$d. an ounce. Find the least number of ounces which can be coined into an exact number of sovereigns, and the number of sovereigns so coined.

14. State the connection between Troy and Avoirdupois weights. A ring weighs 1 dwt. 4 gr., and is worth £1 2s. If 1050 of such rings be packed in a box weighing $3\frac{1}{2}$ lb., what would it cost to convey them 144 mi. at the rate of 5s. per long ton per mile, insurance being demanded at the rate of $\frac{1}{8}$ %?

15. *A* can walk a certain distance in $4\frac{1}{2}$ hr., taking 3 steps 32 in. long each, in 2 sec. How many steps, each 1 yd. long, must he take in one minute to walk half the distance in 2 hr.?

16. A sheet of paper $5\frac{1}{2}$ ft. long, and $2\frac{1}{2}$ ft. wide, is cut into strips 1 in. wide. How many such sheets of paper will it take to go round the earth, a distance of 25000 mi.?

17. Find a decimal multiplier which will convert Troy ounces per inch into tons per mile. Use it to find the weight in tons per mile of wire, which weighs $\frac{3}{100}$ of an ounce Troy to the inch run.

18. How much alloy must be mixed with 30 oz. of gold, worth £4 4s. 6d. an ounce, to reduce the value to £3 10s. 5d. an ounce, if the alloy is worthless ?

19. *A* runs a mile race with *B* and loses. Had he run $\frac{1}{3}$ faster, he would have won by 11 yd. Compare their rates of running.

Metric System.

1. A French metre = 1.0936 of a yard, and a centimetre is the hundredth part of a metre. Find a centimetre in decimals of an inch to 4 places.

2. Express $69\frac{1}{2}$ mi. in metres, 32 m. being taken to be equivalent to 35 yd.

3. A French metre contains 39.371 English inches. Express to three decimal places an English mile in metres.

4. If 8000 m. be equal to 5 mi., and if a cubic fathom of water weighs six tons, and a cubic metre of water 1000 kg., find the ratio of a kilogramme to a pound avoirdupois. (Long ton.)

5. An acre is .40467 ha., and a pound sterling is 25.25 francs. An estate measuring 1927 ha., is sold for 10100000 francs. What is the selling price per acre in £ s. d. ?

6. A decimetre is equal to 3.937 in., and a cubic decimetre of water weighs 1 Kg. If a cubic inch of water weighs 252.45 gr., express a Kg. in pounds avoirdupois, correct to two decimal places.

7. A train is just 27 min. in passing through the Mont Cenis Tunnel, the length of which is 11220 m. Find the speed of the train in miles per hour, a metre being 39.37 in. long.

8. Find the difference between $\frac{13}{24}$ of 3 Kg. 1 Hg. 6 Dg. 1 dg. 5 cg., 2 mg., and $\frac{16}{19}$ of 2 Kg. 2 Dg. 7 dg. 7 mg.; and express the result in kilogrammes.

9. Express 7 ha. 25 a. 7 ca. as a decimal of a square kilometre.

10. If a gallon of water weighs 10 lb., find its volume in cubic centimetres.

11. When cloth is sold @ 18 francs 60 centimes the metre, find the corresponding price per yard in dollars and cents, supposing \$1 = $5\frac{1}{6}$ francs.

12. The pressure of the atmosphere is $14\frac{3}{4}$ lb. to the square inch. Find the pressure in kilogrammes to the square centimetre.

13. How many kilogrammes are there in a bar of gold 10 cm. long, 30 mm. wide, and 25 mm. thick, gold being 19.36 times as heavy as water.

14. Alcohol being $\frac{4}{5}$ as heavy as water, find the number of kilogrammes of alcohol required to fill a tank measuring 1.5 m., by 3.2 m., by 80 cm.

15. The circumference of the earth is 40000000 m., the length of a metre being 39.37 in. Calculate the diameter of the earth in miles, assuming that the ratio of the circumference of a circle to its diameter is 355 to 113.

Problems Relating to Work Done.

1. If 24 men can do a piece of work in 12 da., of 10 hr. each, how many men can do three times as much in 10 da., of 8 hr. each?

2. *A* can mow 5 ac. of grass in 3 da., *B* 7 ac. in 9 da., *C* 11 ac. in 12 da. In how many days can they jointly mow 121 ac.?

3. Two pipes together fill a cistern in 1 hr. One of them alone fills it in $1\frac{1}{2}$ hr. How long will it take the other to fill it?

4. If 9 men or 16 women could do a piece of work in 144 da., in what time would 7 men and 9 women do it, working together?

5. *A* and *B* can do a piece of work in 8 da., *B* and *C* can do it in 12 da., and *A*, *B*, and *C* can do it in 6 da. In how many days can *A* and *C* do it?

6. *A* and *B* can do a piece of work in 4 da., *B* and *C* in $5\frac{3}{4}$ da., and *A* and *C* in $4\frac{3}{5}$ da. In what time can each do the work separately?

7. If 6 men and 2 boys can reap 13 ac. in 2 da., and 7 men and 5 boys can reap 33 ac. in 4 da., how long will it take 2 men and 2 boys to reap 10 ac.?

8. *A* and *B* walk a race of 25 mi. *A* gives *B* 45 min. start. *A* walks uniformly a mile in 11 min., and catches *B* at the 20th milestone. Find *B*'s rate, and by how much he lost in time and space.

9. At billiards *A* can give *B* 5 points in a game of 50, and *C* 10 points in 50. How many points can *B* give *C* in a game of 90?

10. *A* can do a piece of work in 6 da., which *B* can destroy in 4. *A* has worked for 10 da., during the last 5 of which *B* has been destroying. How many days must *A* now work alone, in order to complete his task?

11. Two cisterns of equal dimensions are filled with water, and the taps for both are opened at the same time. If the water in one will run out in 5 hr., and that in the other in 4 hr., find when one cistern will have twice as much water in it as the other has.

12. If 3 men, 4 women, 5 boys, or 6 girls, can perform a piece of work in 60 da., how long will it take one man, 2 women, 3 boys, and 4 girls, all working together?

13. *A* and *B* start to run a race. Their speeds are as 17 to 18. *A* runs $2\frac{1}{3}$ mi. in 16 min. 48 sec. *B* finishes the course in 34 min. Determine the length of the course.

14. A soldier has 5 hr. leave of absence. How far may he ride on a coach which travels 10 mi. an hour, so as to return to the camp in time, walking at the rate of 5 mi. an hour?

15. *A* can do one-half of a piece of work in 1 hr., *B* can do three-fourths of the remainder in 1 hr., and *C* can finish it in 20 min. How long would *A*, *B*, and *C* together take to do it?

16. Two men, *A* and *B*, start from Cambridge, at 4 and 5 o'clock, a.m., respectively, to walk to London, a distance of 50 mi. *B* passes *A* at the twentieth milestone, and reaches London at 5 p.m. When will *A* arrive there?

17. If 12 men or 18 boys can do $\frac{3}{4}$ of a piece of work in $6\frac{1}{2}$ hr., in what time will 11 men and 9 boys do the rest?

18. *A* and *B* can do a piece of work in 6 da., *B* and *C* in 8 da., *A*, *B*, and *C* in 4 da. How long would *A* and *C* take to do it?

19. If two boys and one man can do a piece of work in 4 hr., and two men and one boy can do the same in 3 hr., find in what time a man, a boy, and a man and a boy together, respectively, could do the same.

20. A piece of work has to be finished in 36 da., and 15 men are set to do it, working 9 hr. a day. But after 24 da. it is found that only three-fifths of the work is done. If 3 additional men are then put on, how many hours a day will all have to work so as to finish the task in time?

21. A work can be accomplished by *A* and *B* in 4 da., by *B* and *C* in 6 da., and *A* and *C* in 8 da. Find in what time it would be accomplished by all working together.

22. A man and a boy are to work on alternate days at a piece of work which would have occupied the boy alone 13 da. If the boy take the first day the work will be finished half a day later than if the man commences. Find how long they would take to do it, working together.

23. If 3 men and 5 women do a piece of work in 8 da., which 2 men and 7 children can do in 12, find how long 13 men, 14 children, and 15 women, working together, will take to do it.

24. Five men do .6006 of a piece of work in 2.12 hr. How long will 6 boys take to finish it, it being known that 3 men and 7 boys have done a similar piece of work in 3 hr.?

25. *A* does $\frac{5}{6}$ of a piece of work in 20 da., and then gets *B* to help him. They work together for 2 da., when *B* leaves and *A* finishes the work in half a day more. How long would *B* have taken to do the whole?

26. Two gangs of 6 and 9 men are set to reap two fields of 35 and 45 ac., respectively. The first gang works 7 hr. in the day, and the latter 8 hr. If the first gang complete their work in 12 da., in how many days will the second gang complete theirs?

27. If a piece of work can be done in 50 da. by 35 men working at it together, and if, after working at it for 12 da., 16 of the men were to leave the work, find the number of days in which the remaining men could finish the work.

28. I have to be at a certain place in a certain time, and I find that, if I walk at the rate of 4 mi. per hour, I shall be five minutes too late, if at the rate of 5 mi. per hour, I shall be 10 min. too soon. How far have I to go?

29. If, in a meadow of 20 ac. the grass grows at a uniform rate, and 133 oxen consume the whole of the grass on it in 13 da., or that 28 oxen 5 ac. of it in 16 da., how many oxen can eat up 4 ac. of it in 14 da.?

30. Two pipes, *A* and *B*, would fill a cistern in 25 min. and 30 min., respectively; both are opened together, but at the end of $8\frac{2}{3}$ min. the second is turned off. In how many minutes will the cistern be filled?

31. *A* can do a certain work in 10 da., *B* can do it in 15 da., and *C* in 12 da. They all begin to work together at it at the

same time, but *A* stops in $1\frac{1}{3}$ da. and *C* in $\frac{2}{5}$ da. before it is finished. *B* finishes the work. What part is done by each?

32. *A* can perform a work in 10 da. by getting 2 da. assistance from *B*; *B* can do the same work in 8 da. by getting 2 da. assistance from *A*. In what time could they do it, both working together?

33. A contractor engages what he considers a sufficient number of men to execute a piece of work in 84 da.; but he ascertains that three of his men do, respectively, $\frac{1}{6}$, $\frac{1}{7}$, and $\frac{1}{8}$ less than an average day's work, and two others $\frac{1}{8}$ and $\frac{1}{10}$ more, and in order to complete the work in the 14 weeks, he procures the help of 17 additional men for the 84th day. How much less or more than an average day's work on the part of these 17 men is required?

Problems Relating to Clocks.

1. A clock which loses 4 min. in 12 hr. is 10 min. fast at midnight on Sunday. What o'clock will it indicate at 6 o'clock on Wednesday evening?

2. A watch, which is 5 min. 40 sec. fast on Monday at noon, is 2 min. 51 sec. fast at midnight on the following Sunday. What did it lose in a day?

3. A clock which gains $7\frac{1}{2}$ min. in 24 hr. is 12 min. fast at midnight on Sunday. What o'clock will it indicate at 4 o'clock on Wednesday afternoon?

4. A clock gains $3\frac{1}{2}$ min. a day. How must the hands be placed at noon so as to point to true time at 7 hr. 30 min., p.m.?

5. One clock gains 4 min. in 12 hr., and another loses 4 min. in 24 hr. They are set right at noon on Monday. Determine the time indicated by each clock when the one appears to have gained $16\frac{1}{2}$ min. on the other.

6. How often between 11 and 12 are the hands of a clock an integral number of minute spaces apart?

7. What are the two exact times when the hands of a watch are equally distant from the fig. III.?

8. Two clocks begin to strike 12 together. One strikes in 35 seconds, the other in 25. What fraction of a minute is there between their seventh strokes?

9. Two clocks strike 9 together on Tuesday morning. On Wednesday morning one wants 10 min. to 11 when the other strikes 11. How much must the faster be put back that they may strike 9 together on Wednesday evening?

10. A watch set accurately at 12 o'clock indicates 10 min. to 12 at 12 o'clock next day. What is the exact time when the watch indicates 12 o'clock on that day?

11. Two boys, *A* and *B*, come into school punctually by their own watches, which are quite right at 9 o'clock on Monday morning. *A*'s watch gains two minutes, and *B*'s watch loses a minute and a half every day. Find how much later *B* will be than *A* at Friday afternoon school, 2 p.m.

12. What is the first time after 2 that the minute-hand will be twice as far from the figure 12 as the hour-hand is?

13. What is the second time after 4 that the hands of a watch will include an angle of 40°?

14. A person, being asked the time of day, replied that $\frac{1}{7}$ of the time past noon was equal to $\frac{1}{11}$ of the time to midnight. Required, the time.

15. A person asked the hour of the day, and was told the time past noon was $\frac{4}{5}$ of the time to midnight. What was the time?

16. A watch has three hands moving around the same axis. What is the first time after 12 when the second-hand is midway between the other two? The first time that the hour-hand is midway between the other two? The first time the minute-hand is midway between the other two?

Problems Relating to the Sum and Difference of Two Rates.

1. A boat's crew row over a course of a mile and a quarter against a stream which flows at the rate of 2 mi. an hour, in 10 min. The usual rate of the stream is half a mile an hour. Find the time which the boat would take in the usual state of the river.

2. A man rows down a river 18 mi. in 4 hr., with the stream, and returns in 12 hr. Find the rate at which he rows, and the rate at which the stream flows.

3. If a crew, which can row from *A* to *B* in 60 min., can row from *B* to *A* in 55 min., compare the rates of the stream and boat.

4. On a stream *B* is intermediate to and equidistant from *A* and *C*; a boat can go from *A* to *B* and back in 5 hr. 15 min., and from *A* to *C* in 7 hr. How long would it take to go from *C* to *A*?

5. A steamer whose speed upon a lake is $10\frac{1}{2}$ mi. per hour, plies on a river whose velocity is $1\frac{1}{2}$ mi. per hour, between two cities. The round trip takes 21 hr. How far are the cities apart?

6. A man who rows 4 mi. an hour in still water takes $1\frac{1}{5}$ hr. to row 4 mi. up a river. How many minutes will it take him to row 4 mi. down the river?

7. A crew can row up a stream a certain distance in 64 min. and back again in 60 min. Determine the distance, the rate of the stream being half a mile per hour.

8. A boatman can row with the current from *A* to *B*, a distance of 30 mi., in 4 hr. When the stream is $\frac{4}{5}$ as strong it takes him 10 hr. to row from *B* to *A*. What was the rate of the current?

9. *A* and *B* row on a river, starting together from the same point, *A* down, *B* up stream. In $3\frac{3}{4}$ min. they are $\frac{5}{8}$ mi. apart. *A* turns to follow *B*, and at the end of $12\frac{1}{2}$ min. from that time the boats have together rowed $1\frac{7}{8}$ mi. in all. If the speed of *A*, *B*, and the stream are constant, how many miles per hour does the river flow?

10. *A* rows at the rate of 6 mi. per hour in still water, and he finds it takes him just 3 times as long to row a distance up as to row it down the river. Find the rate of the stream.

11. Find the rate of 2 trains 150 ft. and 180 ft. long, respectively, which pass each other going the same way in 15 sec., and going in opposite directions in 3 sec.

12. A railway train having left a terminus at noon is overtaken at 6 p.m. by another train, which left the same terminus at 1 p.m. If the former train had been 10 mi. farther on the road when the latter started, it would not have been overtaken till 8 p.m. Find the rates of the trains.

13. A train 88 yd. long overtakes a man walking along the line at the rate of 4 mi. an hour and passes him in 10 sec.; 20 min. after it overtakes another man and passes him in 9 sec. Where will the train be when the last man overtakes the first?

14. A train going 30 mi. an hour passes a man walking in the same direction at 3 mi. per hour in 10 sec. Find the length of the train.

15. If a train 110 yd. long meets a person walking on the railway at the rate of 5 mi. per hour, and passes him in 8 sec., what is the rate of the train in miles per hour ?

16. A train 110 yd. long, moving $\frac{1}{2}$ mi. a minute, meets another on a parallel track, moving 40 ft. a second, and passes it in 8 sec. Find the length of the second train.

Scales of Notation.

1. Reduce 725 and 29 to the septenary scale, and find their quotient in that scale.

2. Extract the square root of 3106571 in the scale of 8.

3. Transform 17424 from the denary to the duodenary scale of notation, and find the square root of it in the duodenary scale.

4. Change the numbers 180 and 150 from the scale of 10 to that of 7, and in the latter scale find their product and its cube root.

5. Find the square root of 3124321 in the scale of 6.

6. Find the cube root of 240334012123 in the scale of 5.

7. Reduce $49.2291\dot{6}$ from the scale of 10 to that of 12.

8. Transform 7056.263 from the octenary scale to the denary one.

9. How many times is the greatest number of 3 digits in scale four contained in the greatest number of 4 digits in scale eight ?

10. Which of the weights, 1 lb., 2 lb., 4 lb., 8 lb., etc., must be used in weighing 273 lb. ?

11. Show that 6325 lb. can be weighed by using the weights 1 lb., 3 lb., 9 lb., 27 lb., etc., each weight to be used only once.

Averages.

1. The average price of wheat in 1836 was 62s. 3d., and from 1836 to 1842, inclusive, was 63s., and from 1837 to 1843, inclusive, it was 64s. 2d. Find the average price of wheat in 1843.

2. In a school of 19 children, 7 are boys, and 12 are girls. Of the boys, 3 are 8 yr. old, 2 are 11, and 2 are 12. Of the girls, 3 are 9 yr. old, 2 are 10, 3 are 13, and 4 the average age of the boys. Find the average age of the class.

3. Prove from the ordinary multiplication table that the average of the 8 numbers surrounding any number is that number.

4. The populations of 3 towns in 1881 were, respectively, 5850, 6375, and 4560. In 1891 the two former had increased 6 % and 8 %, respectively, and the latter had decreased 10 %. Find the average population in 1891.

5. The average weight of a crew of 8 men is 160 lb. 6 oz. Three of them weigh 508 lb. What is the average weight of the others?

6. The mean height of 5 mountains is 9473 ft., and the mean height of 6 mountains is 9584 ft. Find the height of the sixth mountain.

7. *A*'s salary was one-third more than *B*'s. They were each promoted, with increase of salary, *B*'s increase being $9 for $4 of *A*'s, and each had then $1440 per annum. Required, their original salaries.

8. A market-woman bought a number of eggs at 3 for 2c., and a number, exceeding the former by 5 doz., at a cent apiece, thus buying the whole at the average price of 11c. a dozen. How many did she buy altogether?

9. At the end of the term a teacher finds that there are 10 boys for every 7 there were at first, and that there are 8 girls for every 9. There being now 19 scholars for every 15 at first, compare the original number of boys and girls.

Percentage.

1. The rent of a farm is $720, and the taxes are 14$\frac{2}{5}$ % on the rent. Find the amount of rent and taxes together.

2. A bankrupt owes $7850, and pays 37$\frac{1}{2}$c. in the dollar. How much did his creditors jointly lose?

3. A person's half-yearly income is derived from the proceeds of $4550 at a certain rate per cent., and $5420 @ 1 % more than the former. His whole income is $453. Determine the rates.

4. A creditor agreeing to receive $281.25 for a debt, finds that he has been paid at the rate of 62$\frac{1}{2}$c. in the dollar. How much was the debt?

5. A person pays one tax of 10d. in the £, and another of 5 % on his income. His remaining income is £545. What was his original income?

6. If, when 25 % is lost in grinding wheat, a country has to import 10000000 qr., but can maintain itself on its own produce if only 5 % be lost, find the quantity of wheat grown in the country.

7. A bankrupt's assets are $2700, out of which he pays 75c. in the dollar on half his debts, and 60c. on the other half. What is the amount of his debts?

8. A man having a flock of sheep sold 8 % of them to *A*, 90 to *B*, $3\frac{1}{2}$ % of the remainder to *C*, and 29 to *D*. He then had 550 left. How many had he at first?

9. The receipts of a railway company are apportioned in the following manner: 48 % for the working expenses, 10 % on one-fifth of the capital, and the remainder, $32000, for division among the holders of the rest of the stock, being a dividend at the rate of 4 %. Find the capital and the receipts.

10. *A* began business with a certain capital. The first year he gained 20 %, which he added to his capital. The second year he gained $37\frac{1}{2}$ %, which he also added to his capital. The third year he lost 40 %, and now found himself $200 worse than when he began business. Find the capital with which he began.

11 I spent 25 % more than my income in a certain year. For each of the next four years I saved $6\frac{2}{3}$ % of it, and then I found that I had lived within it, and had $50 besides. What was my income?

12. A bankrupt can pay 40c. in the $. If his assets were $500 more, he could pay 45c. Find his debts and his assets.

13. A man for 5 yr. spends £40 a year more than his income. If he, at the end of that time, reduce his expenditure 10 %, in 4 yr. he will have paid off his debts and saved £120. Find his income.

14. Eleven and one-half yards of cloth $1\frac{1}{4}$ yd. wide are required for a dress. How many yards must be bought if the shrinkage in sponging is $12\frac{1}{2}$ % in length and 10 % in width?

15. My purse and the money in it are worth $82. If I spend 15 % of the money and sell my purse for three times its value I shall then have $74. Find the value of my purse.

16. If the increase in the number of male and female criminals be 1.8 %, while the decrease in the number of males alone is 4.6 %, and the increase in the number of females is 9.8 %, compare the number of male and female criminals, respectively.

17. 12 bu. of wheat and 25 bu. of barley cost $19; but if wheat were to rise 4 % and barley to fall 10 % in price, the same quantities would cost $18.36. What is the price of each per bushel?

18. The population of a certain city is now 185220. Three years ago it was 160000. Find its population three years hence, supposing its population to increase at the same rate per cent. per annum.

19. A train increases its average speed per minute 12½ % per minute. In 4 min. it travels 1972 yd. How far does it go in the first two minutes.

Profit and Loss.

1. Find the gain or loss per cent. in buying oranges @ $2.50 per hundred and selling them at 8 for 12c.

2. A woman buys a certain number of apples at 3 a penny, and the same number at 2 a penny; she then mixes them and sells them at 5 for twopence. How much does she gain or lose per cent ?

3. A person, by disposing of goods for $182, loses 9 %. What ought they to have been sold at to realize a profit of 7 % ?

4. The cost price of a book is $4.75, expense of the sale 6 %, profit 24 %. What is the retail price ?

5. At what per cent. in advance of cost must a merchant mark his goods so that after throwing off 20 % of the marked price he may make a profit of 25 % ?

6. By selling a house for $3700 I lost 7½ %. What must I have sold it for to have gained 12½ % ?

7. A merchant sells tea to a tradesman at a profit of 60 %, but the tradesman, becoming a bankrupt, pays 37½c. in the dollar. How much does the merchant gain or lose by the sale ?

8. A baker's outlay for flour is 70 % of his gross receipts, and other trade expenses 20 %. The price of flour falls 50 %, and other trade expenses are thereby reduced 25 %. What reduction should he make in the price of a 15c. loaf, allowing him still to realize the same amount of profit ?

9. If a tradesman adds to the cost price of his goods a profit of 12½ %, what is the cost price of an article which he sells for $3.82½ ?

10. A grocer buys 4 cwt. 3 qr. 14 lb. of sugar @ £1 16s. 8d. per cwt. (long cwt.), and sells it @ 4½d. per lb. How much does he gain or lose per cent. ?

11. If, by selling an article for $38.25, 8 % is lost, what per cent. is gained or lost by selling it for $57 ?

12. A man wishing to sell a horse asked 25 % more than it cost. He finally sold it for 15 % less than his asking price, and gained $5.75. How much did the horse cost, and what was the asking price?

13. I bought 20 lb. of opium by Avoirdupois weight @ 55c. per ounce, and sold by Troy weight @ 60c. per ounce. Did I gain or lose, and how much?

14. A man buys an article and sells it again so as to gain 5 %. If he had bought it @ 5 % less, and sold it for $1 less, he would have gained 10 %. Find the cost price.

15. If 7 % be gained by selling goods for $69.55, what will be gained or lost by selling them for $61.75?

16. Which is the more profitable, to buy flour @ $6.50 on 6 mo., or $6.30 cash, money being worth 8 %?

17. If 9 t. 7½ cwt. of iron be sold for $1260, and the gain on it be 20 %, what was the cost per cwt.?

18. A dealer sends out 250 lb. of tea @ 80c. per lb., and allows 2½ % on the price for the expense of carriage. Supposing the whole amount of carriage to come to $9.30, how much will the customer have to pay?

19. By selling tea @ 72c. a pound a grocer clears $\frac{1}{5}$ of his outlay. He then raised the price to 90c. What does he clear per cent. by the latter price?

20. If 6 % be gained by selling a horse for $132.50, how much per cent. is lost by selling him for $115?

21. A man sells two horses for $100 each, and by so doing gains 25 % on one horse and loses 25 % on the other. What did the horses cost him? Does he gain or lose on the whole?

22. Three-fourths the selling price of goods is 20 % less than cost. Find the gain per cent. at which the goods are sold.

23. A grocer has 225 lb. of tea, of which he sells 45 lb. @ 72c. per pound, and only gains 8 % at this price. He now raises the price so as to gain 10 % on the whole outlay. What is the price when raised?

24. A woman buys a certain number of eggs at 21 a shilling, and the same number at 19 a shilling; she mixes them together and sells them at 20 a shilling. How much does she gain or lose per cent. by the transaction?

25. I have a certain sum of money wherewith to buy a certain number of nuts, and I find that if I buy at the rate of 40 for 10c. I shall spend 5c. too much; if at the rate of 50 for 10c., 10c. too little. How much money had I?

26. Bought two kinds of cloth, one @ $\$2\frac{1}{3}$ per yard, the other @ $\$1\frac{2}{3}$ per yard. Sold all at the same price, which was as much per cent. below the cost of one kind, as it was per cent. above the cost of the other. Find the selling price and the gain or loss per cent.

27. Bought equal quantities of two kinds of cloth @ 20c. and 40c. per yard, respectively. I sold all at the same price per yard, and thereby gained as much per cent. on the one kind as I lost per cent. on the other kind. Find (*a*) the selling price, and (*b*) the gain or loss per cent.

28. A machinist sold 24 grain-drills for $125 each. On one-half of them he gained 25 %, and on the remainder he lost 25 %. Did he gain or lose on the whole, and how much?

29. Bought land @ $30 an acre. How much must I ask an acre that I may abate 25 % from my asking price, and still make 20 % on the purchase money?

30. *A* sold a house at a loss of 25 %. If he had received $500 more for it he would have gained $2\frac{7}{9}$ %. Find the cost of the house.

31. I buy two lots for $4000, and sell one so as to lose $7\frac{1}{2}$ %, and the other so as to gain 5 %, and on the whole I neither gain nor lose. Find the cost of each lot.

32. *D* sold one-fourth of his goods at a loss of 20 %. By what increase per cent. must he raise this selling price to gain 20 % on the entire transaction?

33. I invest and sell at a loss of 25 %. I invest the proceeds again, and sell at a gain of 25 %. Do I gain or lose on the two speculations, and how many per cent.?

34. I invest and sell @ 12 % gain. I invest the proceeds and sell at an advance of 15 %. I invest the proceeds again, and sell at a loss of 25 %, and quit with $1254.80. What was my capital at first?

35. A manufacturer sells @ 20 % profit, the wholesaler @ 25 % profit, and the retailer @ 40 % profit. Find the cost to the manufacturer of an article which cost the consumer $2.10.

Commission, Taxes, Etc.

1. Gave $20050 to a broker to invest, with instructions, after deducting his brokerage, @ ¼ %, to invest the balance in Government bonds. What will be the sum invested, and how much will be the brokerage?

2. Mr. *A*. sent $3681 to his agent, with instructions to deduct his commission, @ 2¼ %, and invest the balance in flour @ $7.50 per barrel. If the cost of freightage and insurance amounts to $119, at what must the flour be sold per barrel so as to make a profit of 20 %?

3. An agent received $21.70 for collecting a debt of $2480. What rate was his commission?

4. I send to my agent in Montreal $3060 to invest in tea, @ 75c. per pound. He deducts his commission of 2 %, and invests the balance. At what must I sell per pound so as to make a clear profit of 25 %, after paying freightage $30, and insurance at the rate of ⅓ %?

5. I sent my agent flour to sell on a commission of 3 %, with instructions to deduct a second commission at the rate of 4 %, and to invest the balance in silks. If the agent's total commission is $700, how many yards of silk does he buy @ $1¼ per yard?

6. Having sold a consignment of cotton on 3 % commission, I am instructed to invest the proceeds in city lots, after deducting my purchase commission of 2 %. My whole commission is $265. What is the price of the city lots?

7. A commission merchant sold a consignment of 1400 bbl. of pork @ $12.50 per barrel. After deducting $73.24 for transportation, $19.50 for storage, and his commission, he remits to his employer $16777.26 as the net proceeds of the sale. What was his rate of commission?

8. An agent sold wheat, @ 5 % commission, and invested the proceeds in cotton, @ 2½ % commission. His total commission being $525. Find the value of the wheat sold.

9. A merchant lost a cargo at sea which he had insured. The broker offered him a sum of money for his loss, which the merchant refused as being 10 % below the estimated value of his loss. The broker then offered $379.75 more than he offered at first, and the whole amount of the second offer was 5½ % in excess of the estimated value. What was that value?

10. Find the sum paid for insurance @ $\frac{1}{2}$ % on a house worth $10000, @ $\frac{2}{3}$ % on furniture worth $2500, if the insurance is on $\frac{4}{5}$ of the value of the property insured.

11. A building is assessed for $\frac{4}{5}$ of its value, and the rate of taxation is 17$\frac{1}{2}$ mills on the dollar. What will be the amount of the tax, if it costs 52\frac{1}{2}$ to insure the building for $\frac{2}{3}$ of its value, @ 1$\frac{3}{4}$ %?

12. If $10.50 be a person's income tax @ 1$\frac{1}{2}$c. on the dollar, how much in the dollar is it when his income tax is $12.25?

13. A school rate of 5 mills per dollar, and a general purpose rate of 8 mills on the dollar, produce a tax of $101.40. Find the assessed value of the property.

14. *A* pays $3.60 more tax than *B*, their incomes being equal. Living in different towns, they are rated at 1$\frac{1}{2}$c. and 1$\frac{1}{4}$c. in the dollar, respectively. What is *A*'s income?

Interest.

1. What is the difference between simple interest, compound interest, and discount? Find the difference between the simple interest and the true discount on $1900 for 1$\frac{3}{4}$ yr. @ 8 %.

2. (*a*) Find the difference between the simple and compound interest of 416.66\frac{2}{3}$ for 2 yr. @ 8 %.

(*b*) Find the rate of interest when the discount on $211.60 due at the end of 1$\frac{1}{2}$ yr. is $27.60.

3. What sum will amount to $3213 in ten years @ 8 %, simple interest?

4. At what rate will the simple interest on $125 amount to 13.12\frac{1}{2}$ in 1$\frac{1}{2}$ yr.?

5. What principal will give $616, simple interest, in 5$\frac{1}{2}$ yr. @ 6$\frac{2}{3}$ %?

6. A person invests $750 at simple interest, and at the end of 3 yr. and 8 mo. he finds that he possesses $956.25. At what rate per cent. per annum was his profit?

7. Find the simple interest on 2733\frac{1}{3}$ @ 4 % for 3 yr. and 9 mo.; and determine what sum will amount to $926.10 in 3 yr. @ 5 %, compound interest.

8. Show that the simple interest on $625 for 8 mo. @ 7 % is equal to that on $1093.75 at 8 % for 4 mo.

9. How many years' purchase should I give for an estate so as to get 3$\frac{1}{3}$ % interest for my money?

10. In how many years will $320 double itself @ $7\frac{1}{2}$ %, simple interest?

11. Find the principal sum on which the simple interest in $2\frac{1}{4}$ yr. @ $6\frac{1}{3}$ % per annum is $1068.75.

12. The compound interest on a certain sum @ 4 % for 2 yr. exceeds the simple interest for the same time at the same rate by $6. What is the sum?

13. Compound interest reckoned quarterly @ 2 % is equal to what interest reckoned yearly?

14. The sum of $327 is borrowed at the beginning of a year at interest, and after 9 mo. have passed $400 more is borrowed at a rate of interest double that which the former sum bears. At the end of the year the interest on both loans is $26.35. What is the rate of interest in each case?

15. A contractor sends in a tender of $20000 for a certain work. A second sends in a tender of $19000, but stipulates to be paid $2000 every three months. Find the difference between tenders, supposing the work in both cases to be finished in two years, and money to be worth $7\frac{1}{2}$ %, simple interest?

16. How long will it be before $2500, put out at compound interest @ 10 % per annum, will obtain 1727.58\frac{7}{8}$ as interest?

17. If the difference between the simple and compound interest on a sum of money for two years @ 5 % be $3, find the sum?

18. A banker borrows money @ $3\frac{1}{2}$ %, and pays the interest at the end of the year. He lends it out @ 5 %, but receives the interest half-yearly, and by this means gains $200 per year. How much does he borrow?

19. At what rate will $157.50 amount to $189 in 5 yr.?

20. A sum of money amounts in 10 yr. @ $7\frac{1}{2}$ %, simple interest, to 787\frac{1}{2}$. In how many years will it amount to $990?

21. A note of $1000, dated January 1, 1896, and bearing interest @ 5 % per annum, has the following endorsements: May 13, $240; Aug. 19, $300; Oct. 25, $180. Required, the balance due Jan. 1, 1897.

22. A sum of money was put out at compound interest. The first year's interest was $6000, and the fourth year's $6749.184. Find the sum and the rate per cent.

Discount.

1. What is the present worth of a bill of \$170, due in 4 mo., reckoning money @ 6 % per annum ?

2. Find the interest on \$880 for $1\frac{1}{4}$ yr. @ $4\frac{1}{2}$ %, and the discount on \$929.50 for $2\frac{1}{4}$ yr. @ $2\frac{1}{2}$ %.

3. If \$3 is the discount off \$333 for 2 mo., what was the rate per cent. ? What should be the discount off \$333 for 1 yr. ?

4. The mathematical discount on a sum of money for 2 yr. is \$360. The interest on the same sum for the same time is \$400. Find the sum and the rate per cent.

5. (i) What is the difference between Interest and Discount ? Which of the two is greater ?

(ii) Find the difference between the interest and discount on \$1639 for $4\frac{3}{4}$ mo. @ $6\frac{3}{19}$ %.

6. Find the difference between the true and bank discounts on a note of \$10400, due in 6 mo. (days of grace included) @ 8 % per annum.

7. If \$40 is a proper discount off \$360 for 8 mo., what should be the 12 mo. interest on \$360 ?

8. Find the difference between the discount on \$1622.50 for 14 mo. @ 7 % per annum, and the interest on \$1760 for 15 mo. @ 6 % per annum.

9. If I pay \$750 now for a debt of \$771.09$\frac{3}{8}$ not yet payable, and money be considered worth $7\frac{1}{2}$ % per annum, when will the debt be due ?

10. Find the difference between the interest and discount on \$1265 for 73 da. @ 6 %.

11. Two persons, *A* and *B*, meet to settle their accounts. *A* had $3\frac{1}{4}$ yr. previously lent *B* \$500, and *B* has a bill of \$360.50 against *A*, for which he is to allow nine months' discount. If the *interest* in each case is 4 % per annum, what has *B* to pay *A* ?

12. A tradesman marks his goods with two prices: the one for ready money, the other for 6 mo. credit, the rate of interest being 5 % per annum. If the credit price of an article be \$2.05, what ought its ready-money price to be ?

13. The discount on \$566.50 for 9 mo. is \$16.50. Find the rate of interest.

14. Show that the interest on \$15840 for 3 mo. @ 8 % is equal to the discount on \$3696 for 15 mo. @ $7\frac{1}{2}$ %.

15. The interest on a certain sum at simple interest is $360, and the discount $340 for the same time and rate. What is the sum?

16. Show that the difference between the interest and the discount on the same sum for the same time is the interest of the discount.

17. A tradesman makes a deduction of 10 % for ready money on a bill of $28 due in 12 mo., receiving $25.20. Find the difference between this sum and the present worth of the debt, reckoning interest @ 10 %.

18. If $10 is a proper discount off $210 for 3 mo., what should be a proper discount off the same sum for 1 yr.?

19. A farmer bought a horse for a bill of $292, due in 1 mo., and sold him for a bill of $348, due in 4 mo. What did he gain per cent., money being worth 4½ %?

20. *A* owes *B* $2725, and offers to pay him at a certain rate of discount instantly, instead of at the end of two years, when the debt will be due. *B* can place out the money which he will receive @ 5 % interest, and by that means gain $25 on the transaction. At what rate is the discount calculated?

21. A man discounts a bill of £180, drawn at 4 mo. @ 60 % per annum, and insists on giving in part payment 5 doz. of wine, which he charges @ 4 guineas a dozen, and a picture, which he charges @ £19. How much ready money does he pay? If the cost to the man of the wine and the picture be only one-fourth of the sum he has charged for them, what is the real interest the man has been charged?

22. The discount on a certain sum, due 9 mo. hence, is $20, and the interest on the same sum for the same time is $20.75. Find the sum and the rate of interest.

23. A banker, in discounting a bill due in 3 mo. @ 8 %, charges $16 more than the true discount. Find the amount of the bill.

24. If the discount on a sum due at the end of 2½ yr. is $\frac{80}{87}$ of the simple interest, at what rate is that calculated?

25. Two bills for $273.75 and $456.87½ are due on the 2nd and 22nd July, respectively. What is their value on the 12th July, interest being reckoned at the rate of 5 % per annum?

26. If the discount on a bill due 8 mo. hence @ 7½ % per annum be $48.75, what is the amount of the bill?

27. The difference between the interest and the discount on a certain sum of money for 6 mo. @ 4 %, is $2. What is the sum?

28. What must be the face of the note for 3 mo., made on 18th Aug., so that discounted @ $7\frac{1}{2}$ % on the day of making at the bank, the proceeds may be $14315?

29. If $5 be allowed as discount off a bill of $130 for a certain time, what should be the discount if the bill had one-half as long to run (i) at simple interest, (ii) at compound interest?

Equation of Payments.

1. A debt is due at the end of $4\frac{1}{2}$ mo.; $\frac{1}{4}$ is paid immediately, and $\frac{1}{4}$ at the end of 3 mo. When ought the remainder to be paid?

2. If I owe $2000, to be paid in 4 mo. time, and I pay $500 now, what extension of time ought to be allowed me for the payment of the remainder, reckoning money to be worth 8 % per annum, simple interest?

3. Suppose *A* owes $1357, to be paid in 12 mo., but that he pays $\frac{1}{9}$ of it in 6 mo. and $\frac{1}{18}$ in 9 mo., what time should be allowed for the payment of the remainder?

4. At true discount, find the equated time of payment of $305, due 4 mo. hence, and $415 due 9 mo. hence, money being worth 5 % per annum?

5. A certain sum of money is due in 6 mo. hence. If $\frac{3}{16}$ of it be paid in 3 mo., and $\frac{7}{16}$ in 5 mo., in what time (i) according to true discount @ 5 %, and (ii) according to mercantile discount, ought the remainder to be paid?

6. Determine the date on which the cash balance of the following account should be paid :—

1899.		Dr.	1899.		Cr.
June 5	To goods, 3 mo..	$237	June 20	By cash...	$150
July 18	" " 2 mo..	1365	July 25	" " ...	1000
Aug. 10	" " 5 mo..	128	Nov. 5	" " ...	200

7. A debt is to be paid as follows: One-fifth now and one-fifth every certain interval until all is paid. The equated time of payment being $6\frac{2}{3}$ mo., find the interval between the payments.

Stocks.

1. (i) Which is the better investment, the $3\frac{1}{2}$ per cents. @ 91, or the 4 per cents. @ 103?

(ii) How much must a man invest in the former that he may have a yearly income of $4851, after paying an income tax of 2c. in the dollar?

2. A man has $3430 stock in the $3\frac{1}{2}$ per cents. @ $85\frac{1}{2}$. When the stock rises 2 % he transfers his capital to the 4 per cents. @ 98. Find the alteration in his income.

3. *A* invests $552 in the $3\frac{1}{4}$ per cents. when they are @ 92; *B* invests $679 in the 3 per cents. when they are @ 97. Find the difference of their incomes.

4. A person invests £10000 in 3 per cents. @ 75, and when they rise to 78 he sells out and invests the produce in bank shares @ £208 each, which pay a dividend of £8 per share. Show that his income is not altered.

5. A person invests £5000 in Turkish 6 % stock @ 80. Find the rate of interest he gets for his money. When his stock has risen to 104 he sells out, and buys £20 railway shares @ £18, which pay dividend at the rate of $4\frac{1}{2}$ %. Find the alteration in his income.

6. A man, by selling out of a 3 % stock @ 99, gains 10 % on his investment. At what price did he buy, and what was his income, supposing that he realized $15345?

7. A person invests the present value of £2358, due 2 yr. hence @ 4 %, in gas shares, which pay at the rate of 9 %. He gives £144 for each share of £100. What is his annual income, and what rate per cent. does he make of his money invested in the gas shares?

8. How much money must one invest in 3 % Consols, when they are @ 10 % below par, in order to have an income of £2000 a year?

9. Two persons buy, respectively, with the same sums into the 3 and $3\frac{1}{2}$ per cents., and get the same amount of interest. The 3 per cents. are @ 75. At what price are the $3\frac{1}{2}$ per cents?

10. A man invested $14350 in 6 % stock @ $107\frac{1}{2}$, the brokerage being $\frac{1}{8}$ %. What will be his clear income after an income tax of 5 % is deducted?

11. A man receiving a legacy of $34510 invested one-half in a 6 % stock @ 101, and the other half in a 5 % stock @ $84\frac{1}{2}$,

paying brokerage @ $\frac{1}{2}$ %. What annual income did he secure from his legacy?

12. A speculator bought 43 shares in a mine @ 35$\frac{1}{4}$, and kept them till they dropped to 11$\frac{1}{2}$, when he sold out and bought with the proceeds 6 % railway stock @ 28 % premium. Find his annual income from the latter investment.

13. What sum must a man invest in the 6 % county bonds @ 101$\frac{1}{2}$, in order to have a clear income of $1424.40, after paying an income tax of 1$\frac{1}{2}$ % on all over $400?

14. A person having $9790 in the Toronto city 6 % bonds sells out @ 98$\frac{1}{2}$, and invests the proceeds in Bank of Montreal stock @ 177$\frac{1}{2}$, which pays a dividend of 12 % per annum. Find the change in his income, brokerage in each transaction being $\frac{1}{2}$ %.

15. What sum of money must be left in order that, after a legacy duty of 10 % has been paid, the remainder being invested in the Dominion 3 per cents. @ 91$\frac{1}{8}$, may give a yearly income of $450, brokerage @ $\frac{1}{8}$ %?

16. By investing a certain sum of money in the 6 per cents. @ 91$\frac{1}{2}$, a man obtains an income of $320. What would he obtain by investing an equal sum in the 5 per cents. @ 80?

17. *M* invests one-third of his property in bank stock, one-sixth in consols, and the remainder in railway shares. When he sells out he makes a profit of 5 %, 3 %, and 2 %, respectively, on the investments, and realizes £6190. Required, the amount of his property originally.

18. A merchant in Toronto instructed his agent in Montreal to sell a consignment of flour @ $7.50 per barrel and invest the proceeds in Montreal bank stock @ 174$\frac{1}{2}$, which pays half-yearly dividends of 7 %. If the merchant's first dividend is $445.50, and commissions of 1 % and $\frac{1}{2}$ % be allowed on the transactions, respectively, how many barrels of flour were sold?

19. A person possessing £10000 3 % consols, sells out when they are @ 93$\frac{5}{8}$, and invests the proceeds in 4 % stock @ 101$\frac{7}{8}$. Find the change in his income, allowing $\frac{1}{8}$ % commission on each transaction.

20. A person invests $6825 in 6 % stock @ 91; he sells out $5000 stock when it has risen to 93$\frac{1}{2}$, and the remainder when it has fallen to 85. How much does he gain or lose by the transaction? If he invests the produce in M. B. S., which pays a dividend of 12 %, @ 175, what is the difference of his income?

21. If *A* has $38940 to invest, and can buy Toronto city 4 % bonds @ 98½, or Montreal Corporation Consolidated 4½ % stock @ 117½, how much will the one transaction be better than the other, brokerage being ½ % ?

22. When the New York gold market is @ 104¾, what would I get for $2304.50 currency ?

23. A person invests $9450 in 5½ % stock, so as to receive an income of $787.50. What was the price of the stock ?

24. *A* invests half his capital in the 3 per cents. @ 90, and the other half in the 5 per cents. @ 110. His income from both investments being $1376.70, find his capital.

25. How much 3 % stock @ 89 must a man sell, so that investing the proceeds in 4 % stock @ 92, his income may be increased by $60 ?

Sharing, Partnership, Etc.

1. Divide $13230 between 2 men, so that one may receive a third as much again as the other.

2. *A* is to receive $1.25 a day every day he works, and to forfeit 80c. every day he is idle. At the end of 75 da. his wages amount to $69.15. How many days was he idle ?

3. *A*, *B*, and *C* start on a tour, each with $200 in his pocket, and agree to divide their expenses equally. When they return *A* has $37.50, *B* $50.82, and *C* $16.71. What ought *A* and *B* to pay *C* to settle their accounts ?

4. A gunboat's crew, consisting of a lieutenant, a gunner, and 15 seamen, captured a prize worth £399 7s. The lieutenant's share is 10 times, and the gunner's share 3 times as much as that of each seaman. What is the value of each person's share ?

5. Divide $87.50 between two men, so that one may receive half as much again as the other.

6. The number of disposable seamen at Portsmouth is 800, at Plymouth 756, and at Sheerness 404. A ship is commissioned, whose complement is 490 seamen. How many must be drafted from each place so as to take an equal proportion ?

7. Divide $2849 among *A*, *B*, and *C*, in the proportion of .7, .28, and .056.

8. Gunpowder being composed of 33 parts of nitre, 7 of charcoal, and 5 of sulphur, find how many pounds of each will be required to make 30 lb. of powder.

9. *A*, *B*, and *C* are partners. *A* receives two-fifths of the profits, *B* and *C* dividing the remainder equally. *A*'s income is increased by $220 when the rate of profit rises from 8 % to 10 %. Find the capital of *B* and *C*.

10. *A*, *B*, and *C* rent a meadow for $43. *A* puts in 10 horses for 1 mo., *B* 12 oxen for 2 mo., and *C* 20 sheep for 3 mo. How should the expense be divided if the quantities eaten by a horse, an ox, and a sheep during the same time be in the ratio of 4, 3, and 1?

11. *A* and *B* receive $1.37½ for digging a garden. They work at it together for 4½ hr.; *B* then left, and *A* finished the work in 3½ hr. How should the pay be divided?

12. Divide $1986.50 among *A*, *B*, and *C*, in the proportion of $2.\dot{3}$, $1.1\dot{5}$, and $.5\dot{2}\dot{4}$, respectively.

13. If 20 men, 40 women, and 50 children receive $4200 among them for seven weeks' work, and 2 men receive as much as 3 women or 5 children, what sum does a woman receive per week?

14. Divide $350 among 4 persons, so that *B* may have three times as much as *A*, *C* half as much again as *A* and *B* together, and *D* as much as *A*, *B*, and *C* together?

15. What is the average annual profit of a business when a partner, entitled to $\frac{2}{7}$ of the profits, receives as his share for 2 yr. and 4 mo. the sum of $7890.50?

16. Two ships are built. Twice as many ship-carpenters are employed about the first as about the second. The first is built in 9 mo., the second in 8 mo. The wages of each man of the first set are 25c. per hour, and they work 12 hr. a day. The wages of each of the second set are 18c. per hour, and they work 10½ hr. a day. The cost of the first in carpenters' wages was $30000. What was that of the second?

17. A person leaves $12670 to be divided among his five children and three brothers, so that, after the legacy duty has been paid, each child's share shall be twice as great as each brother's. The legacy duty on a child's share being one per cent., and on a brother's three per cent., find what each will receive.

18. A legacy of $146000 is left to three sons in the proportion of $\frac{1}{6}$, $\frac{1}{7}$, and $\frac{1}{8}$, respectively. How much will each receive?

19. Two men invest $300 and $100 in a machine. It works 5 mo. for each of them. Determine what one must pay the other if they would have made 30 % on the money by letting the machine.

20. The estate of a bankrupt, value $21000, is to be divided among four creditors, whose claims are, *A*'s to *B*'s as 2 to 3, *B*'s to *C*'s as 4 to 5, *C*'s to *D*'s as 6 to 7. What must each receive?

21. In England gunpowder is made of 75 parts nitre, 10 sulphur, and 15 charcoal. In France of 77 parts nitre, 9 sulphur, and 14 charcoal. If half a ton of each be mixed, what weight of nitre, sulphur, and charcoal will there be in the compound?

22. The wages of 5 men, 3 women, and 1 child amount to $34, a man receiving twice as much as a woman, and a woman three times as much as a child. What will be the wages of 6 men, 2 women, and 5 children?

23. The British standard gold for coinage consists of 11 parts of pure gold, and 1 part of alloy (usually a mixture of silver and copper). How much pure gold and how much alloy are contained in a guinea, which weighs 5 dwt. 9 gr.?

24. Three tramps meet together for a meal. The first has 5 loaves, the second 3, and the third, who has his share of the bread, pays the other two 8 half-pence. How ought they to divide the money?

25. *A*, *B*, *C*, and *D* enter into partnership. *A* and *B* contribute $1390, *B* and *C* $1590, *C* and *D* $1810, *A* and *D* $1610, *A* and *C* $1500. They gain $1152. What is the share of each?

26. In a constituency, in which each elector may vote for two candidates, half of the constituency vote for *A*, but divide their votes among *B*, *C*, *D*, *E*, in the proportions of 4, 3, 2, 1. Of the remainder, half vote for *B*, and divide their votes among *C*, *D*, *E*, in the proportions of 3, 1, 1. Two-thirds of the remainder vote for *D* and *E*, and 540 do not vote at all. Find the order on the poll, and the whole number of electors.

27. The sum of £177 is to be divided among 15 men, 20 women, and 30 children, in such a manner that a man and a child may receive together as much as two women, and all the women may together receive £60. What will they each, respectively, receive?

28. A mixture of soda and potash, dissolved in 2540 grains of water, took up 980 grains of aqueous sulphuric acid, and the weight of the compound solution was 4285 grains. Find how much potash and how much soda the mixture contained, assuming that aqueous sulphuric acid unites with soda in the proportion of 49 grains to 32, and with potash in the proportion of 49 to 48.

29. *A*, *B*, *C*, and *D* engage in business with a joint capital of \$45000. At the end of a certain time *A* receives \$2000, *B* \$2800, *C* \$1686, and *D* \$1014. How much capital did *D* put in?

30. A farm of 73 ac. is divided so that a man receives $3\frac{1}{2}$ ac., a woman $1\frac{1}{2}$ ac., and a child 1 ac. There are 5 men to every 6 women, and 2 children to every man. How many men are there?

31. Divide \$202.40 among 3 men, 8 women, and 10 children on the supposition that a man does as much work as 3 women or 5 children.

32. Three merchants form a company. The first, *A*, puts in \$960 for 6 mo. The second, *B*, a sum for 12 mo. The third, *C*, \$640. *A* received \$1200 for his stock and profit; *B*, \$2400 for his; and *C*, \$1040 for his. What was *B*'s stock and *C*'s time?

33. A dealer imports equal weights of tea, coffee, and cocoa. The value per pound of the tea is half as much again as that of the coffee, and $1\frac{4}{5}$ as much as that of the cocoa. The whole weighs 54 cwt. (long cwt.), and costs £672. At what price per pound is each article imported?

34. A warehouse of five storeys is let in flats. Each flat, except the top one, lets for $\frac{3}{4}$ of the rent charged for all the flats above it, and the rent of the whole warehouse is \$9604. What is the rent of the top flat?

35. If one part of \$1600 is put out @ $4\frac{1}{2}$ % per annum, and the other @ $5\frac{1}{4}$ % per annum, and if the yearly interest is \$78.45, find the parts.

36. Three partners in trade contribute, respectively, \$2190, \$1460, and \$3650, and agree that each is to have 5 % on these sums, and the remaining gain to be divided in proportion to the shares. Find the share of each, the whole gain being \$1000.

37. Divide the number 35 into two such parts, that 14 times the first part added to 7 times the second part will give 350.

38. In a certain constituency having 3868 voters, *A* received 23 votes for every 25 votes received by *B*, and was defeated by 148 votes. How many did not vote?

Alligation.

1. Two equal wine-glasses are filled with mixtures of spirit and water in the ratios of 1 of spirit to 3 of water, and 1 of spirit to 4 of water. When the contents are mixed in a tumbler, find the strength of the mixture.

2. I buy wheat @ 39s. a quarter, and some of a superior quality @ 6s. per bushel. In what proportion must I mix them so as to gain 25 % by selling the mixture @ 57s. 6d. per quarter?

3. A dealer purchases a liquid @ 4s. per gallon, and dilutes it with so much water that, when he sells the compound @ 3s. a gallon, he gains 20 % on his outlay. How much water is there in every gallon of the compound sold?

4. A grocer mixes 18 lb. of coffee @ 30c. a pound with 12 lb. of chicory @ 5c. a pound. At what price must he sell the mixture to gain 25 %?

5. A grocer buys 1¾ cwt. of tea @ 60c. per pound and 2¼ cwt. of tea @ 50c. per pound, and mixes them. He sells 2½ cwt. @ 55c. per pound. At what rate must he sell the remainder to gain 20 % on his outlay?

6. If a cask contains 3 parts wine and 1 part water, how much of the mixture must be drawn off and water substituted for the mixture in the cask to become half-and-half?

7. A grocer buys some tea @ 48c. per pound, and some @ 66c. In what proportion must he mix them that when he sells @ 72c. per pound, he may be making a profit of 20 %?

8. How much sugar @ 7c., 8c., 9c., and 14c. per pound must be taken to form a mixture of 360 lb. which will sell for 15c. per pound, at a gain of 25 %?

9. If pure gold is worth $18 per ounce, how much alloy (worthless) must be mixed with it so that it may be sold @ $16 per ounce at a gain of 33⅓ %?

10. If 5 turkeys and 9 geese are worth $16.40, and 9 turkeys and 5 geese are worth $20, find the cost of one turkey and one goose.

11. Bought 1000 bu. of barley, part @ 65c. per bushel and the rest @ 68c. The total cost was $669.20. How many bushels of each kind were bought?

12. How many gallons of water must be added to 80 gal. of alcohol 87½ % strong, so that the mixture may be 66⅔ % strong?

13. A mixture of coffee and chickory weighs 22 lb., and is worth $5.60. If the proportions of coffee and chickory are reversed it is worth $3.20. Chickory being worth 5c. per pound, what is the price of coffee?

14. A dealer buys milk @ 6½c. per quart, and after diluting it with water obtains 60 % profit on his outlay by retailing the

mixture @ 8c. per quart. What proportion of water does he add to the milk ?

15. One cask contains 20 gal. of brandy; another contains 30 gal. of water. How much must be transferred from one cask to the other so that the mixture may be of equal strength ?

16. If 5 oz. of gold 18 carats fine are mixed with 7 oz. 15 carats fine, how much pure gold will there be in an ornament made from the mixture and weighing 4 oz. ?

17. A banker bought 100 shares of stock ($50) at an average of 10 % below par, and sold it at an average of 10 % above par; some at a discount of 20 %, some at a discount of 15 %, some at par, and some at a premium of 15 %. Required, the number of each kind.

18. There are three kinds of tea, valued, respectively, @ 32c., 36c., and 42c. per pound. If a mixture be made containing 6 lb. more of the second than of the first, what total quantity must be taken that the value of the mixture may be 38c. per lb. ?

19. *A*'s farm cost him, on the average, $60 an acre. He gave for 100 ac. of it $50 an acre, and for the rest of it $85 an acre. How many acres are there in his farm ?

20. A stock-dealer bought 270 head of sheep for $1365, paying $4, $4½, $5½, and $7½ a head. How many did he buy at each rate ?

21. A grocer bought tea, and in order to gain 40 % he must sell it @ 42c. per lb. He mixes it with other tea @ 27c. a pound in the proportion of 7 lb. of the first to 3 lb. of the second, and sells it in 10-lb. packages for $3.88. Find his gain per cwt.

22. A wine in a cask is worth $3.20 per gallon, but after adding 2 gal. of water the mixture is worth $3 per gal. How much pure wine was in the cask ?

Exchange.

1. The course of exchange on Paris being 5.17 francs per dollar, how much will a merchant in Montreal have to pay for goods in Paris which cost 2285.14 francs ?

2. What is the face of a bill on London which cost $254.86 in Toronto @ $4.88 per £ ?

3. *A* paid a broker $1513.89 for a bill of exchange on Berlin for 6400 marks. At what quotation was the bill purchased, allowing ⅛ % for brokerage ?

4. If for a sovereign one can buy 11 gulden 12 kreutzers or 25.5 francs, and for one 20-franc piece 9 gulden 20 kreutzers, how much per cent. is gained by buying French gold with English gold before buying German money ?

5. When £170 will purchase 4233 francs, what is the *course* of exchange between London and Paris ? And if 503 gold pieces of 20 francs contain as much pure gold as 400 sovereigns, what is the *par* of exchange between London and Paris ?

6. A draft on Dublin for £360 cost $1736.10. What was the course of exchange, commission charged at the rate of $\frac{1}{4}$ % ?

7. Find the face of a draft which cost $876.75, payable 30 da. after sight, if exchange is $\frac{3}{4}$ % premium and interest is 6 %.

8. Find the cost of a 60-day draft on New York for $2500, exchange being $\frac{5}{8}$ % premium and interest being 6 %.

9. Smith sold on commission goods to the amount of $2375, and, having deducted his commission @ 3 %, he remitted a draft @ 60 days for $2282.07. What was the rate of exchange ?

10. If at Toronto exchange on Liverpool is $4.885 per £1, and at Paris on Liverpool 25.402 francs per £1, what is the exchange on Toronto at Paris per dollar ?

11. A Toronto merchant owes 2000 marks in Frankfort. Should he remit direct from Toronto or through London, exchange at Toronto on Frankfort being $23.5 per 100 marks, on London $4.875 per £1, and at London on Frankfort 20.75 marks per £1 ?

Ratio and Proportion.

1. If .3 of an estate is worth $7500, what is the value of .48 of the estate ?

2. If an income of $1200 pays $18 for income tax, how much must be paid on an income of $750 when the tax is half as much again ?

3. How many hours a day must 42 boys work, to do in 45 da. what 27 men can do in 28 da. of 10 hr. long, the work of a boy being half that of a man ?

4. If 14 men can mow 35 ac. of grass in 6 da. of 10 hr. each, in how many days of 12 hr. each can 3 men mow 24 ac. ?

5. Two trains start at the same time, the one from London to Norwich, the other from Norwich to London. If they arrive in Norwich and London, respectively, 1 hr. and 4 hr. after they pass each other, show that one travels twice as fast as the other.

6. A person can read a book containing 220 pages, each of which contains 28 lines, and each line on an average 12 words, in $5\frac{1}{2}$ hr. How long will it take him to read a book containing 400 pages, each of which contains 36 lines, and each line on an average 14 words?

7. If 15 masons, working 10 hr. a day, can build a wall 6 ft. high, and 100 yd. long, in 6 da., how long will it take 7 masons, working 9 hr. a day, to build a wall 9 ft. high, and 140 yd. long?

8. If 36 men, working 8 hr. a day for 16 da., can dig a trench 72 yd. long, 18 yd. wide, and 12 yd. deep, in how many days will 32 men, working 12 hr. a day, dig a trench 64 yd. long, 27 yd. wide, and 18 yd. deep?

9. If the time past 7 a.m. is to the time past 9 p.m. as 3 to 11, find the time?

10. A cistern can be filled in 54 min. by a pipe running $3\frac{7}{8}$ gal. per minute. In how many minutes can it be filled by another pipe, running $2\frac{1}{4}$ gal. per minute?

11. A contractor, having engaged to lay 12 mi. of railway in 180 da., finds that 90 men have finished 4 mi. in 100 da. How many more men must be employed to finish the work in the given time?

12. Divide 456 gal. into 4 measures, so that the first shall be to the second as 3 : 5; the second to the third as 7 : 9; and the third to the fourth as 15 : 17.

13. If 54 men, in 63 da. of 10 hr. each, dig a trench $67\frac{1}{2}$ yd. long, $5\frac{1}{4}$ yd. wide, and $2\frac{3}{4}$ yd. deep, how many hours a day must 62 men work, to dig a trench $46\frac{1}{2}$ yd. long, $3\frac{2}{3}$ yd. wide, and $2\frac{1}{3}$ yd. deep, in $19\frac{1}{4}$ da.?

14. The weights of equal quantities of silver and platinum are as 10.5 and 21.5; and-32 cu. in. of silver with 100 cu. in. of platinum weigh as much as $127\frac{19}{39}$ cu. in. of gold. What number represents proportionally the weight of gold?

15. If 75 men can do a piece of work in 12 da., of 10 hr. each, how many men will do a piece of work twice as large in $\frac{1}{10}$ of the time, supposing that they work the same number of hours in the day, and that 2 of the second set can do as much work in an hour as 3 of the first set?

16. If 24 pioneers, in 10 da., of $6\frac{1}{4}$ hr., dig a trench $6820\frac{7}{8}$ yd. long, 9 yd. wide, and 5 yd. deep, how many hours a day must 10 pioneers work to dig a trench $9820\frac{1}{8}$ yd. long, $4\frac{7}{8}$ yd. wide, and $1\frac{3}{5}$ yd. deep, in 63 da.?

Mensuration.

1. The weight of the water contained in a rectangular cistern 8 ft. long, 7 ft. wide, is $93\frac{3}{4}$ cwt. (long cwt.). If a cubic foot of water weighs 1000 oz., find the depth of water in the cistern.

2. How many yards of matting, $2.\dot{4}$ ft. broad, will cover a floor that is $27.\dot{3}$ ft. long, and $20.1\dot{6}$ ft. broad?

3. The length of a rectangular field which contains 4 ac. 3 ro. 14 po. $26\frac{1}{2}$ sq. yd., is 260 yd. 1 ft. 4 in. What is its breadth?

4. A street being 850 ft. long, and the width of the pavement on each side being 5 ft. 3 in., find the cost of paving it @ $37\frac{1}{2}$c. per square foot.

5. A log of timber is 18 ft. long, 1 ft. 4 in. wide, and 15 in. thick. If a piece containing $2\frac{1}{2}$ solid feet be cut off the end of it, what length will be left?

6. If 8 guineas be expended in purchasing Brussels carpet $\frac{3}{4}$ yd. wide, @ 3s. 6d. a yard, for a room 20 ft. long, and 16 ft. 9 in. broad, how much of the floor will remain uncovered?

7. What will be the cost of papering a room 21 ft. long by 15 ft. broad and 11 ft. high, which has two windows, each 9 ft. high and 3 ft. wide, a door 7 ft. high and 3 ft. 6 in. wide, and a fireplace 4 ft. high by 4 ft. 6 in. wide, with paper 2 ft. 3 in. wide, @ 9s. a piece; the price of putting it on being 6d. per piece, and each piece containing 12 yd.?

8. A room is 22 ft. 6 in. long, 20 ft. 3 in. wide, and 10 ft. 9 in. high. Find the cost of carpeting the room @ $1.20 a square yard, and of papering the walls @ 20c. a square yard.

9. The external dimensions of a box without a lid are, length, 4 ft., breadth, 3 ft., depth, 2 ft., and the thickness of the sides and bottom is the same, namely, 1 in. If the cost of a cubic yard of the material is 9s., and the cost of making the box is $\frac{1}{11}$ of the cost of the material, what will the box cost?

10. A rectangular court is 50 yd. long, and 30 yd. broad. It has paths joining the middle points of the opposite sides of 6 ft. in breadth, and also paths of the same breadth running all round it. The remainder is covered with grass. If the cost of the pavement be $12\frac{1}{2}$c. per square foot, and of the grass 70c. per square yard, find the whole cost of laying out the court.

11. If the price of 9760 bricks, of which the length, breadth, and thickness are 20 in., 10 in., and $12\frac{1}{2}$ in., respectively, be

$213.50, what will be the price of 100 bricks which are one-fifth smaller in every dimension ?

12. A tank is 8 ft. long, 5 ft. 4 in. wide, 4 ft. 6 in. deep. Find the number of gallons it contains, having given that 1 cu. ft. of water weighs 1000 oz., and that a pint of water weighs a pound and a quarter.

13. A level reach in a canal, 14 mi. 6 fur. long, and 48 ft. broad, is kept up by a lock 80 ft. long, 12 ft. broad, and having a fall of 8 ft. 6 in. How many barges might pass through the lock before the water in the upper canal was lowered one inch ?

14. Find the cost of painting the walls of a square room 14 ft. high, and 18 ft. long, with two doors 8 ft. by 4, and three windows 10 ft. by 5, the amount saved by each window being £2 16s. 3d. What additional height would increase the cost by nine shillings ?

15. A hollow cubical box, made of material which is $1.\dot{3}$ in. in thickness, has an interior capacity of 50.653 cu. ft. Determine the length of the outside edge of the box.

16. A rectangular piece of ground, 72 yd. by 45 yd., is to be laid out in 4 plots of grass, each 27 ft. by $13\frac{1}{2}$ ft., and a pond in the centre 6 yd. square, to contain 252 cu. yd. of water. Find the expense of graveling the remainder, @ $2\frac{2}{3}$c. per square yard, and the depth of the pond.

17. The contents of a cistern is the sum of two cubes, whose edges are 10 in. and 2 in., and the area of its base is the difference between two squares, whose sides are $1\frac{1}{9}$ ft. and $1\frac{2}{9}$ ft. Find its depth.

18. A picture gallery consists of three large rooms. The first is 20 yd. long, 20 yd. broad, and 6 yd. high. The other two are 20 yd. long, 20 yd. broad, and 5 yd. high. Supposing the walls to be covered with pictures, except the doors, which are 8 ft. high, and 3 ft. wide, and of which each room has two, what will be the number of pictures, the average size being 8 ft. by 3 ft. ?

19. The breadth of a room is twice its height and half its length, and the contents are 4096 cu. ft. Find the dimensions of the room.

20. How many bricks, 9 in. long, $4\frac{1}{2}$ broad, and 4 thick, will be required for a wall, 60 ft. long, 20 ft. high, and 4 ft. thick, allowing $6\frac{1}{4}$ % of the space for mortar ?

21. The breadth of a room is two-thirds of its length, and three-halves of its height, and the contents are 5832 cu. ft. Find the dimensions of the room.

22. A plate of gold, 3 in. square, and one-eighth of an inch thick, is extended by hammering so as to cover a surface of 7 yd. square. Find its proper thickness.

23. The flooring of a room, 14 ft. 3 in. long by 13 ft. 4 in. broad, is composed of $\frac{1}{2}$ in. planks, each 8 in. wide, and 10 ft. long. How many will be required, and what will be the weight of the whole, if 1 cu. in. of wood weighs half an ounce?

24. A cistern without a top is 27 ft. long, 22 ft. wide, and 6 ft. 6 in. deep. What will it cost to paint it inside and out, @ $4\frac{1}{2}$c. a square yard?

25. A room, whose height is 11 ft., and length twice its breadth, takes 143 yd. of paper 2 ft. wide for its four walls. How many yards of gilt moulding will be required?

26. The length, breadth, and height of a wooden box are 4 ft., $2\frac{1}{2}$ ft., 3 ft., respectively. Find the cost of painting the outside, @ 1s. 3d. a square yard.

27. A room is 21 ft. long, 15 ft. 6 in. wide, 10 ft. high. It contains 3 windows, the recesses of which reach to the ceiling, and are 4 ft. 6 in. wide. There are in it 4 doors, each 6 ft. 6 in. high, and 3 ft. 3 in. wide. The fireplace is 6 ft. wide, and 4 ft. high. A skirting, 1 ft. 8 in. deep, runs round the walls. Find the expense of papering the room, @ 5c. a square foot.

28. A sphere of lead, 3 in. in diameter, is melted, and recast into three spheres, one $1\frac{1}{2}$ in. in diameter, and another 2 in. Find the diameter of the third one.

29. How many times as large is a hole bored by a 2-in. bit as one bored by a $\frac{1}{8}$-in. bit?

30. If a pipe, $1\frac{1}{3}$ in. in diameter, fill a cistern containing 48 gal. in a given time, what is the capacity of a cistern that a pipe, $2\frac{1}{2}$ in. in diameter, will fill in the same time?

31. How many times as large is a water-pipe, 20 in. in diameter, as one 6 in. in diameter?

32. How many times can a keg, 12 in. in diameter at the bung, be filled from a similarly-shaped barrel whose bung-diameter is 2 ft. 6 in.?

33. If a cannon ball 6 in. in diameter weighs 81 lb., what must be the diameter of a similar ball to weigh $\frac{8}{9}$ of a pound?

34. How many cast-iron balls, 4 in., 6 in., or 8 in. in diameter, can be placed in a cubical vessel whose edge is 2 ft.; and how many gallons of water will it contain after it is filled with balls?

35. If there is a cistern whose dimensions are 6 ft., 5 ft., and 4 ft., find the sides of another to contain 3 times as much, and whose sides will be proportional to those of the first one.

36. The parallel sides of a trapezoid are, respectively, 37½ ft. and 22½ ft. in length, and the non-parallel sides are, respectively, 16⅔ ft. and 18¾ ft. long. The latter sides are produced to meet. Find the respective lengths of the produced sides between the points of meeting and the shorter of the parallel sides of the trapezoid.

37. Two sides of a triangle are 218 ft. and 241 ft. long, respectively, and the perpendicular from the included angle on the third side is 120 ft. Find the third side.

38. One side of a right-angled triangle is 3925 ft. in length, and the difference between the hypothenuse and the other side is 625 ft. Find the hypothenuse and the other side.

39. The sides of a triangular field *ABC* are, *AB* 60 rods, *BC* 100 rods, *CA* 80 rods. A straight road is cut from *A* to *BC*, meeting it at right angles. Also, another from *A* to the middle point of *BC*. Find the area of the field between the roads.

40. The four sides of a field are, 75 yd., 100 yd., 125 yd., and 200 yd., respectively. The first two sides form a right angle. Find the area of the field.

41. The sides of a field in the shape of an isosceles triangle are the sides of three other square fields. The area of the larger is 24 sq. ch. greater than that of the triangle, while each of the others is 13 sq. ch. greater. The difference between the base and each of the equal sides is one chain. Find the area of the triangle.

42. A square is inscribed in a circle whose radius is 42 in. Find the area of the four segments of the circle outside the square.

43. What are the width and depth, inside measurement, of a rectangular box whose length inside is 5¼ ft., and contains 229635 cu. in., the width being ⅘ of the depth?

44. The difference between the areas of two squares inscribed and circumscribed about a circle is 338 sq. ft. Find the radius of the circle.

45. A rectangular piece of ground, covering ½ an acre, is 55 yd. long. Just within the fence surrounding the entire plot is a shrubbery, 10 ft. wide. Find the area of the shrubbery.

46. A rectangular field, containing 9 ac., is surrounded by a road 66 ft. wide. The area of the road being 4.3 ac., find the length and width of the field.

47. A field, containing 1 ac., is in the shape of a triangle. Its base being 137½ yd. long, find its altitude.

48. How much shorter would a path be from one corner of a rectangular field, 442 yd. long, and 120 yd. wide, than if it went along the side and end of the field?

49. A room is 18 ft. long, 13 ft. wide, and 9 ft. high. What is the distance from any corner of the floor to the farthest corner of the ceiling?

50. An electric light is 15 ft. above the ground. A man, 6 ft. high, finds his shadow is 7½ ft. long. How far is he standing from the foot of the post on which the light is placed?

51. The sides of a triangle are 164 in., 225 in., and 349 in. If squares are described on the sides so as to fall outside of the triangle, find (i) the perimeter of the figure, and (ii) its area.

52. Find the area of an equilateral triangle 20 ft. in altitude.

53. A square space containing 992.25 sq. yd. is to be lengthened by 1.5 yd. in one dimension, and shortened by 1.5 yd. in the other. Find the change in its area.

54. A square and a rectangular field have the same perimeter, 140 yd. The length of the rectangular field is 2½ times its width. Find the difference of their areas.

55. A square plot contains 2025 sq. yd. Find the area of a rectangular field of the same perimeter and whose length is 3½ times its width.

56. A square and a rectangular field each contain 10 ac. Find the difference in their perimeters, the length of the rectangular one being 4 times its width.

57. The diagonals of a rhombus are 25 in. and 15 in. Find its area.

58. Each side of a rhombus is 24 in. long, and one of the diagonals is also 24 in. in length. Find its area.

59. Each side of a rhombus is 65 ft. long, and one of the diagonals is 104 ft. in length. Find its area.

60. The radius of a circle is 126 in. Find the length of a tangent to the circle drawn from a point 130 in. from the centre.

61. Find the side of the largest square that can be inscribed in a circle 12⅚ ft. in circumference.

62. A circular fish-pond has a road running round it. The outer circumference of the road is 1144 yd. long, and the inner one is 1100 yd. in length. What is the area of the road?

63. The area of the basement of a circular building is 14454 sq. ft. The wall is 35 in. thick at the foundation. Find the surface covered by the base of the wall.

64. The area of a sector of a circle is 616 sq. ft. The angle of the sector is 40°. Find the perimeter of the sector.

65. A gardener lays out a flower-bed as follows. He marks out a square, whose side is 14 ft. Then, on each side of the square, and outside of it, he lays out a semicircle, with the side as diameter. Find the area of the flower-bed and its perimeter.

66. A circle, square, and equilateral triangle have the same perimeter, viz., 88 in. Find the area of each.

67. One extremity of a string is fastened to a corner of a board of the shape of an equilateral triangle, the side being 7 in. long, and the string is then wound around the triangle. It is then unwound, being kept stretched. Find the length of the distance moved over by the free end in one complete revolution.

68. The length of a triangular prism is 18 ft., and the edges of the triangular end are 13 ft., 14 ft., and 15 ft. Find the whole area of the prism.

69. How fast is a locomotive going when the small wheel, which is 4 ft. in diameter, makes 120 revolutions more per minute than the driving wheel, which is 7 ft. in diameter.

70. A water wheel, which is 17 ft. 6 in. in diameter, makes 8 revolutions per minute. Find the number of miles per week a point on the circumference travels, if the wheel is in motion 11 hr. per day, during 6 da.

71. The side of a square field is 48 rods. Find the side of a square field 3 times as large as it.

72. The radius of a sphere is diminished by $\frac{1}{6}$ of itself. By what fraction of itself is the volume diminished?

73. Find the volume of a cubical box, if the distance from one of the corners at the base to the extreme opposite corner on the top, is 8 ft.

74. At what distance from the base must a cone, 20 in. high, be cut parallel to the base that the volumes of the two parts may be equal?

75. Four men bought a grindstone 60 in. in diameter. How much of the diameter must be ground off by each man, one grinding his part first, then another, and so on, that each may have an equal share of the stone, allowing 6 in. for the axle?

CHAPTER XXI.

Examination Papers.

BRITISH COLUMBIA.

TEACHERS' EXAMINATION.

ARITHMETIC.—For all Classes and Grades, 1899.

1. Prove (i) the difference between the interest and the discount on a sum of money is the interest on the discount for the same time and rate, and (ii) $(1.05)^3$ is the amount of $1 for 3 yr. @ 5 %, compound interest.

2. A party of men were employed to do a work in 25 da. At the end of 20 da. only $\frac{5}{7}$ of the work was done, and 3 more men had then to be employed to complete the work in due time. How many men were employed at first?

3. What is the rate per cent. when $10 is the true discount allowed off $210 for 4 mo., and what discount ought to be allowed off $210 for 16 mo., at the same rate?

4. A man invests $6000 in 3 % stock @ 75. He sells out @ 80, and invests $\frac{1}{3}$ of the proceeds in $3\frac{1}{2}$ % stock @ 96, and the remainder in 5 % stock at par. Find his income from the latter investments.

5. The par of exchange between Paris and London is 25.2215 francs for £1, and that between St. Petersburg and London is 38.177 pence for 1 rouble. Find the par of exchange between Paris and St. Petersburg.

6. For what sum must property, worth $2940, be insured @ 2 % to cover $\frac{2}{3}$ of its value and the premium paid?

7. An agent sold cotton on a commission of 4 %, and invested the net proceeds in sugar on a commission of $1\frac{1}{2}$ %. His total commission was $220. Find the value of the cotton.

8. A cask contains 2 gal. of wine, and 3 gal. of water. Another cask contains 3 gal. of wine, and 1 gal. of water. How many gallons must be drawn from each cask so as to produce by their mixture 1 gal. of wine and 1 gal. of water?

Mensuration.—For Second Class, Grades A and B, 1899.

1. Prove (i) a rule for finding the area of a triangle, and (ii) the area of a circle is half the product of the circumference multiplied by the radius.

2. A rectangular section of land is half as long again as it is broad, and it contains 240 ac. Find the cost of fencing it @ 17s. 6d. per chain.

3. Find the length of a side of a regular polygon of twelve sides, inscribed in a circle whose radius is 10 ft.

4. The radius of the outer boundary of a ring is 14 in., and its area is 462 sq. in. Find the circumference of the inner boundary.

5. The radius of a circle is 25 in., and the chord of a segment is 14 in. Find the area of the segment.

6. Make a sketch and find the area of the field *ABCD*. *BC* is parallel to *AD*; $AB = BC = CD = 325$ links; and $AD = 733$ links.

7. The parallel sides of a trapezoid are 16 and 24 ft., respectively, and the perpendicular distance between them is 8 ft. Determine the position of the straight line that divides it into two equal trapezoids.

8. The radius of the base of a right circular cone is 10 in., and its height is 20 in. Find the area of the whole surface of the cone.

Mensuration.—For First Class, Grades A and B.

1. Investigate the approximate accuracy of Simpson's Rule.

2. Show that the length of a ring is equal to the sum of the inner boundary and the circumference of the cross section.

3. The content of a rectangular box, whose length is twice its breadth, and whose breadth is twice its depth, is 1 cu. yd. Find its dimensions, and also the cost of gilding its whole outside surface @ 1s. 9d. per square foot.

4. Compare the volume of a sphere which touches the six sides of a cube with that of a sphere which passes through the angular points of the cube.

5. If a sphere, the diameter of which is 4 in., is depressed in a conical glass full of water, the diameter of which is 5 in., and altitude 6 in., how much water will run over?

6. Water passes through a pipe 6 cm. in diameter, at a velocity of 7.6 dm. per second. How many litres flow through in 11 sec.?

7. A cow is tied to the outside of a square enclosure (60 ft. to the side) 20 ft. from a corner, by a rope 100 ft. long. Find how much ground she can procure grass from.

8. A block of marble, in the form of a pyramid, on a square base, weighs 18 t. (long), the perpendicular height being twice the diagonal of the base.. Find the side of the base, a cubic inch of marble weighing 1.6 oz. avoirdupois. .

NORTH-WEST TERRITORIES.

TEACHERS' EXAMINATION.

Arithmetic and Mensuration.—Third Class, July, 1899.

1. Distinguish numeration and notation, and give examples illustrating the systems most frequently used by English-speaking communities.

2. Explain by means of diagrams, or otherwise, the truth or falsity of the following statements :—

(i) $\frac{3}{4} = (\frac{1}{4} \text{ of } 1) \times 3.$ | (ii) $\frac{3}{4} = 3 \div 4.$ | (iii) $\frac{3}{4} = \frac{1}{4}$ of 3.

3. The square of the sum of two numbers is .25 If one of the numbers is seventy-five ten-thousandths, find the product of the numbers. What per cent. of the larger number is the smaller?

4. If $2\frac{1}{3}$ ft. of rosewood, $\frac{1}{2}$ in. in thickness, were used in making a closed box 10 in. long, 8 in. wide, and 6 in. deep (outside measurement), find the value of the wasted material, at the rate of $158.40 per M.

5. Make and solve a problem in interest, having given all the data necessary to find the rate per cent.

6. Explain the meaning of the terms bankrupt, capital, dividend, bank discount, trade discount.

7. The stock remaining in a bankrupt's store, if sold at cost, would realize $6000, and the creditors, after allowing 10 % of the proceeds for expenses, would receive $66\frac{2}{3}$c. on the dollar. When the stock was sold $\frac{2}{3}$ of it brought 20 % less than its value, and the remainder 10 % less. Again allowing 10 % for expenses, find

(i) The merchants' loss on the goods sold.
(ii) The creditors' loss on the dollar.
(iii) The creditors' loss in dollars.
(iv) The expense of winding up the business.

8. Smith and Jones form a partnership and invest in their business capital in the proportion of 2 to 3. It is agreed that at the end of the first year Smith is to receive 40 % of the profits for conducting the business, and that the remaining profits will be divided according to the capital invested by each. On this basis of calculation Smith receives $2560 and Jones $1440. Find the amount invested by each, if the profits were 25 % of the capital.

9. A western farmer forwarded a car of wheat (600 bu.) to a commission merchant in Fort William with instructions to sell and return the proceeds, after deducting freight and other charges. The agent's statement contained the following, among other items: freight charges $76, elevator and other expenses $23.20, commission $10.80. If the farmer received $250, find (i) the rate of commission charged, (ii) the selling price of a bushel of wheat at Fort William, (iii) the selling price of a bushel of wheat at the farmer's market town if he would have lost 20 % by selling it there.

10. On Sept. 5th, 1898, the following note was presented at the Bank of Montreal, Regina, and discounted at 10 %:—

$100. REGINA, May 29th, 1898.

Eight months after date I promise to pay to the order of D. H. Ganton the sum of One Hundred $\frac{00}{100}$ Dollars, with interest at six per cent. Value received.

F. F. CUNNINGHAM.

(i) Who had the right to present the note for discount?

(ii) What were the proceeds of the note?

(iii) If the proceeds were immediately placed in the P. O. Savings Department at 3 % interest, find the loss to the depositor at the time of the maturity of the note.

11. If a cubic foot of water weighs 1000 oz. and lead has a specific gravity of 11.5, find the cost of a lead pipe 14 ft. long and $\frac{1}{2}$ in. thick, the bore being $1\frac{3}{4}$ in. in diameter, and lead pipe being worth 7c. per pound.

12. Two cylinders have the same height, but the radius of the base of one cylinder is six times that of the other. (i) Compare their volumes. (ii) Compare their lateral surfaces. (iii) Compare the height of the smaller cylinder to that of a right cone which contains an equal volume but whose base is twice as wide.

ARITHMETIC AND MENSURATION.—Second Class, July, 1899.

1. Symbolize and solve the following: Divide the square root of ten times the cube root of two hundred and sixty-two, and one hundred and forty-four one-thousandths by the cube root of ten times the square root of forty, and ninety-six one-hundredths.

2. Discuss the truth or falsity of the following statements :—

(i) A vulgar fraction represents one number divided by another.

(ii) A vulgar fraction represents the quotient of the numerator divided by the denominator.

(iii) $.\dot{4}$ is a decimal fraction.

(iv) $\frac{1}{4} \div 4$ represents an impossible operation, and is never required in the solution of problems.

3. A quantity of milk, which cost 48c. per gallon, is watered until it is reduced in value to 5c. per pint. After $\frac{2}{3}$ of it is sold, it is still further watered, until its value is reduced to 4c. per pint.

(i) Find the proportion of water to milk in the last mixture.

(ii) If the retail price was 14c. per quart, find the gain per cent. realized.

4. A rancher shipped a car load of cattle to the Kootenay at a cost for freight and feed of $38.50. His personal expenses amounted to $17.50, and the consignment was insured for $\frac{5}{6}$ of its value @ $1\frac{3}{5}$ %. The cattle were sold at a loss of $12\frac{1}{2}$ %, the owner receiving $19 less than the original cost of the cattle. Find the cost of the cattle, and the premium paid for insurance.

5. A merchant purchased goods on July 10th, amounting, as per catalogue prices, to $1400, subject to trade discounts of 20 % and 5 %, if paid at three months, and a further 5 % for cash.

(i) Why do wholesale firms quote their prices in the above form ?

(ii) To what rate of interest is the 5 % off for cash equal ?

(iii) If the merchant were to discount a note for three months @ 6 % for the credit amount of the above account, by how much would the proceeds be greater or less than the cash price ?

6. Make and solve a problem in commission in which an agent is instructed to sell and buy, and the value of the goods purchased is to be calculated.

7. A merchant buys 150 lb. of coffee @ 24c. per pound, and $13\frac{1}{3}$ % as much chicory @ 8c. per pound, paying a specific duty of 6c. per pound on the coffee, and an *ad valorem* duty of $12\frac{1}{2}$ % on

the chicory. He mixes all the chicory with coffee in the proportion of $\frac{1}{3}$ to 1, and sells the mixture at a loss of $27\frac{3}{11}\%$. Find

(i) The price at which the mixture was sold.

(ii) The amount paid in duties.

(iii) The selling price per pound of the coffee which remains, so that the merchant may not lose anything on the transaction.

8. The rate of taxation in a municipality last year was $12\frac{1}{2}$ mills, and a rebate of 10% was allowed on all taxes paid before a certain date. A resident who owns two lots, one of which is assessed 20% higher than the other, pays his taxes in time to take advantage of the rebate. This year the assessed value of the more expensive lot has been increased 25%, and the rate of taxation has been raised 20%. If the owner of the above lots pays this year's taxes, so as to receive the rebate on the more valuable lot only, his taxes will amount to $10.50 more than they did last year. Find the value of the lots.

9. A rectangular block of lead 6 in. thick, is hammered down until its length is increased 20%, and its width $12\frac{1}{2}\%$. Find the thickness of the block in its new form.

10. A reservoir, whose sides slope uniformly, is 60 ft. square on top, and 40 ft. square at the bottom. If it is 15 ft. deep, and the earth taken from it is spread uniformly over a field containing 10 sq. ch., find the depth of this earth if it occupies $1\frac{1}{2}$ times as much space as packed earth.

11. Take a rectangular sheet of paper 8 in. by 6 in., and find

(i) The volume of the cylinder that can be formed.

(ii) The area of the largest isosceles triangle that can be formed.

(iii) The surface area of the largest right-cone that can be formed from the largest right-angled triangle contained in the sheet of paper.

NOTE.—Illustrate your explanations with drawings.

DEPARTMENT OF EDUCATION, MANITOBA.

COMMERCIAL DIPLOMA.

ARITHMETIC.—July, 1899. Part I.

Time—Fifteen minutes.

NOTE.—Candidates will place results on this paper. At the end of the 15-minutes, examiners will collect the envelopes and distribute the regular paper. Both papers will be placed in the same envelope.

1. Find the totals as indicated.

$6294.40	$3986.60	$8299.75	$......
349.98	679.87	1237.84	......
759.57	1287.93	7789.25	......
2988.40	9642.44	885.97	......
32.66	944.32	337.89	......
753.72	842.89	1027.63	......
29.30	1978.64	12.89	......
9876.35	8927.59	8.68	......
762.39	923.77	17.56	......
8349.66	79.28	8.99	......
468.95	929.75	9429.36	....
700.67	20.88	642.61	.....
99.75	190.85	748.26	......
421.13	68.79	45.62	......
346.28	898.23	842.76	......
99.29	298.36	399.87	......
645.75	942.88	87.37	......
75.88	293.84	699.32	......
1022.39	9764.29	25.87	......
769.73	937.41	736.75	......
$.......	$.......	$.......	$.....

Part II.

1. Find the amount of the following bill :—

205 bu. 15 lb. wheat @ 52c.
18 bu. 17 lb. oats @ 45c.
10 bu. 6 lb. barley @ 48c.
56 bu. 10 lb. beans @ 90c.
4 bu. 7 lb. Indian corn @ 96c.
320 lb. 15 lb. potatoes @ 64c.
25 bu. 20 lb. carrots @ 63c.

2. At $18.50 per thousand find the cost of

25 boards, 12 ft. long, 16 in. wide, 1 in. thick.
10 boards, 16 ft. long, 12 in. wide, $\frac{3}{4}$ in. thick.
20 planks, 18 ft. long, 14 in. wide. 2 in. thick.
75 scantlings, 12 ft. long, 6 in. wide, 5 in. thick.

3. Three men, *A*, *B* and *C*, worked together 10 da., and their wages amounted to $85. *B* received 25c. per day more than *A*, and *C* received as much as *A* and *B* together. Find the daily wages of each.

4. A merchant bought a quantity of apples @ \$1.75 per barrel. At what price per barrel must he sell, so as to make a profit of 25 % on the transaction, supposing that there is a waste of 8 %?

5. Which is the better investment, 5 % bonds @ 95, or 6 % bonds @ $102\frac{1}{2}$, if the brokerage on the first is $1\frac{1}{2}$ %, and on the second $1\frac{2}{3}$ %? Explain fully.

6. What is the value of a 90-day draft for \$1200, exchange being $\frac{1}{8}$ % premium, and the current rate of interest being 6 %?

7. If 4 men in 5 da. of 8 hr. each, can dig a ditch 120 yd. long, 3 ft. wide, and 4 ft. deep, in how many days of 10 hr. each can 12 men dig a ditch 300 yd. long, 4 ft. wide, and $4\frac{1}{2}$ ft. deep?

8. *A*, *B* and *C* were partners in business. *A*'s capital was $\frac{2}{3}$ of *B*'s, and *B*'s was $\frac{4}{7}$ of *C*'s. *A*'s capital was in 8 mo.; *B*'s, 9 mo.; *C*'s, 10 mo. The net gain amounted to \$2674. What was the share of each?

Examination of Teachers.—Third Class, 1899.

1. (i) How many thirds in $\frac{1}{2}$? How often is .56 contained in 2.8?
(ii) If you know the dividend, the quotient and the remainder, how will you find the divisor? Illustrate.

2. If I buy sugar @ 15 lb. for a dollar, and sell @ 7c. a pound, find my gain or loss per cent.

3. *A*'s farm is $\frac{1}{7}$ less than *B*'s in size, but $\frac{1}{20}$ better in quality. How do the farms compare in value?

4. If 6 men can do a work in $5\frac{1}{2}$ da., how much time would be saved by adding 4 men more?

5. *A*, *B* and *C* have equal amounts of money. *A* increases his by $\frac{1}{6}$; *B* increases his by $12\frac{1}{2}$ %, and *C*'s loss is equal to the average gain of the other two. Find the gain or loss per cent. of the three together.

6. If money is worth 8 %, find the cash value of an article that is sold on a half-year's credit for \$20.80.

7. A rectangular field contains 40 ac. It is half a mile long. Find its width. Find the distance between opposite corners. How far will a farmer walk in plowing it if the furrow is 8 in. wide? If the farmer sows $1\frac{1}{2}$ bu. to the acre, and the value of the grain is 64c. a bushel, find the value of the grain used in seeding. If the total cost of seeding and cultivating is \$230, and the cost of harvesting, etc., is 60 % more than this, at what price per bushel must the grain be sold to give a profit of \$360, the yield being 25 bu. to the acre?

8. Solve the following problems in simple interest :—

	Principal.	Amount.	Rate.	Time.	Interest.
(i)	\$600	\$630	4 %	?	?
(ii)	\$500	?	?	4 yr.	\$30.00
(iii)	?	?	2 %	6 mo.	\$2.50

9. Solve the following problems in commission :—

	Goods sold.	Rate.	Commission.	Net proceeds.
(i)	\$4500	2 %	?	?
(ii)	?	?	\$105	\$1895

Examination of Teachers.—Second Class, 1899.

1. Find a multiplier that will reduce pence per pound, Troy (5760 gr.), to francs per pound Avoirdupois (7000 gr.), if 3 pence = 5c. and 3 francs = 58c.

2. Find the length of the edge of a cube whose surface is equal to that of a circle with diameter 30 ft. (The area of a circle is obtained by multiplying the square of the radius by 3.1416.)

3. After 4 % of a flock of sheep had been killed by dogs, and 68 had been sold to a butcher, $\frac{4}{7}$ of the original number were left. How many sheep were in the original flock ?

4. *A* owes *B* \$1500, due in 1 yr. 10 mo. He pays him \$300 cash, and gives his note @ 6 mo. for the balance. What is the face of the note, money being worth 6 % per annum ?

5. I sold an article @ 20 % gain. Had it cost \$300 more, I should have lost 20 % by selling at the same price. Find the cost.

6. The stocks of three partners, *A*, *B*, and *C*, are \$350, \$220, and \$250, respectively, and their gains are \$112, \$88, and \$120, respectively. Find the time the stock of each was in trade, *B*'s time being 2 mo. longer than *A*'s time.

7. What proportions of vinegar, $\frac{3}{4}$ pure and $\frac{2}{5}$ pure, should be taken to give a mixture of $\frac{1}{2}$ pure ?

8. An agent sold for his principal goods to the amount of \$9000, upon which he charged a commission of $2\frac{1}{2}$ %. He invested the remainder, less a commission of 2 %, in dry goods. Find the amount invested in dry goods and his total commission.

9. The French gram is the weight of a cubic centimetre of water; a kilogram is 1000 grams. Find the weight in kilograms of a body weighing 6500 lb., given that a cubic foot of water weighs 1000 oz., and the centimetre is .3937 of an inch.

10. The stocks of two companions are @ 106 and 101. The former pays 4 % and the latter $3\frac{1}{2}$ %, half-yearly. Which is the better investment, and by how much per cent. ?

EDUCATION DEPARTMENT, ONTARIO.

ANNUAL EXAMINATIONS, 1900.

ARITHMETIC AND MENSURATION.

Part I.—Junior Leaving or Public School Leaving.

1. If a bushel of wheat costs 80c., how many bushels can be bought for $10.40 ?

Work this so as to explain clearly how you can get an answer in bushels when the quantities given are expressed in dollars and cents.

2. (i) What number will represent the quantity, 3 gal. 2 qt., when the unit of measurement is 2 pints ?

(ii) Find a cube such that the volume of 729 such cubes will equal that of a wall 1 ft. 6 in. thick, 4 ft. 6 in. high, and 13 ft. 6 in. long.

3. (i) If a square metre contains 1550 sq. in., find the length of a metre in inches to three decimal places.

(ii) If 1 litre equal 1.760775 pints, find the number of litres in a gallon correct to five decimal places.

4. The average attendance at a school for any period is obtained by dividing the aggregate attendance by the number of teaching days for that period. The average attendance for the first half-year is 385, the aggregate for this period being 46200; the aggregate and average for the second half-year are, respectively, 33200 and 415. Find the average attendance for the whole year.

5. A person's salary is four-fifths of his total income, the other one-fifth being interest on money which he has lent at 7 %. It costs him 60 % of his total income to live, and the rest ($700) is placed in the savings bank. If the rate of interest falls from 7 % to 5 %, and his salary and living expenses remain the same, how much less each year can he put in the bank ?

6. A note for $1215, payable December 16th, is discounted at a bank on the 12th of September @ 6 %. What are the proceeds ?

7. The wheel of a bicycle turns 12 times for 5 revolutions of the crank. Find the speed in miles per hour when the crank revolves once a second, the diameter of the wheel being 28 in.

8. The length of the side of an equilateral triangle is 2000 ft. Find the area of the triangle in acres.

ARITHMETIC.—Part I.—Junior Matriculation.

1. Simplify $\{(.05)^3 - (.02)^3\} \div \{(.05)^2 + (.05)(.02) + (.02)^2\}$.

2. Reduce to its simplest form·
$$(3\tfrac{4}{5} + 5\tfrac{1}{9} - \tfrac{1}{45})(4\tfrac{1}{5} - 3\tfrac{1}{4}) \div \{1\tfrac{5}{11} + 2\tfrac{1}{8} - (2\tfrac{9}{16} - \tfrac{1}{8} - \tfrac{1}{22})\}.$$

3. A man owns $.\dot{1}\dot{8}$ of a certain mine. He sells $.2\dot{5}\dot{2}$ of his share for $5000. What is the value of the mine?

4. A person sold to *A* $\frac{4}{5}$ of his land; to *B* $\frac{5}{6}$ of the remainder; to *C* $\frac{6}{7}$ of what then remained, and received $55 for what he had left @ $75 an acre. Find the number of acres he had at first.

5. A merchant buys goods amounting, per catalogue price, to $540.80, subject to 25 % and 10 % off, and he sells at catalogue prices. Find the rate per cent. profit.

6. What per cent. is realized on money by investing in a $3\frac{1}{2}$ % stock @ $140, dividends payable yearly?

7. A young man deposits $100 in a savings bank at the beginning of each year, making his first deposit on January 1st, 1895. How much will there be to his credit on December 31st, 1899, the bank paying 3 % per annum?

8. $500. TORONTO, January 15th, 1899.

Ninety days after date I promise to pay Henry Graham, or order, the sum of Five Hundred Dollars, with interest at six per cent. per annum. Value received.

JOHN RYAN.

This note was discounted on Feb. 20th, 1899, @ 8 %. Find its cash value on that date.

9. I buy a house, agreeing to pay $500 at the end of each year for 2 yr. What would be the cash value equivalent to these two payments, money being worth 6 % per annum?

10. I invested $9536 in Bank Stock @ $74\frac{3}{8}$, brokerage $\frac{1}{8}$, and sold @ 80, brokerage $\frac{1}{8}$. What did I gain?

ARITHMETIC AND MENSURATION.—Part II.—Junior Leaving.

1. (i) Prove that $.8\dot{6} = \frac{13}{15}$.

(ii) Prove that the interest on any sum of money is equal to the true discount on the same sum, together with the interest on the true discount, time and rate being the same throughout.

2. A man buys a house, agreeing to pay $200 at the end of each year for a period of three years. Find its cash value, money being worth 5 % per annum.

3. Find the surface and the volume of a right circular cone, given the diameter of the base 42 ft., and the slant height 26 ft. 3 in. ($\pi = \frac{22}{7}$.)

4. A man invested a certain sum of money in a 6 % stock @ $115\frac{7}{8}$, brokerage $\frac{1}{8}$, and one-and-a-half times as much in a 5 % stock @ $89\frac{7}{8}$, brokerage $\frac{1}{8}$. His income from the two investments being $940, how much did he invest in each kind of stock ?

5. A Toronto merchant owes 18000 francs in Paris. He buys a draft on London when Sterling Exchange is @ 8, and when exchange between London and Paris is 25.20 francs per £. What does he pay for the draft ?

6. A hollow iron sphere, 2 in. thick, has an internal capacity of 22 cu. ft. 792 cu. in. Find the weight of iron in it, if that metal is 7.7 times as heavy as an equal volume of water. (C. ft. water weighs 1000 oz.)

7. *A*, *B*, and *C* form a trading company for a year, investing, respectively, $3000, $2000, $1000. *A* is appointed manager @ $60 a month, and *B* secretary @ $30 a month, salaries to be reduced in proportion as the capital invested is reduced. At the end of 6 mo. *A* withdraws from the business, and *C* is appointed manager, and 2 mo. later *B* withdraws, and *C* becomes secretary. How should the year's gain of $1150 be divided ?

8. A town borrows $20000, to be repaid with interest, in 20 equal annual instalments. Find the amount of each payment, money being worth 5 % per annum, given $(1.05)^{20} = 2.6533$.

9. A merchant in London, England, consigns to his agent in Montreal, 5000 yd. of cloth, invoiced @ 6s. a yard, and instructs him to sell the cloth and invest the proceeds in apples, after deducting the following charges: commissions 5 % each, freight $325, and an *ad valorem* duty of 25 %. The cloth is sold @ $2 a yard, and apples are bought @ $2 a barrel. Find the merchant's gain, if the apples net him in London 10s. a barrel, Sterling Exchange at par.

QUEBEC.

ASSOCIATE IN ARTS EXAMINATION.

OPTIONAL ARITHMETIC.—June, 1899.

1. Express $.03\dot{7}\dot{5}$ as a vulgar fraction, proving your method.

2. The interest on a certain sum for the first year is $122, the compound interest for the second year is $128.10. Find that for the third year.

3. What will it cost to cover with lead a tank whose depth is 3 ft. 6 in., whose length is 7 ft. 10 in., and whose breadth is 5 ft. 4 in., the lead costing 10s. $1\frac{1}{2}$d. per square yard?

4. What must be the price of 5 % stock in order to gain 8 % on money invested?

5. The diameter of a 50c. piece being $1\frac{3}{16}$ in., and the average thickness $\frac{1}{16}$ in., and 1000 such coins being melted down and formed into a cube, find an edge of the cube.

6. The height of a cone is 12 in., the diameter of the base 10 in. Calculate the surface and volume.

7. Express log. 24 in terms of log. 2 and log. 3. With the aid of logarithms multiply 2616 by .0325, and find the eighth root of 196.83.

NEW BRUNSWICK.

ENTRANCE AND PRELIMINARY.

Arithmetic.—Class III.—1899.

1. Find the length of the shortest line that can be exactly measured by each of the following measures, viz., a yard measure, a ten-foot pole, and a two-rod chain.

2. How long will it take \$450 @ 8 % per annum, to yield \$21.30 interest?

3. If an agent received \$1092 to buy pork, how many pounds @ $6\frac{1}{2}$c. per pound can he buy, and retain his commission of 5 % for buying?

4. In the number 28672, the value expressed by the first two digits on the left is how many times the value expressed by the fourth digit, counting from the left?

5. Three men can dig a certain drain in 8 da. They work at it 5 da., when one of them falls ill, and the other two finish the work in five days more. How much of the work did the first man do before he fell ill?

6. Find the altitude in yards of a triangle whose area is 37 ac., 30 rd., $12\frac{1}{2}$ yd., and whose base is 900 yd.

7. 10 yd. of muslin $1\frac{1}{4}$ yd. wide, cost \$1.30. What is the cost of 12 yd., $1\frac{1}{8}$ yd. wide?

Class II.

Note.—Any two of questions 3, 5 and 7, and all the others, make a full paper.

1. Show clearly that you understand each principle involved in finding the sum of two fractions. (Use $\frac{5}{6}$ and $\frac{3}{4}$.)

2. What per cent. must be assessed on $15000 to produce $294, after paying 2 % for collecting?

3. At what time between 4 and 5 o'clock is the minute-hand exactly two minute spaces ahead of the hour-hand of a watch marking correct time?

4. What is the compound interest of $2400 for $1\frac{1}{2}$ yr. @ 10 % per annum, paid half-yearly, and at what rate, simple interest, would it amount to the same sum in the same time?

5. A room, whose height is 12 ft., and length $1\frac{1}{6}$ times its width, takes $178\frac{2}{7}$ yd. of paper 1 ft. 9 in. wide to cover its walls. What will it cost to cover the floor with carpet 27 in. wide, costing $1.75 per yard?

6. Express the square root of (.0864 × 753) ÷ .00391, correctly to the nearest integer.

7. 3 women can do as much as 5 children, and a man twice as much as a woman. If the total wages of 6 men, 10 women, and 15 children be £20 13s. 4d., how ought the wages to be divided?

Class I.

1. From the list price of a line of goods a purchaser is allowed a trade discount of 20 %; a further discount of 10 % off the trade price for taking a quantity, and a still further discount of 5 % off his bill for cash. Find his gain per cent. by selling @ 10 % less than the list price.

2. Two pipes, *A* and *B*, would fill a cistern in 25 min. and 30 min., respectively. Both are opened together, but at the end of $8\frac{2}{5}$ min. the second is turned off. In how many minutes will the cistern be filled?

3. I own $6000 of bank stock, paying an annual dividend of 5 %. How much will my annual revenue from the bank stock be reduced by selling enough of it @ 72 to pay a debt of $3735?

4. The external dimensions of a rectangular covered box, made of inch stuff, are 7, 8 and 9 ft. Find (i) the capacity of the box, (ii) the quantity of lumber in it, and (iii) the side of a cube equal to the interior of the box. (Note: Answer to (iii) to be true to two places of decimals.)

5. What must be the face of a note for 3 mo., made on the 18th day of August, so that discounted (bank discount) @ $7\frac{1}{2}$ %

per annum on the day of making at the bank, the proceeds may be \$14315 ?

6. A cylinder, whose radius is 14 in., is 12 ft. long. Find the number of cubic feet in it; also the side of a square equal in area to the curved surface of the cylinder.

NOVA SCOTIA.

COUNTY ACADEMY ENTRANCE.

ARITHMETIC.—July, 1899.

NOTE.—Each numbered question of equal value. Answers without the work necessary to find them will be assumed to be merely guesses, and therefore of no value.

1. Divide 93456780123 by 89657. (Answer of no value if not *exactly* correct).

2. Bought $3\frac{1}{2}$ yd. of doeskin @ \$1.40, and $3\frac{3}{8}$ yd. of pilot cloth @ \$2.40. How many yards of flannel @ $8\frac{2}{5}$c. a yard will pay the bill ?

3. What will 37 sq. yd. 4 sq. ft. 120 sq. in. of plastering cost, if 9 sq. yd. 2 sq. ft. cost \$3.50 ?

4. A web of Tweed was marked @ \$1.10 per yard, which was 25 % above cost. What was the profit on the sale of 9 yd. @ \$1.05 per yard ?

5. If \$390 is assessed upon a school section, the assessable property of which is placed at \$78000, what should be the proportion paid by a man whose property is valued at \$6500, and another whose property is valued at only \$975 ?

6. A note for \$200 payable 90 da. after its date, the 25th day of May, was discounted at a bank on the 22nd of June. Find the net proceeds.

7. Find the square root of 2 to four decimal places. What useful application can be made of the extraction of the square root ?

8. (i) The area of a circle is expressed by a formula PR^2. If $P = 3.1416$, and $R =$ the radius, what is the area of a circle 12 ft. in diameter ?

(ii) If $a = 10$, $b = 8$, $x = 12$, $y = 4$, find the value of

$$-a + b\sqrt{(x+y)} - (a-b)\sqrt[3]{(x+y)}.$$

9. (i) If $W = 5a + 4b - 6c$, $X = -3a - 9b + 7c$, $Y = 20a + 7b - 5c$ and $Z = 13a - 5b + 9c$, evaluate $W - X - Y + Z$.

(ii) Divide $1 + 6x^5 + 5x^6$ by $1 + 2x + x^2$.

10. Discuss the *metric* system, or say anything you choose about it, to show your knowledge of it.

Grade D.—July, 1900.

1. Simplify $6.0\dot{3}\dot{8} + .0\dot{8}7\dot{5} - 4.003\dot{7}6\dot{3}$, giving the answer *exactly* correct in its simplest decimal form.

2. Multiply £5 19s. $11\frac{1}{4}$d. by $14\frac{3}{11}$.

3. What must be the depth of a cubical tank in inches in order to contain as much as a tank 1368 in. long, 912 broad, and 76 deep?

4. What is the difference in time between Cape of Good Hope, in longitude 18° 29′ E., and Quebec, in longitude 71° 13′ 45″ W.?

5. The temperature of a room is 20° C. What should a Fahrenheit thermometer indicate?

6. Sent a commission merchant $2472 to be invested in flour, his own commission being 3 %. How many barrels of flour @ $4.80 per barrel can he purchase for me?

7. I am offered a house for $2838, to be paid at the end of 18 mo. If the rate of interest is 5 %, what is the offer equivalent to in cash?

8. What is gained by investing $10000 in British consols @ $103\frac{1}{2}$, and selling immediately at 105, brokerage in each transaction $\frac{3}{4}$ %?

9. A commission merchant sold a consignment of fruit in Halifax for $960, and after deducting his commission of 4 %, he purchased a sight draft on New York @ $\frac{1}{2}$ % premium. What was the face of the draft?

Grade C.

1. Reduce the difference between £$.42708\dot{3}$ and $.23\dot{4}\dot{5}$ of £6 17s. 6d. to the decimal of £5.

2. How much must I pay for Bank Stock which pays a dividend of 11 % per annum, so that I may make 5 % on my investment?

3. What sum will amount to $500 in 4 yr. @ 6 %, (i) simple interest, and (ii) compound interest?

4. A man bought on Sept. 14, 1897, $400 worth of goods at 6 mo. credit. On Nov. 25, he paid $115, and on Dec. 10, $96. When, in equity, should he pay the balance?

5. If 3 men can do $\frac{5}{12}$ of a piece of work in 5 da. of 12 hr. each, how many men will it take to do $\frac{4}{9}$ of the work in 6 da. of 8 hr. each ?

6. What are the (i) amount, and (ii) present worth of an annuity of $400 for 3 yr. @ 5 %, compound interest ?

7. What must be the face of a note, drawn May 4, @ 3 mo., so that if discounted immediately @ 6 %, the proceeds will be $300 ?

8. A bill of £90.10 is due in Edinburgh, Scotland, for school apparatus imported. What must be paid for a bill of exchange to liquidate this debt, when Sterling Exchange is quoted @ $109\frac{3}{4}$?

9. A circular path, 10 ft. wide, is laid at a distance of 40 ft. from the centre of a statue. What is the cost of its paving @ $1.20 per square yard ?

PRINCE EDWARD ISLAND.

PRINCE OF WALES COLLEGE AND NORMAL SCHOOL ENTRANCE EXAMINATION.

ARITHMETIC.—July, 1897.

1. In a school district a tax of $500 is to be raised. If the amount of taxable property is $30000, what will be the tax on the dollar, and what is *A*'s tax, whose property is valued @ $1500 ?

2. A merchant buys goods; the cost of freight is 8 %, and that of insurance 12 % on the original outlay. He is obliged to sell them at a loss of 7 %; but if he had received £20 8s. more for them he would have gained $1\frac{1}{2}$ %. Find the original outlay.

3. The true discount on a note of $945, drawn at 9 mo., is $70. Find the rate per cent.

4. A bankrupt has book debts equal in amount to his liabilities; but on $24000 of them he realizes only $66\frac{2}{3}$c. in the dollar, and the expenses of the bankruptcy are 5 % on the book debts. He pays 65c. on the dollar. Find the amount of his liabilities.

5. Find the cost of papering a room 29 ft. 6 in. wide, 36 ft. 6 in. long, and 13 ft. 6 in. high, with paper $2\frac{3}{4}$ ft. wide, and costing $2.20 per piece of 12 yd. long, the parts not requiring paper, making up $\frac{1}{5}$ of the whole.

6. 5 lb. of coffee and 4 lb. of tea cost $4.60. There is an advance of 25 % on the price of the coffee and a decline of $13\frac{1}{3}$ % on that of tea, and 5 lb. of the former and 4 lb. of the latter still cost $4.60. Find the price of each at first.

7. A man bought a store and contents for $4720. He sold the same for 12½ % less than he gave, and then lost 15 % of the selling price in bad debts. Find his entire loss.

8. A man can row a boat from *A* to *B* (a distance of 30 mi.) in 7½ hr. if there be no current. How long would it take him to row it if there was a current of 1½ mi. per hour from *B* to *A*?

Class II.—December, 1897.

1. (i) What is a decimal fraction? (ii) Upon what does the value of a decimal depend? (iii) What are circulating decimals? (iv) What is the difference between decimal fractions and vulgar fractions?

2. The cost of carpeting a room 20 ft. long, 18 ft. wide, with Brussels ¾ yd. wide @ $1.50 a yard, the strips running lengthwise, is $83. How much is the waste on each strip in matching figures?

3. I sold a horse for a certain sum and gained 20 %. I purchased another with the money and sold it at a loss of 20 %. My whole loss was $4.80. Find the cost of the first horse.

4. A man's income is derived from the proceeds of $4550 at a certain rate per cent., and $5420 at one per cent. more than the former rate. His whole income being $453, find the rates.

5. A commission merchant sold flour @ 5 % commission, and invested the proceeds in sugar @ 5 % commission. The commission on the flour was $69.50 more than the commission on the sugar. What was the value of each?

6. What must be the face of a note, so that when discounted at a bank for 4 mo. and 9 da. @ 9 %, it will give $240?

7. What sum must be insured on a house worth $665, so that in case of loss the owner may receive 4/7 of this sum, and also ⅚ of the premium, which was @ 6 %?

8. What is gained on 240 shares of stock bought so that by selling @ 16 % discount, 16⅔ % is gained on the investment?

ANSWERS.

Examples i.—Page 12.

1. 43. 2. 389. 3. 4962. 4. 28645. 5. 46583. 6. 7711.
7. Two hundred and seventy-three.
8. Three thousand six hundred and forty.

Examples ii.—Page 13.

1. 123030 quinary. 2. 13056 octenary. 3. 133053 senary.
4. 31450 septenary. 5. 3530 senary.

Examples iii.—Page 14.

1. 3131 quaternary. 2. 34504 nonary. 3. 110 binary.
4. 1353 senary. 5. 643250 septenary. 6. 272508 nonary.
7. $267\frac{1}{5}$ octenary. 8. 51266 nonary; 33756 denary.

Examples iv.—Page 15.

1. 1112210221 ternary. 2. 2163 septenary. 3. 1266 denary.
4. 1034 nonary. 5. 103659 denary. 6. 1209 denary.
7. 19904 denary.

Examples v.—Page 17.

1. $\frac{15}{40}$ duodenary. 2. $\frac{5}{14}$ denary. 3. $\frac{3}{101}$ quaternary.
4. $\frac{54}{110}$ septenary.
5. (i) .2; (ii) .4; (iii) $.\dot{6}$; (iv) .8; (v) $.\dot{5}\dot{2}$.
6. (i) $.\dot{1}21\dot{0}$; (ii) $.\dot{3}$; (iii) .6; (iv) $.\dot{7}24\dot{9}$; (v) $.\dot{4}63\dot{1}$.
7. (i) $.2\dot{1}$; (ii) .5; (iii) $.8\dot{3}$; (iv) t; (v) $.6\dot{5}\dot{2}$.
8. (i) $\dot{2}\dot{1}$; (ii) .513; (iii) .875; (iv) $.t6$; (v) .7.
9. .13 senary. 10. .213 senary. 11. .043 senary.
12. $.0003\dot{1}235\dot{0}$ senary. 13. .625. 14. .546875.
15. $.911458\dot{3}$.

Examination Papers.—Page 17.

I.

3. 2400 denary. 4. 1111101 binary. 5. $111450\frac{2}{4}$ senary.

II.

2. 61560. 3. 203 duodenary. 4. 3797. 5. 3201 octenary.

III.

1. 5, 14, 23, 32, 41, 50. 2. 776001. 3. $\frac{1}{3}$ terminates in scales of 3, 6, 9, and 12; recurs in the other scales. $\frac{5}{6}$ terminates in scales of 6, and 12; recurs in the other scales. $\frac{3}{4}$ terminates in the scales of 2, 4, 6, 8, 10, 12; recurs in the other scales. $\frac{7}{36}$ terminates in scales of 6 and 12; recurs in the other scales.
4. 2053.353515625. 5. 242*te*54.

IV.

1. 7*t* 20 undenary. 2. *t*7405 duodenary. 3. 178 nonary; *t*7774.
5. 3102 quaternary; 210.

V.

1. Quinary. 2. Nonary. 3. G. C. M. 32 septenary; L. C. M. 2230 septenary. 4. 1440 min.; 2640 octenary; 109*t* undenary. 5. 1440 times.

VI.

1. $\frac{31}{6t}$; $63\frac{42}{59}$. 2. 1, 4, 8, 32; 128, 8. 3. 729, 243, 81, 27, 1.
4. 14217 octenary. 5. 167.6.

Examples vi.—Page 20.

1. 83706951. 2. 94114466. 3. 77480828.
4. 785102497. 5. 852250164. 6. 767700901.

Examples vii.—Page 21.

1. 231136774864. 2. 577243911204. 3. 155981257688.
4. 178193323164. 5. 293664165.

Exmples viii.—Page 22.

1. 771029178122. 2. 44204861121436. 3. 556223507840.
4. 57413580099499. 5. 34628453681550. 6. 40100374881714.

Examples ix.—Page 23.

1. $109017\frac{7372}{8976}$. 2. $50047\frac{3701}{4785}$. 3. $12377\frac{21074}{54809}$.
4. $41733\frac{12701}{19085}$. 5. Quot. 735, Rem. 484460055.

Examples x.—Page 24.

1. 896. 2. 574. 3. 63092. 4. 162. 5. 626.

EXAMPLES XI.—PAGE 27.

1. 1227184. 2. 13832604. 3. 7612605.
4. 8764583454. 5. 52522716. 6. 9486681237.
7. 51661885186625. 8. 38796672242944. 9. 6889.
10. 992016. 11. 990021. 12. 99900016.
13. 9937018. 14. 9971303300.

EXAMPLES XII.—PAGE 27.

1. 394840; 1974200; 9871000. 2. 482735; 2413685; 12068425.
3. 69739; $13947\frac{4}{5}$; $2789\frac{141}{25}$.
4. $86385\frac{1}{3}$; $37022\frac{2}{7}$; $28795\frac{1}{9}$; $23559\frac{7}{11}$.

EXAMPLES XIII.—PAGE 29.

1. $87388\frac{5}{9}$; $7944\frac{41}{99}$; $787\frac{284}{999}$. 2. 157395; 31479; $6295\frac{4}{5}$.
3. 69739; $13947\frac{80}{100}$; $2789\frac{560}{1000}$.
4. $4649\frac{80}{300}$; $1992\frac{380}{700}$; $1549\frac{680}{900}$; $23559\frac{700}{1100}$.

EXAMINATION PAPERS.—PAGE 30.

I.

2. 4 × 907. 3. 9900; Rem., 100. 4. 27 hr.
5. 5504940730.

II.

1. 619161890. 2. 8670344882024. 3. 7283.
4. $14541. 5. 7684 and 978.

III.

1. $269. 2. 72 da. 3. 252.
4. 296237. 5. $13300, $11900, $10500.

IV.

1. 7000. 2. 450 lb. 3. 31239. 4. $232. 5. $30.

V.

2. 9999000025. 3. $37569. 4. 11796 steps.
5. $21000; $5400.

VI.

2. 170680900742874252. 3. 2796203. 4. 786543. 5. 31116.

VII.

1. 39. 2. 689000. 3. 123; 7317. 4. 496; 1087.
5. 9801, 998001, 99980001, 9999800001, 9604, 996004, 99960004, 999960004, 9409, 994009, 99940009, 9999400009.

VIII.

1. 623309. 2. 37632. 3. 24725. 4. 377. 5. 567761.

EXAMPLES XV.—PAGE 35.

1. 23. 2. 37. 3. 22. 4. 365. 5. 10. 6. 37.

EXAMPLES XVI.—PAGE 36.

1. 1800. 2. 25200. 3. 34320.
4. 27324. 5. 36036. 6. 2340.

EXAMINATION PAPERS.—PAGE 36.

I.

1. 3327. 2. 35 times. 3. 44496 rails. 4. 7.
5. 84, 36 and 132.

II.

2. Bags of 1, 2, or 3 bu. each; bins of 300, 200, or 150 bu.
3. $1650. 4. 60 min. 5. 982832.

III.

1. 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 25, 27, 30, 36, 45, 50, 54, 60, 75, 81, 90, 100, 108, 135, 150, 162, 180, 225, 270, 300, 324, 405, 450, 540, 675, 810, 900, 1350, 1620, 2026, 2700, 4050, 8100.
2. 29.
3. 3391 and 2699 are prime; 14787 and 1477 are composite.
4. 60 hr.; *A*, 300 mi.; *B*, 240 mi.; *C*, 180 mi. 5. 40 gr.

IV.

2. 203. 3. $9\frac{3}{4}$ mi. 4. 70560. 5. 24 firkins.

V.

2. 60. 3. 3 and 6. 4. 44 times; 9284 trees. 5. 3366000.

VI.

1. 446, 475. 2. 46, 47, 48, 49, 50. 4. 12 lb. 5. 1474 marbles.

EXAMPLES XVII.—PAGE 41.

1. $\frac{67}{9}$. 2. $\frac{1096}{47}$. 3. $\frac{6277}{29}$. 4. $\frac{173019}{1000}$.
5. $\frac{394}{9}$. 6. $\frac{1043}{19}$. 7. $\frac{1358}{15}$. 8. $\frac{64012}{71}$.
9. $\frac{6589}{13}$. 10. $\frac{70706}{101}$. 11. $\frac{498735}{707}$. 12. $\frac{249088}{409}$.

EXAMPLES XVIII.—PAGE 41.

1. $42\frac{7}{10}$. 2. $3\frac{477}{1000}$. 3. $31\frac{46}{137}$. 4. $928\frac{55}{71}$.
5. $26\frac{2}{23}$. 6. $35\frac{13}{25}$. 7. $13\frac{51}{73}$. 8. $3\frac{961}{1941}$.
9. $9\frac{538}{607}$. 10. $92\frac{58}{83}$. 11. $34\frac{798}{2237}$. 12. $17\frac{292}{709}$.

Examples xix.—Page 43.

1. $\frac{3}{10}$. 2. $\frac{9}{35}$. 3. $\frac{1}{5}$. 4. $\frac{1}{3}$. 5. $\frac{3}{4}$. 6. $\frac{7}{8}$.
7. $\frac{13}{15}$. 8. $\frac{29}{517}$. 9. $\frac{9}{13}$. 10. $\frac{211}{347}$. 11. $\frac{14}{23}$. 12. $\frac{29}{39}$.

Examples xx.—Page 44.

1. $\frac{21}{28}$, $\frac{20}{28}$. 2. $\frac{24}{54}$, $\frac{15}{54}$, $\frac{14}{54}$. 3. $\frac{231}{385}$, $\frac{220}{385}$, $\frac{210}{385}$.
4. $\frac{100}{240}$, $\frac{156}{240}$, $\frac{51}{240}$, $\frac{38}{240}$. 5. $\frac{408}{714}$, $\frac{630}{714}$, $\frac{364}{714}$, $\frac{455}{714}$.
6. $\frac{30}{90}$, $\frac{54}{90}$, $\frac{15}{90}$, $\frac{25}{90}$. 7. $\frac{324}{1080}$, $\frac{200}{1080}$, $\frac{84}{1080}$, $\frac{33}{1080}$.
8. $\frac{180}{420}$, $\frac{340}{420}$, $\frac{348}{420}$, $\frac{161}{420}$, $\frac{282}{420}$.

Examples xxi.—Page 45.

The fractions are arranged in *descending* order.

1. $\frac{4}{5}$, $\frac{9}{13}$, $\frac{3}{7}$. 2. $\frac{5}{6}$, $\frac{7}{9}$, $\frac{12}{17}$. 3. $\frac{13}{15}$, $\frac{9}{11}$, $\frac{17}{21}$.
4. $\frac{7}{40}$, $\frac{11}{70}$, $\frac{3}{20}$. 5. $\frac{7}{33}$, $\frac{9}{43}$, $\frac{11}{53}$. 6. $\frac{5}{34}$, $\frac{7}{51}$, $\frac{2}{17}$.

Examples xxii.—Page 46.

1. $\frac{19}{35}$. 2. $\frac{37}{42}$. 3. $\frac{13}{21}$. 4. $\frac{58}{63}$. 5. $\frac{125}{156}$.
6. $\frac{31}{32}$. 7. $\frac{1018}{1485}$. 8. $\frac{3043}{3465}$. 9. $\frac{2231}{2880}$. 10. $\frac{11}{18}$.
11. $\frac{119}{120}$. 12. $1\frac{179}{196}$. 13. $32\frac{71}{144}$. 14. $22\frac{583}{1080}$. 15 $30\frac{13}{400}$.

Examples xxiii.—Page 47.

1. $\frac{1}{30}$. 2. $\frac{48}{133}$. 3. $\frac{1}{156}$. 4. $\frac{48}{119}$. 5. $\frac{1}{756}$.
6. $\frac{13}{418}$. 7. $\frac{218}{1911}$. 8. $\frac{1}{450}$. 9. $\frac{1061}{7659}$. 10. $\frac{237}{350}$.

Examples xxiv.—Page 48.

1. $2\frac{9}{16}$. 2. $2\frac{5}{9}$. 3. $14\frac{73}{84}$. 4. $10\frac{29}{66}$. 5. $5\frac{21}{25}$.
6. $2\frac{2}{7}$. 7. $5\frac{8}{25}$. 8. $8\frac{66}{83}$. 9. $53\frac{13}{16}$. 10. $9\frac{17}{27}$.

Examples xxv.—Page 49.

1. $\frac{5}{21}$. 2. $\frac{135}{308}$. 3. $\frac{2}{5}$. 4. $\frac{98}{165}$. 5. $\frac{9}{76}$.
6. $\frac{275}{1104}$. 7. $\frac{1287}{2320}$. 8. $\frac{32}{405}$. 9. $\frac{50}{243}$. 10. $\frac{13}{225}$.

Examples xxvi.—Page 50.

1. $\frac{4}{7}$. 2. $\frac{5}{6}$. 3. $\frac{5}{12}$. 4. $\frac{12}{29}$. 5. $\frac{2}{5}$.
6. $\frac{49}{54}$. 7. $\frac{29}{77}$. 8. $\frac{5}{6}$. 9. $\frac{20}{27}$. 10. $\frac{133}{375}$.

Examples xxvii.—Page 50.

1. $65\frac{5}{9}$. 2. $178\frac{5}{7}$. 3. $43\frac{1}{3}$. 4. 225. 5. 156. 6. 247.
7. $1\frac{16}{119}$. 8. $1\frac{2}{5}$. 9. $1\frac{25}{27}$. 10. $9\frac{1}{2}$. 11. $1\frac{3}{7}$. 12. $1\frac{5}{6}$.

Examples xxviii —Page 52.

1. $27\frac{3}{5}$. 2. 744. 3. $718\frac{3}{4}$. 4. $\frac{16}{35}$. 5. $1\frac{1}{3}$. 6. 4. 7. $2\frac{3}{4}$. 8. 26.

EXAMPLES XXIX.—PAGE 53.

1. $\frac{1}{35}$.　2. $\frac{1}{756}$.　3. $\frac{1}{24}$.　4. $\frac{1}{5460}$.
5. $3\frac{1}{3}$.　6. $142\frac{1}{2}$.　7. $8350\frac{1}{3}$.　8. 24.

EXAMPLES XXX.—PAGE 56.

1. $1\frac{1}{23}$.　2. $\frac{77}{206}$.　3. $\frac{12}{95}$.　4. $\frac{165}{254}$.　5. $1\frac{7}{19}$.
6. $\frac{23}{64}$.　7. $\frac{26}{73}$.　8. $1\frac{7}{23}$.　9. $2\frac{28}{31}$.　10. $\frac{12}{19}$.

EXAMPLES XXXI.—PAGE 57.

1. $\frac{21}{25}$.　2. $\frac{2}{11}$.　3. $1\frac{4}{19}$.　4. $\frac{3}{5}$.　5. $5\frac{37}{41}$　6. $\frac{1}{2}$.
7. $\frac{685}{756}$.　8. $20\frac{103}{120}$.　9. $1\frac{11}{108}$.　10. $1\frac{1}{17}$.　11. $7\frac{7}{20}$.　12. $\frac{3}{4}$.

EXAMPLES XXXII.—PAGE 57.

1. $1\frac{43}{88}$.　2. $1\frac{6}{35}$, $\frac{20}{63}$.　3. $\frac{1}{9}$, $6\frac{2}{3}$.　4. $\frac{13}{37}$, $8\frac{7}{13}$, 3.
5. $4\frac{1}{5}$.　6. $11\frac{1}{12}$, 20.　7. $\frac{15}{284}$, $\frac{1}{4}$.　8. 3.
9. $7\frac{1}{9}$　10. 3.　11. $10\frac{146}{171}$.　12. $1\frac{1}{2}$, $1\frac{1}{4}$.
13. $\frac{3}{4}$, $\frac{7}{51}$.　14. $\frac{304}{357}$, $1\frac{2}{7}$.　15. 1, $\frac{3}{4}$.　16. 5, 18.
17. 66.　18. $14\frac{42}{193}$.　19. $\frac{75}{77}$.　20. 1.
21. $\frac{1}{2}$.　22. $\frac{226}{299}$.　23. 2.

EXAMINATION PAPERS.—PAGE 60.

I.

2. 49\frac{1}{8}$.　3. $5\frac{82}{157}$.　4. $13860.　5. $3\frac{27}{70}$ and $\frac{57}{70}$.

II.

2. $\frac{23}{44}$, $\frac{29}{56}$, $\frac{7}{51}$, $\frac{8}{35}$.　3. $\frac{728}{1000}$.　5. Ship, $24000; cargo, $36000.

III.

2. 18\frac{2}{3}$.　4. $\frac{473}{18500}$.　5. *A*, 20; *B*, 48; *C*, 84.

IV.

2. $\frac{27}{35}$.　3. $34\frac{5}{17}$.　4. Horse, $120; carriage, $105; harness, $25.
5. *A*, $4334; *B*, $1474; *C*, $3080.

V.

2. $3\frac{22}{39}$.　3. $2015; 465 sheep; 390 calves; 806 pigs.
4. $180.　5. 18 ft.

VI.

2. $\frac{3}{15}$ $\frac{5}{15}$, $\frac{7}{15}$.　3. 1.　4. $40.　5. $\frac{5}{36}$ ft.

VII.

2. 1333\frac{1}{3}$, $\frac{1}{30}$.　3. 1000000.　4. 30 min.; *A*, 6 times; *B*, 5 times; *C*, 4 times.　5. *A*, $\frac{5}{36}$; *B*, $\frac{5}{28}$; *C*, $\frac{10}{231}$; *D*, $\frac{15}{77}$.

VIII.

2. $\frac{1}{6}$; $\frac{8}{91}$. 3. 36. 4. 30 min.; 4500 rods; 3600 rods; 3000 rods.
5. 252.

IX.

1. 72 ft., 64 ft. 2. $10\frac{1}{4}$ bu. 3. $1\frac{3}{7}$. 4. 1350 men; Irish, 540; Scotch, 405; English, 405. 5. $2520.

EXAMPLES XXXIII.—PAGE 66.

1. $\frac{1}{2}$. 2. $\frac{1}{4}$. 3. $\frac{3}{4}$. 4. $\frac{3}{8}$. 5. $\frac{243}{1000000}$.
6. $\frac{29}{4000000}$. 7. $\frac{74}{5}$. 8. $\frac{20847}{200}$. 9. $\frac{125001}{2500}$. 10. $\frac{100001}{1000}$.
11. .9. 12. .37. 13. .4579. 14. .003. 15. 172.95.
16. .0000059. 17. .025679. 18. 3.25793. 19. .0019.

EXAMPLES XXXIV.—PAGE 68.

1. .7. 2. .2464. 3. .0012. 4. 758.279832.
5. 385.260863. 6. 8741.2062. 7. 6964.72672. 8. 970.17047.

EXAMPLES XXXV.—PAGE 69.

1. 51.211. 2. 1.543. 3. 48.2293. 4. .001. 5. .0607.
6. 579.1274. 7. .0000014. 8. .004385. 9. 9.9998. 10. .00101.

EXAMPLES XXXVI.—PAGE 71.

1. 35.25. 2. 18.9326. 3. .100345.
4. .00041588. 5. 12.08980432. 6. .9.
7. 14977.92625425. 8. .0000465131.
9. .057746898828045. 10. 203.175662750726562.
11. .00984126. 12. 1.01. 13. .00031304.
14. .15205806. 15. .1009981674. 16. 20.570824.
17. 150.0625.

EXAMPLES XXXVII.—PAGE 74

1. 12. 2. 14400. 3. .0013.
4. 12700. 5. 43.078. 6. 10000.
7. 430. 8. 147. 9. .0000002004.
10. 98.476. 11. .0065839. 12. 876540000.
13. .0000771039. 14. 299846000. 15. .20162.
16. 2469300000. 17. 3596. 18. .00000029.
19. 1290. 20. 3.59. 21. 457.61.
22. 76.371. 23. 905741000.

EXAMPLES XXXVIII.—PAGE 75.

1. 23.28125. 2. 1.119296875. 3. 3.4608.
4. 33035.448... 5. .00192. 6. .0001736.

EXAMPLES XXXIX.—PAGE 76.

1. 26.654875. 2. .0010902475. 3. 14498.8.
4. .00001614. 5. 175.0309875. 6. .0000926.
7. 154468.75. 8. 25000000. 9. .00001.
10. .0000005005005.

EXAMPLES XL.—PAGE 77.

1. 18478.260. 2. .249. 3. .092.
4. 8658146.964. 5. .095. 6. 32714.285.

EXAMPLES XLI.—PAGE 80.

1. .35. 2. .44. 3. $.\dot{8}5714\dot{2}$. 4. $.\dot{0}\dot{1}$. 5. $.\dot{0}0\dot{1}$.
6. $.\dot{0}243\dot{9}$. 7. $.\dot{5}2380\dot{9}$. 8. $.21\dot{6}$. 9. $.012\dot{3}\dot{6}$. 10. $2.3\dot{4}\dot{5}$.

EXAMPLES XLII.—PAGE 83.

1. .09484. 2. .002521. 3. 165.6995. 4. 235.104.
5. 26.38702. 6. 1.611. 7. .0374. 8. 426.104.
9. 170.3367. 10. .928.

EXAMPLES XLIII.—PAGE 84.

1. $\frac{2}{3}$. 2. $\frac{3}{11}$. 3. $\frac{5}{111}$. 4. $\frac{347}{1111}$.
5. $\frac{8}{1111}$. 6. $\frac{447}{1111}$. 7. $\frac{6}{11111}$. 8. $\frac{1}{11111}$.

EXAMPLES XLIV.—PAGE 85.

1. $\frac{421}{990}$. 2. $\frac{1178}{2475}$. 3. $4\frac{19}{75}$. 4. $\frac{211}{49500}$. 5. $7\frac{2009}{9990}$.
6. $2\frac{1751}{3300}$. 7. $53\frac{9}{3700}$. 8. $18\frac{36053}{99990}$. 9. $267\frac{641}{1800}$.

EXAMPLES XLV.—PAGE 87.

1. $15.8\dot{4}3\dot{0}$. 2. $20.51\dot{6}6202\dot{5}$. 3. $1.7\dot{7}8005\dot{2}$. 4. $.02\dot{0}6724\dot{9}$.
5. $20\frac{676}{891}$. 6. $\frac{13}{3960}$. 7. $\frac{310}{369}$. 8. $\frac{37}{297}$.

EXAMINATION PAPERS.—PAGE 87.

I.

3. 1st. 4. 263 times; $\frac{1}{3}$. 5. .0000006 and .0000009.

II.

1. $\frac{5}{32}$, $\frac{77}{1100}$, $\frac{1820}{2912}$, $\frac{91}{560}$. 2. $14.90. 3. 3.715. 4. $\frac{11}{9}$. 5. .7142.

III.

2. $\$816\frac{2}{3}$. 3. $.1\dot{3}$; $3000. 4. $\$232\frac{2}{3}$. 5. $\frac{13200}{2009}$.

IV.

1. $21.60. 3. 425. 4. $8.75. 5. $82\frac{1}{4}$ yd.

V.

2. $\frac{39371}{2534400}$. 3. 10.7608 mi. 4. $13\frac{23}{48}$.
5. *A*, \$192.23$\frac{1}{2}$; *B*, \$145.53$\frac{1}{2}$; *C*, \$110.94$\frac{1}{2}$.

VI.

3. \$34$\frac{1}{3}$. 4. 7.025, .17141.

EXAMPLES XLVI.—PAGE 90.

1. 497025. 2. 956484. 3. 6225.21. 4. .322624.
5. .133225. 6. $5076\frac{9}{16}$. 7. $4987\frac{57}{64}$. 8. .339889.
9. .56625625. 10. 50653. 11. 2460375. 12. .000004096.
13. .398688256. 14. $9709\frac{1}{27}$. 15. $40707\frac{73}{125}$. 16. 28000.
17. 16200. 18. .25. 19. 30140000. 20. 3673.
21. 47.11.

EXAMPLES XLVII.—PAGE 94.

1. 14. 2. 23. 3. 32. 4. 75. 5. 297.
6. 345. 7. 327. 8. 867. 9. 440. 10. 835.
11. 6031. 12. 4698. 13. 23456. 14. 72500. 15. 2031.
16. 739000. 17. 5678. 18. 437962. 19. 17325.

EXAMPLES XLVIII.—PAGE 95.

1. 4.1. 2. .95. 3. .51. 4. .25. 5. .027.
6. 131.31. 7. 1.001. 8. 210.75 9. 137.65.

EXAMPLES XLIX.—PAGE 96.

1. 4.4721. 2. 5.4772. 3. .9486. 4. .3478.
5. .4110. 6. .1264. 7. .0252. 8. .0347.
9. 4.0305. 10. .9999. 11. .5025. 12. 6.4833.

EXAMPLES L.—PAGE 97.

1. $\frac{6}{7}$. 2. $\frac{8}{11}$. 3. $\frac{17}{25}$. 4. $\frac{37}{87}$. 5. $\frac{123}{427}$.
6. $2\frac{1}{4}$. 7. $2\frac{2}{5}$. 8. $1\frac{10}{13}$. 9. $8\frac{1}{9}$. 10. $6\frac{1}{5}$.
11. $4\frac{1}{5}$. 12. $3\frac{3}{7}$. 13. .7905. 14. .6454. 15. 2.5298.
16. 3.0822. 17. 8.7649. 18. 4.0632.

EXAMPLES LI.—PAGE 97.

1. 855. 2. 797. 3. 2223. 4. 748, 187. 5. 405, 270.

EXAMPLES LII.—PAGE 99.

1. 16. 2. 32. 3. 42. 4. 79. 5. 85. 6. 64.
7. 34. 8. 73. 9. 92. 10. 99. 11. 39. 12. 63.

EXAMPLES LIII.—PAGE 100.

1. 245. 2. 531. 3. 307. 4. 670. 5. 128.
6. 179. 7. 463. 8. 103. 9. 256. 10. 579.
11. 438. 12. 507. 13. 686. 14. 708. 15. 888.
16. 512. 17. 4968. 18. 8765.

EXAMPLES LIV.—PAGE 101.

1. .73. 2. .364. 3. 30.02. 4. $\frac{11}{12}$. 5. $\frac{5}{7}$.
6. $1\frac{5}{7}$. 7. $7\frac{2}{5}$. 8. 1.709. 9. 8.320. 10. .495.
11. 2.516. 12. .822. 13. .908. 14. .693. 15. 1.966.
16. 1.473.

EXAMPLES LV.—PAGE 102.

1. 27. 2. 45. 3. 6.3. 4. 13. 5. 54. 6. 8.1.

EXAMPLES LVI.—PAGE 102.

1. 879. 2. 3420. 3. 93, 31, 62.
4. 55, 110, 220, 440. 6. 35856144, 35832196. 7. 1512.
8. 9.

EXAMPLES LVII.—PAGE 104.

1. 2.2894. 2. 3.1072. 3. 6.0322. 4. 7.2112. 5. .9654.
6. .4308. 7. 2.1006. 8. .8372. 9. 2.0614.

EXAMPLES LVIII.—PAGE 110.

1. 13s. 4d.; £1 11s. 3d.; £3 10s. 9d.
2. 6 fur. 16 po.; 30 sq. po.; $83\frac{1}{3}$ lb.
3. £152 11s. $0\frac{4}{5}$d.; £1 13s. 9d.; 2 mi. 2 fur.
4. £514 16s.; 15s. 9d. 5. £1 2s. $10\frac{1}{2}$d.
6. 13s. 6d. 7. 9 ac. 2 ro. $13\frac{1}{2}$ po.
8. 16 da. 3 hr. 35 min. 9. 2 fur. 6 po. 4 yd. $1\frac{1}{2}$ in.
10. 4 cwt. 2 qr. 11 lb. $10\frac{6}{7}$ oz.

EXAMPLES LIX.—PAGE 111.

1. $\frac{1}{46}$. 2. $\frac{224}{247}$. 3. $\frac{11}{42}$. 4. $\frac{158}{211}$. 5. $\frac{613}{2016}$. 6. $\frac{5}{8}$.
7. $\frac{3}{5}$. 8. $\frac{407}{967}$. 9. $\frac{3}{11}$. 10. $\frac{1}{3}$. 11. $\frac{2880}{17}$. 12. $\frac{187}{119}$.

EXAMPLES LX.—PAGE 113.

1. 12s. 6d. 2. £15 5s. 6d. 3. 2.3436d.
4. 3 qr. 18 lb. 12 oz. 5. .75 oz. 6. £16 0s. 6d.
7. 1s. 5d. 8. £2 16s. 9.375d. 9. 4s. 2d.
10. £2 11s. 8d. 11. 22 lb. $6\frac{2}{5}$ oz. 12. £7 16s. $2\frac{1}{4}$d.
13. 4 t. 16 cwt. 17.4 lb. 14. £26 17s. $10\frac{23}{30}$d. 15. £2 5s. 9.8d.
16. 16s. 7d. 17. £1 14s. 8d.

Examples lxi.—Page 114.

1. .3285. 2. $.00208\dot{3}$. 3. .1875. 4. $.4\dot{3}$.
5. 14.49. 6. .24. 7. 2.64. 8. 1.382890625.
9. $.002\dot{7}$. 10. $1.43\dot{1}\dot{8}$. 11. .3. 12. .00091875.
13. $2.44591\dot{6}$. 14. .5581..... 15. .1406.....

Examination Papers.—Page 115.

I.

1. $366\frac{6}{59}$. 2. 3413 da. 9 hr.
3. 3 da. 2 hr. 20 min. 4. Friday, April 19, 1850.
5. 191; 4 hr. 25 min. 30 sec., a. m., Dec. 4.

II.

2. $13\frac{1307}{1321}$ ft. 3. 49 min. past 1 p.m.; $149\frac{4}{7}$ mi.
4. 19 mi. $1464\frac{27}{62}$ yd. 5. 7488 strokes.

III.

2. 3600; £7 10s.
3. *A*, 3 ac. 1 ro. 20 po. 21 yd. $77\frac{1}{7}$ in.; *B*, 6 ac. 3 ro. 1 po. 11 yd. 7 ft. $118\frac{2}{7}$ in.; *C*, 7 ac. 2 ro. 16 po. 17 yd. 1 ft. $29\frac{4}{7}$ in.
4. 29 yd. 5. 16 t. 4 cwt.; 10 cwt. 3 qr. 5 lb.

IV.

1 $11.37½. 2. 259 bu. 2 pk. 1 gal. $1\frac{193}{231}$ pt.
3. 41 bu. 3 pk. $2\frac{2}{5}$ qt. 4. 47 bags.
5. $96.93.....

V.

1. Loses $2. 2. $13\frac{43}{64}$c.
3. 20 gr. is largest weight. 4. 12 t. 7 cwt. 3 qr. 16.67 lb.
5. 250 lb.

VI.

1. 18s. 4d.
2. G.C.M., 4 ac. 2 ro. 7 po. 3 yd.; L.C.M., 699 ac. 3 ro. 13 po. 8¼ yd.
3. 2 hr. 10 min. 4. Loses £295 3s. 2½d. 5. 80 oz.

Examples lxii.—Page 120.

2. 5007.039 m. 3. 17800.308 m.
4. 60070.807 m. 5. 5760 m., 57600 dm., 5760000 mm.
6. 8.769 m., .08769 Hm., .008769 Km.
7. 5887.816 m. 8. 2999.494 m. 9. 119.73 Km.
10. $2180.40. 11. 3100 posts. 12. 2008 m.
13. $3230\frac{10}{13}$ times.

EXAMPLES LXIII.—PAGE 121.

1. 5.0205 Ha.; 502.05 a.; 50205 ca. 2. 19070005 sq. m.
3. 607.81 sq. m. 4. 2.705608 sq. m. 5. 9900 ca.
6. 786.96 Ha. 7. $14000. 8. 12470.30 sq. m.

EXAMPLES LXIV.—PAGE 121.

1. 715.007078 cu. m.; 715007.078 cu. dm.; 715007078 cu. cm.
2. 7047.08 l.; 7047080 ml. 3. 899.640 st. 4. 253.980 st.
5. 157.3 loads. 6. 7680 l. 7. 3 times.

EXAMPLES LXV.—PAGE 122.

1. 3 Kg. 4 Hg. 7 Dg. 1 g. 0 dcg. 8 cg. 6.8 mg. 2. 4303570 cg.
3. 6230 g. 4. 37.005 Kg.
5. 710500 g.; 710500000 mg.; 7105000 dg. 6. 4500 Kg.
7. 840 Kg. 8. 780 Kg. 9. 7882.4 Kg.
10. 229.5 Kg. 11. .001293 to 1. 12. .8 to 1.

EXAMPLES LXVI.—PAGE 123.

1. 1067.25 dm. 2. 15000 mg.
3. 43.7 mm.; 4.37 cm. 4. 155000 sq. cm.
5. 1086.42 sq. dm. 6. 1725 g.
7. 100 mg.; 10000 dg. 8. 256700 cg.
9. 5000 mg. 10. 567.875 cu.cm.
11. 3720 l. 13. 24855.303 mi.
14. 5168240 g.

EXAMPLES LXVII.—PAGE 124.

1. $591.75. 2. $171. 3. $562.50.
4. $1891. 5. £8061 7s. $3\frac{3}{4}$d. 6. £6022 0s. $7\frac{1}{2}$d.
7. $68.04. 8. $156.20. 9. $42562.50.
10. £231 0s. $8\frac{112}{121}$d. 11. £50 14s. $4\frac{179}{200}$d. 12. £32 2s. $7\frac{1}{2}$d.
13. £496 2s. $6\frac{47}{160}$. 14. £805 9s. $1\frac{1}{5}$d.

EXAMPLES LXVIII.—PAGE 126.

1. (*a*) $1690; (*b*) $10.93½.
2. (*a*) $164.74; (*b*) $74.14½; (*c*) $103.70; (*d*) $33.91¼.
3. $779.69¼. 4. $20.65. 5. $19.02½. 6. $105.

EXAMPLES LXIX.—PAGE 130.

1. 6 da. 2. 1½ da. 3. $47775. 4. 25 men.
5. $543\frac{21}{47}$. 6. 203 mi. 7. $3240. 8. $14\frac{7}{13}$.
9. £31 10s. 10. 54 da. 11. 20c. 12. $408⅓.
13. $9.15. 14. $14.40. 15. £18 15s. $7\frac{46}{47}$d.

EXAMPLES LXX.—PAGE 131.

1. \$7833.33$\frac{1}{3}$. 2. \$5040. 3. 18$\frac{7}{10}$ lb. 4. $\frac{14}{165}$
5. 27 min. 6. \$1182.12$\frac{1}{2}$. 7. 286$\frac{2}{13}$ mi. 8. \$400.
9. £1 17s. 4.128d. 10. The first. 11. 22$\frac{1}{2}$ cwt.

EXAMPLES LXXI.—PAGE 132.

1. 480 ac. 2. \$1152. 3. 11268. 4. 4000.
5. \$10.52. 6. \$54.61$\frac{1}{4}$. 7. \$360. 8. 16 bu.
9. 20 da. 10. \$78.60. 11. 27 laborers. 12. 75 burners.
13. 6 wk. 14. \$396. 15. 7 wk. 16. 4 da.
17. 155$\frac{3}{4}$ qr. 18. 5$\frac{1}{2}$ wk. 19. 84 da. 20. 10 da.
21. 2 da. 22. 660 men. 23. £120.

EXAMPLES LXXII.—PAGE 135.

1. 3$\frac{3}{5}$ hr. 2. 13$\frac{37}{191}$ da. 3. 2$\frac{22}{73}$ da. 4. 2$\frac{2}{3}$ min.
5. 10 da. 6. 4 hr. 7. 18 da. 8. $\frac{29}{36}$.
9. *A*, $\frac{81}{94}$ da.; *B*, 1$\frac{19}{47}$ da.; *C*, 1$\frac{81}{94}$ da. 10. 10$\frac{22}{41}$ da.

EXAMPLES LXXIII.—PAGE 137.

1. (*a*) 20$\frac{9}{11}$ min. past 4; (*b*) 32$\frac{8}{11}$ min. past 6; (*c*) 49$\frac{1}{11}$ min. past 9.
2. (*a*) 5$\frac{5}{11}$ min., and 38$\frac{2}{11}$ min. past 4; (*b*) 20$\frac{9}{11}$ min., and 54$\frac{6}{11}$ min. past 7; (*c*) 10$\frac{10}{11}$ min., and 43$\frac{7}{11}$ min. past 10.
3. (*a*) 38$\frac{2}{11}$ min. past 1; (*b*) 54$\frac{6}{11}$ min. past 4; (*c*) 10$\frac{10}{11}$ min. past 8.
4. (*a*) 13$\frac{1}{11}$ min., and 19$\frac{7}{11}$ min. past 3; (*b*) 34$\frac{10}{11}$ min., and 41$\frac{5}{11}$ min. past 7; (*c*) 51$\frac{3}{11}$ min., and 57$\frac{9}{11}$ min. past 10.
5. (*a*) 9$\frac{3}{13}$ min. past 2; (*b*) 32$\frac{4}{13}$ min. past 7; (*c*) 41$\frac{7}{13}$ min. past 9.
6. (*a*) 45$\frac{9}{11}$ min. past 10; (*b*) 45$\frac{5}{11}$ min. past 10; (*c*) 41$\frac{5}{11}$ min. past 10; (*d*) 30$\frac{10}{11}$ min. past 10; (*e*) 27$\frac{3}{11}$ min. past 10; (*f*) 25$\frac{7}{11}$ min. past 10.
7. 20$\frac{8}{11}$ min., or 33$\frac{9}{11}$ min. past 5.
8. 19$\frac{7}{11}$ min. or 45$\frac{9}{11}$ min. past 6. 9. 18 min. past 2.

EXAMPLES LXXIV.—PAGE 139.

1. 1 mi. 2. 36 mi. 3. 1$\frac{1}{3}$ mi. 4. 1$\frac{2}{3}$ hr.
5. 2 mi. 6. 50 mi.; 30 mi. 7. 2 mi. 8. 6$\frac{2}{3}$ mi.

EXAMINATION PAPERS.—PAGE 140.

I.

1. \$.029$\frac{23}{136}$. 2. \$269.33$\frac{1}{3}$. 3. \$2408.40. 4. 300. 5. 324 da.

II.

1. 18 da. 2. 1200 men. 3. 35 da. 4. 33$\frac{1}{3}$ da.; $\frac{1}{5}$. 5. 60 min.

III.

1. 24 da. 2. 360 da.
3. 60 mi, from *A*'s starting point; 5 hr. and 15 hr. from starting.
4. At 10 hr. 15 min. a.m. on Saturday the watch is 5 min. $36\frac{7}{48}$ sec. too slow. 5. $10\frac{10}{29}$ min. past 5 and $9\frac{21}{31}$ min. to 5.

IV.

1. 52 da. 2. 10 hr. 3. *A* in $9\frac{3}{5}$ hr.; *B* in $6\frac{6}{7}$ hr.
4. 3 hr. $54\frac{2}{5}$ min. p. m., nearly.

EXAMPLES LXXV.—PAGE 142.

1. (*a*) 170; 21.25; (*b*) 5170; $738.\dot{5}7142\dot{8}$; (*c*) 168263; $56087.\dot{6}$; (*d*) 215.7; 26.9625; (*e*) 81.639; 10.204875.
2. 92. 3. 174 lb. 4. 10.4 yr. 5. $6.60. 6. 36.75c.

EXAMPLES LXXVI.—PAGE 143.

1. (*a*) $120; (*b*) 278 horses; (*c*) $202.50; (*d*) 4 ac.; (*e*) 216 books; (*f*) 30 yd.
2. 123 sheep. 3. $187.50. 4. 31970. 5. 148 pupils.
6. 4500 qt. 7. $700.57.

EXAMPLES LXXVII.—PAGE 144.

1. (*a*) $33\frac{1}{3}$ %; (*b*) 20 %; (*c*) $1\frac{1}{2}$ %; (*d*) 40 %; (*e*) $2\frac{1}{2}$ %; (*f*) $11\frac{1}{4}$.
2. 20 %. 3. 33.11 %. 4. $3\frac{3}{4}$ %; $12\frac{1}{2}$ %; 15 %. 5. $1\frac{3}{4}$ %.
6. $11\frac{1}{9}$ %. 7. 1500 %. 8. 64 %.

EXAMPLES LXXVIII.—PAGE 145.

1. 300; $23437\frac{1}{2}$; 4300000. 2. 1875. 3. $8100.
4. 1000. 5. 200. 6. 330.
7. 360 bu. 8. 27900. 9. 60 gal.
10. $3584.

EXAMPLES LXXIX.—PAGE 146.

1. (*a*) $196; (*b*) $256.50; (*c*) $2103.30. 2. $248.
3. $33\frac{3}{5}$. 4. \$$33\frac{1}{3}$. 5. $32\frac{1}{2}$ %. 6. $1.25.
7. $7\frac{9}{13}$ %. 8. 0. 9. $113.40. 10. $16.
11. Loss, $14\frac{1}{2}$ %. 12. $8.

EXAMPLES LXXX.—PAGE 150.

1. 25 %. 2. $45\frac{5}{11}$ %. 3. 4.91 .. %.
4. $33\frac{1}{3}$ %. 5. 50 %. 6. 12 %.
7. 5 %. 8. 23 %. 9. Gained 13.9 . . . %
10. 40c. 11. $9. 12. 24 lb.

13. \$3.50.
14. \$3500.
15. \$2000.
16. \$4.35.
17. £1 1s. 1d. per yd.
18. Gained $3\frac{13}{17}$ %.
19. \$600.
20. \$15.
21. 8 %.
22. 10 %.
23. 10 %.
24. \$300.
25. 3s. $7\frac{1}{4}$d. per lb.
26. 4 lb.
27. \$3.45.
28. \$3.60.
29. $33\frac{1}{3}$ %.
30. \$2.17, nearly.
31. \$40.
32. \$224.
33. \$360.
34. \$400.
35. $66\frac{2}{3}$ %.
36. \$180 ; \$360.
37. \$800.
38. 100 %.

EXAMPLES LXXXI.—PAGE 154.

1. (*a*) \$106.40; (*b*) \$700.
2. $\$167.37\frac{1}{2}$.
3. \$7488.
4. \$26.66.
5. $4\frac{1}{2}$ %.
6. $2\frac{1}{4}$ %.
7. $5\frac{1}{3}$ %.
8. $1\frac{1}{2}$ %.
9. \$38400.
10. \$7840.
11. \$3750.
12. \$712.50
13. \$77.
14. 600 bbl.
15. \$9800.
16. 2 %.

EXAMPLES LXXXII.—PAGE 155.

1. \$3.125.
2. \$747.25.
3. £4488 15s.
4. \$415.25.
5. \$473.
6. \$9.80.
7. \$10000.
8. 4800.
9. \$9930.

EXAMPLES LXXXIII.—PAGE 156.

1. \$56.70.
2. \$0.017.
3. \$2437500.
4. $1\frac{1}{4}$c. on the dollar.
5. $16\frac{1}{2}$ mills on the dollar.
6. \$25.07...
7. \$2300.

EXAMPLES LXXXIV.—PAGE 157.

1. \$916.20.
2. \$616.
3. \$218.75.
4. \$262.50.
5. \$4750.
6. \$1179.
7. 35 %.

EXAMINATION PAPERS.—PAGE 159.

I.

1. 4.065 %.
2. \$225.
3. \$3640.
4. 41825.
5. 10 %.

II.

1. \$1760.
2. \$7119.80.
3. 21.75.
4. \$10935000.
5. *A*, \$40; *B*, \$45.

III.

1. \$8400.
2. 418 bales; \$323.58.
3. As 40 and 31.
4. \$8.
5. 2.97d.

IV.

1. $\$305.78\frac{1}{8}$.
2. 16222.11 lb.
3. 165 ; 255 ; 330.
4. Grain, \$1020 ; groceries, \$950.
5. *A* gets \$842.30 ; *B*, \$918.87 ; *C*, \$1598.83.

V.

1. \$4500.
2. \$2040.
3. 100 bales.
4. \$255.
5. \$3000.
6. $\$46\frac{2}{3}$.
7. $13\frac{1}{3}$ %.

EXAMPLES LXXXV.—PAGE 164.

1 (*a*) \$63.80 ; (*b*) \$412.50 ; (*c*) \$870 ; (*d*) \$729 ; (*e*) \$3441.75 ; (*f*) \$156 ; (*g*) $\$164.02\frac{1}{2}$; (*h*) £4 9s. $2\frac{7189}{7300}$d.
2. \$4789.50.
3. \$11677.20.
4. Gains \$22.50.

EXAMPLES LXXXVI.—PAGE 165.

1. $4\frac{1}{2}$ %.
2. $5\frac{1}{4}$ yr.
3. \$1350.
4. \$375.
5. 20 yr.
6. $5\frac{1}{4}$ %.
7. $12\frac{1}{2}$ yr.
8. 5 %.
9. 97 da.
10. $4\frac{3}{4}$ %, nearly.
11. $9\frac{1}{8}$ %.
12. Lost \$5.
13. 6 %.
14. \$600 ; $13\frac{1}{2}$ yr.
15. October 6.
16. \$320.
17. \$250 ; 6 %.
18. \$360 ; $2\frac{1}{2}$ yr.

EXAMPLES LXXXVII.—PAGE 169.

1. \$869.75.
2. \$902.83.
3. \$82.56.
4. \$4920.78.
5. \$1830.58.
6. \$1257.63.
7. \$3429.58.

EXAMPLES LXXXVIII.—PAGE 172.

1. (*a*) \$4600 ; (*b*) \$70 ; (*c*) \$562.50 ; (*d*) \$1050 ; (*e*) £537 10s.
2. (*a*) \$36 ; (*b*) \$456.80 ; (*c*) \$137.50 ; (*d*) £152 1s. 3d. ; (*e*) £7 19s. 3d.
3. \$6030.
4. \$4.40.
5. 5 %.
6. \$1105.
7. $\$45\frac{5}{6}$, $\$13\frac{2}{21}$.
8. 80 to 83 ; \$32.
9. $\$16\frac{112}{113}$.
10. \$520 ; 6 %.

EXAMPLES LXXXIX.—PAGE 176.

1. (*a*) Dec. 3 ; 108 da. ; \$985.20 ; \$14.80.
   (*b*) June 24 ; 73 da. ; \$602.34 ; \$9.79.
   (*c*) June 20 ; 66 da. ; \$501.55 ; \$7.36.
2. (*a*) \$393.34 ; (*b*) \$1584.44 ; (*c*) \$577.40 ; (*d*) \$722.
3. \$714.28...
4. $\$1333\frac{1}{3}$.
5. \$485.
6. \$1300.
7. $9\frac{13}{43}$ %.
8. \$78 ; \$50.
9. \$838.02... ; \$25.98.
10. \$63.52...
11. \$25.50.
12. \$11.22, nearly.
13. $7\frac{1}{2}$ %.
14. $6\frac{1}{4}$ %.
15. 73 da.
16. \$292.
17. $8\frac{1}{3}$ %.
18. Sept. 11.
19. \$800.

Examples xc.—Page 180.

1. (*a*) \$59.109; (*b*) \$148.037; (*c*) \$248.77; (*d*) \$160.32.
2. (*a*) \$1141.166; (*b*) \$843.648; (*c*) \$1458.6075; (*d*) \$1159.274.

Examples xci.—Page 182.

1. \$125.509. 2. \$238.81. 3. \$1689.73... 4. \$132.405.
5. \$1157.308. 6. \$153.22. 7. \$5.508. 8. £4 3s. $2\frac{74}{125}$d.
9. \$420.25. 10. 92c. 11. \$15400. 12. \$24000.
13. \$1200. 14. 4 %. 15. \$1000; 4 %. 16. \$800; 5 %.
17. $7\frac{49}{400}$ %. 18. 4.88 %. 19. \$9.80. 20. \$16.
21. \$560. 22. \$1200.

Examples xcii.—Page 185.

1. $7\frac{1}{2}$ mo. 2. $4\frac{7}{12}$ mo. 3. March 18.
4. $8\frac{1}{3}$ mo. after Nov. 6. 5. 4 mo. after 14 mo. 6. $12\frac{1}{5}$ mo.
7. 13 mo. 8. 5 mo. 9. $3\frac{1}{2}$ mo.
10. $7\frac{1}{2}$ mo. 11. \$$666\frac{1294}{2491}$. 12. $8\frac{2}{3}$ mo.
13. \$364.01. The equated time is May 5, 1897. All the bills are equivalent to \$362.35, but this will draw interest @ 6 % till June 2. 14. May 28th.

Examples xciii.—Page 188.

1. \$431; May 9. 2. \$190; March 11. 3. \$395; May 28.
4. \$410.63; Aug. 6. 5. Nov. 23. 6. Nov. 27.

Examination Papers.—Page 189.

I.

1. \$0.285. 2. \$716.76. 3. \$314. 4. \$511. 5. \$18.86...

II.

1. \$3121.60. 2. \$781.25. 3. 4.08 %. 4. \$77.90.
5. \$8108.326...

III.

2. \$19.047 : 95c. 3. £9 3s. 4d. 4. $3\frac{1}{8}$ %. 5. \$100.

IV.

1. \$2500. 2. \$71.88... 3. \$$888.88\frac{8}{9}$. 4. \$$41\frac{33}{487}$.

V.

1. \$$7.11\frac{1}{9}$. 2. \$50000. 3. $6\frac{1}{4}$ %; £574 13s.
4. *A*'s by \$224.05 5. \$12000.

EXAMPLES XCIV.—PAGE 199.

1. (*a*) \$8562.40; (*b*) \$9310; (*c*) \$11207.25; (*d*) £3542; (*e*) £523 16s. 9d.
2. (*a*) \$11200; (*b*) \$2700; (*c*) \$850; (*d*) £2400; (*e*) £6000.
3. (*a*) \$58.80; (*b*) \$171.60; (*c*) \$385; (*d*) \$150; (*e*) £276.
4. (*a*) \$600; (*b*) \$680; (*c*) \$550; (*d*) \$3200.
5. (*a*) $6\frac{2}{3}$ %; (*b*) $5\frac{5}{19}$ %; (*c*) $5\frac{10}{13}$ %; (*d*) $4\frac{2}{3}$ %.
6. (*a*) $8\frac{1}{3}$ %; (*b*) $7\frac{1}{4}$ %; (*c*) $7\frac{1}{3}$ %. 7. (*a*) 140; (*b*) 70; (*c*) 270.
8. (*a*) \$9600; (*b*) \$67500; (*c*) \$41540.
9. (*a*) $101\frac{1}{2}$; (*b*) 85; (*c*) $92\frac{1}{8}$. 10. $83\frac{1}{3}$.
11. 6 per cents.; $\frac{4}{5}$ %. 12. £4725. 13. \$450.
14. \$4984.80. 15. 223. 16. 60 %.
17. £24960. 18. 90. 19. Nothing.
20. Increased, \$56.55. 21. \$10692; \$21384. 22. \$8000.
23. $5\frac{3}{5}$ yr. 24. Loss, \$1.70... 25. \$6000.
26. 6 per cents. 27. \$40000. 28. \$44092.
29. $89\frac{3}{4}$. 30. \$23. 31. \$$4\frac{4}{249}$; $4\frac{9}{34}$.
32. \$3200000. 33. \$6000; \$3000. 34. \$21500.
35. \$24044.25.

EXAMINATION PAPERS.—PAGE 202.

I.

1. \$2568. 2. Loss, $8\frac{4}{7}$ %. 3. \$$1.86\frac{12}{43}$ per lb.
4. $21\frac{1}{2}$ %. 5. Loss of \$10.163...

II.

1. \$$2.33\frac{1}{3}$. 2. $11\frac{1}{9}$ %; $8\frac{7}{8}$ %. 3. $9\frac{11}{96}$c. 4. 90. 5. $229\frac{1}{3}$.

III.

1. 4000 lb.; \$$1.08\frac{1}{2}$. 2. \$80.
3. *A*; \$$2.67\frac{6}{7}$; \$$2.68\frac{3}{4}$; 40 %, and $39\frac{23}{43}$ %. 4. \$10. 5. \$102.723..

IV.

1. 12.9 9%, gain. 2. Lost, \$$71\frac{19}{91}$. 3. \$6000; \$8160; \$6528.
4. \$510.59. 5. \$2530.

V.

1. \$1000. 2. 40 shares. 3. Latter; \$136762.50.
4. $43\frac{1}{3}$. 5. Loses \$978; gains \$139.50.

EXAMPLES XCV.—PAGE 207.

1. \$33; \$27. 2. \$250; \$375; \$875; \$1000.
3. \$3300; \$2200; \$1650; \$1320.
4. 9 cwt. of saltpetre; $1\frac{1}{5}$ cwt. of sulphur; $1\frac{4}{5}$ cwt. of charcoal.
5. 120 yd.; 160 yd.; 200 yd.

6. \$240 to *A*; \$80 to *B*; \$320 to *C*. 7. 28; 32; 40.
8. A, £102 3s. 9d.; *B*, £132 16s. $10\frac{1}{2}$d.; *C*, £183 18s. 9d.
9. \$4515; \$10836; \$12040; \$9030.
10. *A*, \$11.50; *B*, \$5.75; *C*, \$9.20.
11. Silver, $201\frac{9}{11}$ gr.; copper, $16\frac{4}{11}$ gr. 12. 113; 339; 678; 791.
13. 30. 14. $57\frac{41}{87}$ ac.; $40\frac{20}{87}$ ac.; $91\frac{83}{87}$ ac.; $10\frac{30}{87}$ ac.
15. *A*, 9s.; *B*, 12s.; *C*, 24s.
16. Men, \$5; women, \$3; boys, \$2.40.
17. Men, \$182.70; women, \$182.70; children, \$152.25.
18. \$6000. 19. *A*, \$700; *B*, \$2500; *C*, \$1800.
20. *A*, \$1050; *B*, \$1200; *C*, \$1250; *D*, \$1500.
21. $\frac{414}{1573}$; $\frac{36}{143}$; $\frac{69}{286}$.
22. \$175.50; \$218.40; \$252.72; \$117.00; \$149.76.
23. 1200 boys. 24. \$2 per oz. 25. \$12; \$8; \$4.80.

EXAMPLES XCVI.—PAGE 210.

1. First, \$44.25; second, \$88.50.
2. *A*, $\$75\frac{177}{205}$; *B*, $\$84\frac{12}{41}$; *C*, $\$101\frac{31}{205}$; *D*, $\$170\frac{142}{205}$.
3. *A*, \$4.50; *B*, \$6.75; *C*, \$11.25.
4. *A*, \$2062.40; *B*, \$2320.20; *C*, \$773.40.
5. *A*, \$1350; *B*, \$1890; *C*, \$2100; *D*, \$3450; *E*, \$247.50; *F*, \$412.50.
6. *A*'s, \$11600; *B*'s, \$9062.50. 7. *A*, $\$656\frac{820}{1649}$; *B*, $\$286\frac{829}{1649}$.
8. *D*, \$20; *E*, \$50. 9. *A*, \$87.50; *B*, \$120; *C*, \$202.50.
10. \$30; \$48; \$28. 11. \$15.30; \$14.25. 12. *A*, \$245; *B*, \$225.
13. Johnston, \$585; Wilson, \$487.50; Miller, \$175.50.
14. *A*, \$3355; *B*, \$5830; *C*, \$1400. 15. *A*, \$34.30; *B*, \$53.90.
16. *A*, \$735; *B*, \$490; *C*, \$367.50; *D*, \$294.
17. *A*'s, $16\frac{5}{7}$ gal., and *B*'s, $25\frac{1}{14}$ gal.
18. *A*, \$118.30; *B*, \$55.90; *C*, \$13. 19. *B*, \$2250; *C*, \$5625.

EXAMPLES XCVII.—PAGE 213.

1. Net gain, \$1974; *A*'s, \$2312; *B*'s, \$2172.
2. Net loss, \$3165; *A*'s, \$2836; *B*'s, \$1154.
3. Net loss, \$3550; *A*'s, \$1010; *B*'s, net insolvency, \$2730.

EXAMPLES XCVIII.—PAGE 218.

1. 18.825 carats. 2. $12\frac{1}{2}$c. 3. 73.25°. 4. $72\frac{2}{7}$%.
5. 5 lb. of first, 7 lb. of second.
6. 30 bu. oats; 20 bu. rye; 20 bu. barley.
7. 50 lb. @ 55c.; 30 lb. @ 75c. 8. 15 gal. water.
9. 12 gal. kerosene. 10. 14 bu. rye; 14 bu. barley.
11. 18 lb. @ 14c.; 18 lb. @ 18c.; 48 lb. @ 30c.
12. 36 lb. @ 33c.; 36 lb. @ 37c.; 48 lb. @ 45c.

13. 60 @ \$1.50, 60 @ \$2, 120 @ \$2.75, 30 @ \$3, and 30 @ \$4.

14. 9 lb. @ 40c., and 9 lb. @ 50c.

15. 10 lb. @ 30c., 10 lb. @ 35c., 20 lb. @ 40c., 50 lb. @ 45c., and 10 lb. @ 50c.

16. 12 bu. corn, 24 bu. rye, 12 bu. barley, 12 bu. oats.

17. \$1.50. 18. 30 men, 5 women, 20 boys.

19. 14 calves, 42 sheep, 98 lambs.

20. 92 geese, 4 pigs, and 4 calves. 21. 130 lb.

EXAMPLES XCIX.—PAGE 224.

1. \$250.625. 2. \$2298.05. 3. \$2725.50, nearly.

4. 109$\frac{1}{2}$. 5. \$44693.20. 6. 2 francs 13 centimes.

7. 1760 copeks. 8. 9 fl. 20 krs. 9. £576 12s. 6d.

10. £1 = \$4.8665. 11. London gives 25 francs 45c. for £1.

12. £1 = 13$\frac{2}{9}$ marcs banco. 13. \$4.86. 14. 2602$\frac{1}{8}$ florins.

15. 53$\frac{1}{2}$d. per milree, nearly. 16. .0102045 oz.; 25.17 francs.

EXAMINATION PAPERS—PAGE 226.

I.

1. 1.2372. 2. \$5774.43.

3. Dir., \$14224.91; cir., \$14476.72; gain, \$251.81. 4. 2.341 % dis.

II.

1. 78$\frac{3}{4}$c. and 66$\frac{3}{4}$c. 2. *A*, \$4912; *B*, \$6168.

3. £1 = 25.35$\frac{15}{23}$ francs. 4. 10 and 4.

III.

1. 33$\frac{1}{3}$ lb. of 8, 10, and 12c., and 100 lb. of 20c.

2. \$1212. 3. \$1257$\frac{1}{7}$. 4. $\frac{81}{256}$. 5. £2 3s. 2$\frac{1}{2}$d., nearly.

IV.

1. \$2211$\frac{9}{11}$. 2. \$43.63.

3. Paris, \$14285.71$\frac{3}{7}$; London, \$14600; Amsterdam, \$14640.

4. 1 lb. @ 8; 8$\frac{1}{2}$ lb. @ 13; 2$\frac{2}{3}$ lb. @ 14.

V.

1. 118$\frac{4}{7}$ %. 2. 8, 10 and 12 mo. 3. £9176$\frac{8}{17}$. 4. .42; 23$\frac{9}{17}$ %.

EXAMPLES C.—PAGE 230.

1. $\frac{4}{9}$ is greater. 2. $\frac{19}{41}$ is greater. 3. $\frac{13}{24}$ is least; $\frac{4}{7}$ is greatest.

4. 45 : 364. 5. 112 : 405. 6. \$31.25. 7. $\frac{13}{40}$.

8. $\frac{1}{2}$; 3$\frac{1}{3}$. 9. 128 : 1. 10. 9 : 13.

12. *A*'s rate : *B*'s as 159 : 120. 13. 2 hr. 36 min. p. m.

EXAMPLES CI.—PAGE 233.

1. 4 : 3 :: 12 : 9. 2. 12$\frac{3}{4}$. 3. .0076. 4. *A* : *C* :: 25 : 39.

5. 21. 6. $\frac{10}{21}$. 7. .048. 8. 28.

9. $\frac{5}{27}$. 10. 17.68. 11. \$1560. 12. $\frac{48}{175}$.
13. *A*, \$552; *B*, \$450; *C*, \$345; *D*, \$230.
14. *A*, \$3000; *B*, \$4500; *C*, \$5625; *D*, \$6562.50.
15. *A*, \$67.50; *B*, \$54; *C*, \$45. 16. 55, 77, 99, 121.

EXAMPLES CII.—PAGE 235.

1. £1285. 2. 10 hr. 40 min. $36\frac{7}{48}$ sec. 3. $\frac{6}{13}$ mi.
4. 3 hr. 25 min. p. m. 5. 10 da.; $12\frac{6}{7}$ da. 6. \$47.13$\frac{1}{3}$. 7. $78\frac{3}{4}$.
8. 8 p. m. Thursday. 9. 7722 stones. 10. 12800.

EXAMPLES CIII.—PAGE 237.

1. 54 men. 2. 1050 men. 3. 18 men. 4. 50 men.
5. Navvies did 6 times as much as soldiers. 6. $12\frac{53}{256}$ dronas.
7. 576. 8. $16\frac{2}{3}$. 9. 155 da. 10. 12 da.
11. 4 men. 12. 30c. 13. $9\frac{1}{3}$ ft. 14. 24 oz.
15. 340 ft. 16. $18\frac{2}{3}$ da. 17. 117 da.

EXAMPLES CIV.—PAGE 240.

1. (*a*) 35 sq. ft. (*b*) 135 sq. ft. (*c*) $300\frac{3}{8}$ sq. ft.
(*d*) 12 sq. ft. (*e*) $452\frac{5}{6}$ sq. ft. (*f*) 224 sq. ft.
(*g*) 608 sq. ft. (*h*) $356\frac{1}{2}$ sq. ft.
2. (*a*) $30\frac{1}{4}$ sq. yd. (*b*) $1406\frac{1}{4}$ sq. yd. (*c*) $315\frac{1}{16}$ sq. ft.
(*d*) $870\frac{1}{4}$ sq. ft. (*e*) 91 sq. ft. 121 sq. in.
(*f*) $11\frac{1}{9}$ sq. ft. (*g*) 502 sq. ft. 73 sq. in.
(*h*) 2232 sq. ft. 81 sq. in.
3. (*a*) 16 ft. (*b*) 7 ft. 5 in. (*c*) 8 ft. 9 in.
(*d*) 11 yd. (*e*) 88 yd. (*f*) 99 yd.
4. (*a*) 103 ft. (*b*) 405 ft. 5. 64 sq. in.

EXAMPLES CV.—PAGE 241.

1. (*a*) $28\frac{8}{9}$ yd. (*b*) 67 yd. (*c*) $142\frac{52}{81}$ yd. (*d*) $46\frac{8}{27}$ yd. (*e*) 58 yd.
2. (*a*) \$33.60. (*b*) \$90.93$\frac{1}{3}$. (*c*) \$83.89$\frac{1}{2}$. (*d*) £11 9s. 8d.

EXAMPLES CVI.—PAGE 243.

1. (*a*) 630 sq. ft. (*b*) 855 sq. ft. (*c*) $875\frac{7}{8}$ sq. ft. (*d*) 798 sq. ft.
2. (*a*) \$25.60. (*b*) \$13.62. (*c*) £6 13s. $2\frac{1}{3}$d. (*d*) £6 6s. $9\frac{9}{10}$d.

EXAMPLES CVII.—PAGE 244.

1. (*a*) 28 sq. per. (*b*) $21\frac{1}{12}$ sq. yd. (*c*) $252\frac{15}{16}$ sq. ft. (*d*) $341\frac{1}{4}$ sq. yd.
2. $137\frac{1}{2}$ yd. 3. $2\frac{1}{2}$ ac. 4. \$100. 5. 1120 yd. 6. 55 ft.

EXAMPLES CVIII.—PAGE 245.

1. (*a*) 105 sq. ft. (*b*) $688\frac{1}{2}$ sq. in. (*c*) 54.192 ac.
2. 27 ft. 3. $27\frac{13}{15}$ chains. 4. $237\frac{1}{3}$ ft.

EXAMPLES CIX.—PAGE 246.

1. 19500 sq. ft. 2. $18\frac{3}{4}$ ac. 3. 36 ft.
4. $28\frac{2}{3}$ sq. ft. 5. 8.284 sq. chains. 6. 245 sq. yd.

EXAMPLES CX.—PAGE 248.

1. (*a*) 58 ft. (*b*) 169 in. (*c*) 185 ft. (*d*) 274 yd.
2. (*a*) 374 in. (*b*) 608 ft. (*c*) 1012 yd. (*d*) 532 in.
3. (*a*) 64 yd: (*b*) 69 ft. (*c*) 532 in. (*d*) 896 ft.
4. 12 in. 5. 346 yd. 6. 17 ft. 7. $112.80.
8. 163.2...yd. 9. 883.875 ft. 10. 76.12 ft. 11. 13 ft.
12. 61.6 ft. 13. 92.95 ft.

EXAMPLES CXI.—PAGE 250.

1. (*a*) $2\frac{2}{5}$ in. (*b*) $4\frac{8}{13}$. (*c*) $6\frac{18}{25}$. (*d*) $8\frac{32}{41}$. (*e*) $10\frac{50}{61}$.

EXAMPLES CXII.—PAGE 251.

1. (*a*) 1026 sq. yd. (*b*) 600 sq. ft. (*c*) 2250 sq. yd.
(*d*) 84 sq. chains. (*e*) 690 sq. ft. (*f*) 19536 sq. chains.
2. 14160 sq. ft. 3. On 14, 12 ft.; on 13, $12\frac{12}{13}$ ft.; on 15, $11\frac{1}{5}$ ft.
4. 133837 sq. ft. 5. 29 ft., 52 ft., 69 ft.

EXAMPLES CXIII.—PAGE 252.

1. (*a*) 44 ft. (*b*) 19.8 ft. (*c*) $24866\frac{2}{7}$ mi. (*d*) 1518 ft.
2. (*a*) $59\frac{1}{2}$ ft. (*b*) 21.7 ft. (*c*) 3 ft. $9\frac{1}{2}$ in. (*d*) 8.05 mi.
3. (*a*) 11 ft. (*b*) $102\frac{2}{3}$ ft. (*c*) 29.92 chains. (*d*) $146\frac{2}{3}$ ft.
4. $51\frac{3}{7}$ ft. 5. 11 in., $13\frac{1}{5}$ in., $17\frac{3}{5}$ in., $82\frac{1}{2}$ in. 6. $6187\frac{1}{2}$ sq. ft.
7. 4136 ft. 8. $75\frac{3}{7}$ in.; or $81\frac{5}{7}$ in. 9. 9 mi. 10. 7 ft.

EXAMPLES CXIV.—PAGE 254.

1. (*a*) 1386 sq. ft. (*b*) 3850 sq. in. (*c*) 7546 sq. in.
(*d*) $471\frac{5}{8}$ sq. ch. (*e*) $9735314\frac{2}{7}$ sq. yd. (*f*) $38028\frac{4}{7}$ sq. ft.
2. (*a*) 63 ft. (*b*) $3\frac{1}{2}$ yd. (*c*) 5 6 yd. (*d*) 21 ch.
3. 110 sq. in. 4. 246.67 yd. 5. 336 in. 6. 2464 sq. in.
7. 19.79 in. 8. 14 yd.; 11704 sq. yd.; 49896 sq. yd.
9. $962\frac{1}{2}$ sq. in. 10. $3\frac{1}{2}$ ft. 11. $282\frac{6}{7}$ in. 12. 42 ft.
13. 702.048 in. 14. Square, 3748096 sq. in.; circle, 4770304 sq. in.

EXAMPLES CXV.—PAGE 255.

1. 36 sq. ft. 2. $5\frac{5}{12}$ sq. ft. 3. $30\frac{4}{5}$ sq. ft. 4. 8 ft.
5. Length, $42\frac{1}{6}$ in.; area, $442\frac{3}{4}$ sq. in. 6. 144°; 88 ft.

EXAMPLES CXVI.—PAGE 256.

1. 24 in. by 32 in. 2. 1 to 660. 3. 36 min. 4. $4\frac{4}{9}$ ac.
5. $1240\frac{1}{3}$ sq. ft. 6. 1280 sq. mi. 7. $67\frac{1}{2}$ sq. ft. 8. $1\frac{9}{16}$ ac.

9. $38\frac{2}{5}$ ft.; $28\frac{4}{5}$ ft. 10. 220 yd. 11. $390.
12. 16 min.; $26\frac{2}{3}$ min.

MISCELLANEOUS EXAMPLES.—PAGE 257.

1. 210 ft. 2. £13 10s. 3. $5.95. 4. $1.20. 5. 12 ft.
6. $17\frac{1}{2}$ ft. 7. 1 ft. $9\frac{1}{3}$ in. 8. $12. 9. 5952 stones.
10. $13.50. 11. 429 yd.; 715 yd. 12. $10511\frac{1}{9}$ sq. yd.; $2955\frac{5}{9}$ sq. yd.
13. 26 yd. 14. $69.20. 15. 2 ft. 16. 300. 17. 11250 sq. ch.
18. 221 ft. 19. 5.6 ft. 20. 39 ft., 60 ft., 63 ft. 21. 22 ft.

EXAMPLES CXVII.—PAGE 260.

1. (*a*) 336 cu. ft. (*b*) $548\frac{5}{8}$ cu. ft. (*c*) $83\frac{73}{96}$ cu. ft. (*d*) $850\frac{185}{216}$ cu. ft.
(*e*) $1058\frac{17}{432}$ cu. ft. 2. $421\frac{7}{8}$ cu. in. 3. $61\frac{3}{4}$ sq. ft.; $31\frac{7}{8}$ cu. ft.
4. 8 in.; 512 cu. in. 5. $5\frac{1}{4}$ ft. 6. 9600. 7. $14088\frac{15}{16}$ t.
8. 500 men. 9. 1 ft. 7 in. 10. $2031\frac{1}{4}$ lb. 11. $\frac{1}{8}$ cwt.
12. $5\frac{1}{2}$ ft. 13. 160. 14. $3\frac{3}{4}$ ft. 15. £38 19s. 2d.

EXAMPLES CXVIII.—PAGE 262.

1. (*a*) $14\frac{2}{3}$ sq. ft. (*b*) $93\frac{1}{2}$ sq. ft. (*c*) $4\frac{7}{12}$ sq. ft. (*d*) 33 sq. ft.
2. (*a*) $38\frac{29}{36}$ sq. ft. (*b*) $945\frac{5}{16}$ sq. in. (*c*) $682\frac{22}{25}$ sq. in. (*d*) $356\frac{8}{9}$ sq. in.
3. $1018\frac{2}{11}$ sq. in. 4. $$28.87\frac{1}{2}$. 5. (*a*) 176 cu. ft. (*b*) $57\frac{3}{4}$ cu. ft.
(*c*) $269\frac{1}{2}$ cu. in. (*d*) $3118\frac{1}{2}$ cu. in. 6. $19\frac{1}{4}$ cu. yd. 7. $247.50.
8. 1188 cu. in. 9. $14\frac{2}{3}$ cu. ft. 10. $27\frac{1}{2}$ cu. ft. 11. $36\frac{2}{3}$ cu. ft.
12. $3\frac{3}{7}$ ft. per hr. 13. 14.4 in. 14. 11.2 in., nearly.
15. 1694 coins. 16. 3 ft. 17. 4752 lb.

EXAMPLES CXIX.—PAGE 264.

1. 1125 sq. ft. 2. 1053 sq. ft. 3. 432 sq. ft. 4. $37\frac{1}{2}$ sq. ft.

EXAMPLES CXX.—PAGE 265.

1. 630 cu. ft. 2. 2250 cu. ft. 3. $342\frac{5}{6}$ cu. ft.
4. 13400 lb. 5. $3466145\frac{31}{81}$ cu. yd. 6. 9680 cu. ft.

EXAMPLES CXXI.—PAGE 266.

1. 66 sq. yd. 2. $554\frac{2}{5}$ sq. ft.; $592\frac{9}{10}$ sq. ft. 3. 242 sq. ft. 4. $6\frac{2}{3}$c.
5. $421\frac{3}{32}$ sq. ft. 6. $2\frac{5}{36}$ ft. 7. $22\frac{44}{63}$ sq. yd. 8. $51\frac{1}{3}$ sq. ft.

EXAMPLES CXXII.—PAGE 267.

1. $282\frac{1}{3}$ cu. ft. 2. $19\frac{1}{4}$ cu. ft. 3. 14 ft. 4. $26\frac{22}{25}$. 5. $55\frac{605}{864}$ cu. ft.

EXAMPLES CXXIII.—PAGE 268.

1. (*a*) $273\frac{7}{9}$ sq. in. (*b*) 3850 sq. in. (*c*) 1386 sq. in. (*d*) 5544 sq. in.
2. 14 in. 3. 21 in. 4. $$5.47\frac{5}{9}$ 5. 46.28.. ft.
6. $196742052\frac{4}{7}$ sq. mi. 7. $16\frac{1}{2}$ ft. 8. $666\frac{2}{3}$ mi.

Examples cxxiv.—Page 269.

1. (*a*) $113\frac{1}{7}$ cu. in. (*b*) $905\frac{1}{7}$ cu. in. (*c*) $1767\frac{6}{7}$ cu. in (*d*) $1437\frac{1}{3}$ cu. in.

2. $822\frac{6}{7}$ cu. in. 3. $179\frac{2}{3}$ cu. ft. 4. 64000. 5. $977\frac{5}{11}$ yd.

6. 4 in. 7. .805 . . . : 1. 8. 1.3817 : 1.

9. $510\frac{5}{7}$ oz. 10. $606\frac{3}{8}$ cu. in. 11. .8284 . . in.

Examples cxxv.—Page 271.

1. $119\frac{1}{4}$ sq. ft. 2. $247\frac{1}{2}$ sq. ft. 3. $3300062\frac{1}{2}$ gal. 4. $23.24 . . .

5. 225500 gal. 6. $28\frac{1}{2}$ cu. ft. 7. 88 cu. ft.

Examples cxxvi.—Page 272.

1. 243 oz. 2. 54 oz. 3. $5 : \sqrt{7}$. 4. 28.845 ft. 5. 64 : 125.

6. $3\frac{3}{4}$ in. 7. 13 ft. 8. .7937 ft. 9. 12 in. 10. $10.

11. 13.39 in. 12. $\frac{169}{343}$. 13. 7 in.

Miscellaneous Examples.—Page 273.

1. 42; 33; 22. 2. $4.40. 3. 41.568 cu. ft. 4. 60 in.

5. $523\frac{2}{7}$ cu. in. 6. 35 in. 7. $20\frac{5}{6}$ in. 8. $6647\frac{2}{3}$ cu. in.

9. $269\frac{1}{2}$ cu. ft. 10. $184\frac{5}{6}$ in. 11. $45\frac{5}{6}$ cu. ft. 12. 584 lb.

13. 12 in. 14. 1828000 cu. ft. 15. $69\frac{1}{7}$ in. 16. $1571\frac{3}{7}$ sq. in.

17. $7\frac{1}{2}$ ft.; $2\frac{1}{2}$ ft. 18. $48\frac{1}{8}$ in. 19. 3823.2638 lb.

Examples cxxvii.—Page 279.

3. $\frac{P^2}{M}$ 5. 22.5 years, nearly.

6. *A*'s share is $\frac{PR^{b+c}}{R^{a+b}+R^{a+c}+R^{b+c}}$.

7. 5 %. 8. $\frac{\log. 2.}{\log. (mn-m+n)-\log. mn}$.

10. $n=20\frac{\log. 6}{\log. e.}$. 11. 17.67 years.

12. 6708.471. 13. 125 years, nearly. 14. $3600.

Examples cxxviii.—Page 284.

1. 706.66\frac{2}{3}$. 2. $13585. 3. $19.50. 4. $900.

5. $100\frac{B-A}{A}$ %. 6. $3\frac{27}{71}$. 7. $aR^{p-m}-bR^{p-n}$.

8. $\frac{Aa}{a+r}$ 9. 11.463 %. 10. $1\frac{1}{2}$ yr.

Examples cxxix.—Page 292.

1. 26666\frac{2}{3}$. 2. $7453.87 . . . 3. 3.926 . . times the annuity.

4. $8916.13. 5. $159.008 . . . 6. $107.73. 7. $1959.54.

8. At simple interest, \$2237.77; at compound interest, \$2173.10.

9. \$802.42. 10. \$16666.66$\frac{2}{3}$ 11. \$2199.95.

12. \$7360.08; \$6404.74; \$2901.83. 13. \$3090.56.

14. $\frac{\pounds 150.76}{1-(1.04)^{-30}}$ 15. $\frac{dr\,R^a}{R^c-1}$.

16. $\frac{P}{1-R^{-n}}(R^{-p}-R^{-q})$ 17. $\frac{a}{b}=2\left\{1-\frac{1}{R^n}\right\}$.

18. $\frac{1}{r}\left\{1-\frac{1}{\left(1+\frac{r}{n}\right)^{mn}}\right\}$ 19. \$53.63. 20. \$2422.85.

Simple Rules.—Page 294.

1. 113. 2. \$210. 3. 7$\frac{9}{13}$ mi. 4. 77190.

5. 42238274625; 1959$\frac{39}{63}$. 6. 90 mi. 7. 400 mi.

8. \$115. 9. 5s. 10. 90 da. 11. 3998936616. 12. *B*.

13. 478, 369, 584. 14. \$46850. 15. \$22950. 16. 37199.

17. Min., 921654; sub., 184796.

Factors, Measures and Multiples.—Page 295.

1. 840 sec. 2. *B*; 16 yd. 3. 460800. 4. 32.

5. 30; 360; 36, 40, 45, 60, 72, 90, 120, 180. 6. 28 bu. 7. 35 mi.

8. 7113120 da. 9. *A*, 21 mi.; *B*, 14 mi.; *C*, 12 mi.; *D*, 10$\frac{1}{2}$ mi.

10. 1141140 rd. 11. 900. 12. 2521777. 13. 14, 21, 35.

14. Div., 561; Quot., 943. 15. \$1152.

16. 65, 78, 91, 143; 13, 65, 91, 858. 17. 27, 29, 31, 33, 35.

Vulgar Fractions.—Page 297.

1. \$4200. 2. 2691$\frac{13}{17}$ yd. 3. \$3733$\frac{1}{3}$. 4. \$9.37$\frac{1}{2}$. 5. \$12705.

6. 4$\frac{1}{21}$. 7. 46. 8. 1520 t. 9. 10861578, nearly.

11. 9000 men. 12. 42000 men. 13. 400 in. 15. 4$\frac{1}{2}$ mi.

16. 1$\frac{1}{6}$ mi. 17. 7. 18. 2s. 6d. 19. 5 hr. 20. 54 sheep.

21. $\frac{27\frac{1}{2}}{71\frac{1}{2}}$. 22. 36400. 23. 45 ft. and 50 ft. 24. \$1575. 25. 6 yr.

Decimals.—Page 299.

1. $\frac{1000010001}{500000}$. 2. $.4\dot{3}\dot{2}$. 3. .0189... 4. .75. 5. 14.

6. $\frac{9001}{400}$; $\frac{13}{1600}$; .21; 2100. 7. $\frac{6667}{3630}$. 8. $\frac{6}{37}$. 9. 1.

10. 937; .02268 in. 11. 2.198 in. 12. 345 boys; 15 masters.

13. .05. 14. Between .0001 and .0002. 15. £62 5s.

16. 8085. 17. .632120558. 18. 3.141592. 19. .00097061.

Involution and Evolution.—Page 300.

1. 12.96; $\frac{23}{49}$. 2. 256. 3. 89$\frac{81}{196}$; .000365. 4. 900.

5. 103.67; 574. 6. 124.001; $\frac{8}{9}$. 7. 70.41; 1.46. 8. .3.

9. .03. 10. 862500. 11. .132 in. 12. 4.5 ft. 13. 45.
14. 4007. 16. 9261000. 17. 49; 8.549. 18. 613089; 616225.

Reduction and Compound Rules.—Page 301.

1. 6. 2. 45 mi. 3. $\frac{331}{352}$. 4. 21 yd. 2 ft. $2\frac{5}{6}$ in. 5. 60 in.
6. 1.2535... lb. 7. $.\dot{7}4\dot{0}$. 8. 4497. 9. 4000 ft. 10. $7442\frac{2}{35}$ lb.
11. 56 yd. 12. £9 3s. 6d. 13. 160 oz.; 623 sovereigns.
14. £1 11s. $4\frac{1}{5}$d. 15. 90. 16. 800000. 17. 2.1723; .065169 t.
18. 6 oz. 19. 40 : 53.

Metric System.—Page 303.

1. .393696 in. 2. $111835\frac{3}{7}$ m. 3. 1609.306... m.
4. 27951 : 12500. 5. £84. 6. 2.20 lb. 7. 15.495... mi.
8. 10.101 g.; .010101 Kg. 9. .072508 sq. Km.
10. 4545 cu. cm. 11. $3.29... 12. 1.039... Kg.
13. 1.452 Kg. 14. 3072 Kg. 15. 7911.5... mi.

Problems Relating to Work Done.—Page 304.

1. 108 men. 2. 36 da. 3. 3 hr. 4. $107\frac{85}{193}$ da. 5. 8 da.
6. *A*, $6\frac{22}{27}$ da.; *B*, $9\frac{13}{19}$ da.; *C*, $14\frac{2}{13}$ da. 7. 4 da.
8. *B* walks a mile in $13\frac{1}{4}$ min.; he lost by $11\frac{1}{4}$ min., and by $\frac{45}{53}$ mi.
9. 10. 10. $3\frac{1}{2}$ da. 11. $3\frac{1}{3}$ hr. 12. $28\frac{4}{7}$ da. 13. $7\frac{1}{3}$ mi.
14. $16\frac{2}{3}$ mi. 15. 48 min. 16. 6 hr. 30 min. p.m. 17. $1\frac{9}{17}$ hr.
18. $4\frac{4}{5}$ da. 19. $7\frac{1}{5}$ hr.; 18 hr.; $5\frac{1}{7}$ hr. 20. 10 hr. 21. $3\frac{9}{13}$ da.
22. $4\frac{1}{3}$ da. 23. $1\frac{11}{13}$ da. 24. $2.\dot{8}523309\dot{}$ hr. 25. 32 da.
26. 9 da. of 8 hr. each. 27. 70 da. 28. 5 mi. 29. 25.
30. 18 min. 31. *A*, $\frac{1}{3}$; *B*, $\frac{14}{45}$; *C*, $\frac{16}{45}$. 32. $5\frac{3}{7}$ da. 33. $\frac{3}{85}$ more.

Problems Relating to Clocks.—Page 307.

1. 5 hr. 48 min. 2. 26 sec. 3. 4 hr. 32 min. 4. $1\frac{3}{32}$ min. to 12.
5. 9 hr. 11 min.; 8 hr. 54 min. 30 sec. 6. 4 times.
7. $13\frac{11}{13}$ min., and $16\frac{4}{11}$ min. past 3. 8. $\frac{1}{11}$ min. 9. $13\frac{29}{31}$ min.
10. 12 hr. $10\frac{10}{143}$ min. 11. 14 min. $43\frac{3}{4}$ sec. 12. 2 hr. 24 min.
13. 4 hr. $29\frac{1}{11}$ min. 14. 4 hr. 40 min. 15. 5 hr. 20 min.
16. $30\frac{390}{1427}$ sec. past 12; $59\frac{13}{73}$ sec. past 12; 1 min. $1\frac{683}{697}$ sec. past 12.

Problems Relating to the Sum and Difference of Two Rates.—Page 308.

1. $8\frac{1}{3}$ min. 2. 3 mi. per hr.; $1\frac{1}{2}$ mi. per hr. 3. 1 : 23. 4. $3\frac{1}{2}$ hr.
5. 108 mi. 6. $51\frac{3}{7}$ min. 7. 16 mi. 8. $2\frac{1}{2}$ mi. per hr. 9. 2 mi.
10. 3 mi. per hr. 11. 45 mi., and 30 mi. per hr.
12. 30 mi. and 25 mi. per hr. 13. 60 mi. distant. 14. 132 yd.
15. $23\frac{1}{8}$ mi. 16. 114 yd.

SCALES OF NOTATION.—PAGE 310.

1. 2054; 41; 34. 2. 1613. 3. *t*100; *e*0.
4. 345; 303, 141501; 42. 5. 1445. 6. 4032. 7. 41.29.
8. 3630.349609375. 9. 65 times. 10. 1 lb. 16 lb., and 256 lb.
11. Put 1 lb. 3^2 lb., and 3^8 lb. in one scale, and 3 lb. and 3^5 lb. in the other.

AVERAGES.—PAGE 310.

1. 70s. 5d. 2. $10\frac{6}{19}$ yr. 4. 5730. 5. 155 lb. 6. 10139 ft.
7. *A*'s, $1200; *B*'s, $900. 8. $2\frac{1}{2}$ doz., and $7\frac{1}{2}$ doz.
9. 7 boys to 3 girls.

PERCENTAGE—PAGE 311.

1. $823.68. 2. $4906.25. 3. 4 %; 5 %. 4. $450. 5. £600.
6. 50000000 qrs. 7. $4000. 8. 750.
9. Capital, $1000000; receipts, $100000. 10. $20000.
11. $3000. 12. $10000; $4000. 13. £1160. 14. $14\frac{38}{63}$ yd.
15. $2. 16. 5 : 4. 17. Barley, 40c.; wheat, 75c.
18. 214415. 19. $870\frac{2}{5}$ yd.

PROFIT AND LOSS.—PAGE 313.

1. 40 %. 2. Loses 4 %. 3. $214. 4. $\$6.17\frac{1}{2}$. 5. $56\frac{1}{4}$ %.
6. $4500. 7. Loses 40 %. 8. 6c. 9. $3.40. 10. $14\frac{6}{11}$ %.
11. $37\frac{5}{51}$ %. 12. $92; $115. 13. Loss, $1. 14. $200.
15. Loss, 5 %. 16. $6.50. 17. $5.60. 18. $4.30. 19. 50 %.
20. 8 % 21. $\$13\frac{1}{3}$. 22. $6\frac{2}{3}$ %. 23. $73\frac{2}{3}$c. 24. Loses $\frac{1}{4}$ %.
25. 70c. 26. $\$1\frac{17}{18}$; loss, $2\frac{7}{9}$ %. 27. $26\frac{2}{3}$c.; loss, $11\frac{1}{9}$ %.
28. Loss, $200. 29. $48. 30. $1800. 31. $1600; $2400.
32. $66\frac{2}{3}$ %. 33. Loss, $6\frac{1}{4}$ %. 34. $1298.96... 35. $1.

COMMISSION, TAXES, ETC.—PAGE 316.

1. $20000; $50. 2. $9.50. 3. $\frac{7}{8}$ %. 4. $96\frac{7}{8}$c. 5. 7760 yd.
6. $5141. 7. $3\frac{3}{5}$ %. 8. $7175. 9. $2450. 10. $\$53\frac{1}{3}$.
11. $63. 12. $1\frac{3}{4}$c. 13. $7800. 14. $1440.

INTEREST.—PAGE 317.

1. $\$32.66\frac{2}{3}$. 2. $\$2\frac{2}{3}$; 10 %. 3. $1785. 4. 7 %. 5. $1680.
6. $7\frac{1}{2}$ %. 7. $410; $800. 9. 30 yr. 10. $13\frac{1}{3}$ yr. 11. $7500.
12. $3750. 13. 8.243... %. 14. 5 %, 10 %.
15. Second is greater by $50. 16. $5\frac{1}{2}$ yr. 17. $1200.
18. $12800. 19. 4 %. 20. 16 yr. 21. $316.06.
22. $150000; 4 %.

DISCOUNT.—PAGE 319.

1. $\$166.66\frac{2}{3}$. 2. $49.50. 3. $5\frac{5}{11}$ %; $\$17\frac{13}{58}$. 4. $3600; $5\frac{5}{9}$ %.

5. $\$.95\frac{1}{16}$. 6. $16. 7. $67.50. 8. $9.50. 9. $4\frac{1}{2}$ mo. 10. 18c.
11. $215. 12. $2. 13. 4 %. 14. $316.80. 15. $6120.
17. $\$\frac{14}{55}$. 18. $35. 19. $17\frac{6}{7}$ %. 20. $4\frac{1}{2}$ %. 21. £150. 22. 5 %.
23. $40800. 24. $3\frac{1}{2}$ %. 25. $\$274\frac{1}{8}$; $\$456\frac{1}{4}$. 26. $1023.75.
27. $5100. 28. $14600. 29. $\$2\frac{28}{51}$; $\$2\frac{2672}{5099}$.

Equation of Payments.—Page 321.

1. $7\frac{1}{2}$ mo. 2. $1\frac{1}{3}$ mo. 3. 13 mo. 4. $6\frac{6}{7}$ mo.
5. $8\frac{62}{89}$ mo.; $8\frac{2}{3}$ mo. 6. March 23, 1900. 7. $3\frac{1}{3}$ mo.

Stocks.—Page 322.

1. 4 per cents; $128700. 2. $2.45. 3. $1.50.
5. $7\frac{1}{2}$ %; £50 less. 6. 90; $465. 7. £136, 9s. 2d.; $6\frac{1}{4}$ %.
8. £60000. 9. 87.5. 10. $760. 11. $2035. 12. $\$23.17\frac{31}{32}$.
13. $24360. 14. $59.40. 15. $\$15208\frac{1}{3}$. 16. $305. 17. £6000.
18. 1500 bbl. 19. Increase, £$66\frac{2}{3}$. 20. Increase, $\$16\frac{2}{7}$.
21. First by $\$88\frac{1}{3}$. 22. $2200. 23. $66. 24. $34947. 25. $6900.

Sharing, Partnership, Etc.—Page 324.

1. $7560; $5670. 2. 12. 3. *A*, $2.49; *B*, $15.81.
4. £142 12s. 6d.; £42 15s. 9d.; £14 5s. 3d. 5. $52.50.; $35.
6. 200; 189; 101. 7. $1925; $770; $154.
8. Nitre, 22 lb.; charcoal, $4\frac{2}{3}$ lb.; sulphur, $3\frac{1}{3}$ lb. 9. $8250.
10. $10, $18, $15. 11. *A*, 88c.; *B*, $49\frac{1}{2}$c.
12. *A*, $1155; *B*, $572; *C*, $259.50. 13. $6.
14. *A*, $17.50; *B*, $52.50; *C*, $105; *D*, $175. 15. $11835.75.
16. $8400. 17. Child, $1920.60; brother, $960.30.
18. $56000; $48000; $42000. 19. $30.
20. *A*, $3200; *B*, $4800; *C*, $6000; *D*, $7000.
21. Nitre, $15\frac{1}{5}$ cwt.; sulphur, $1\frac{9}{10}$ cwt.; charcoal, $2\frac{9}{10}$ cwt. 22. $39.95.
23. Gold, 4 dwt. $22\frac{1}{4}$ gr.; alloy, $10\frac{3}{4}$ gr. 24. $3\frac{1}{2}$d., and $\frac{1}{2}$d.
25. $234; $266.40; $306; $345.60.
26. *A*, 3240; *B*, 2196; *C*, 1944; *D*, 2052; *E*, 1728; 6480.
27. Man, £4 4s.; woman, £3; child, £1 16s.
28. 375 gr. potash; 390 gr. soda. 29. $6084. 30. 10 men.
31. Men, $79.20; women, $70.40; children, $52.80.
32. $1600; 15 mo. 33. Tea, 3s.; coffee, 2s.; cocoa, 1s. 8d.
34. $1024. 35. $740 @ $4\frac{1}{2}$ %; $860 @ $5\frac{1}{4}$ %.
36. $300; $200; $500. 37. 15; 20. 38. 316.

Alligation.—Page 327.

1. 9 of spirits to 31 of water. 2. 2 : 7. 3. 3 pt. 4. 25c.
5. $82\frac{1}{3}$c. 6. $\frac{1}{3}$. 7. 1 to 2.
8. 40 lb. @ 7c., 40 lb. @ 8c., and 40 lb. @ 9c., and 240 lb. @ 14c.

9. 1 oz. alloy to 2 oz. gold. 10. Turkey, \$1.75; goose, 85c.
11. 360 bu. @ 65c.; 640 bu. @ 68c. 12. 25 gal. 13. 35c.
14. 10 parts milk to 3 of water. 15. 12 gal. 16. 2$\frac{17}{24}$ oz.
17. 4 @ 80, 8 @ 85, 8 @ 100, 80 @ 115. 18. 13 lb. 19. 140 ac.
20. 30 @ \$4, 135 @ \$4$\frac{1}{2}$, 75 @ 5$\frac{1}{2}$, 30 @ \$7$\frac{1}{2}$. 21. \$9.70.
22. 30 gal.

Exchange.—Page 329.

1. \$442. 2. £52, 4s. 6$\frac{6}{61}$d. 3. 94$\frac{1}{2}$c. for 4 marks. 4. 6$\frac{1}{4}$ %.
5. 24.9 francs; 25.15 francs. 6. \$4.81, nearly. 7. \$875.
8. \$2489.72 ... 9. 2 % premium. 10. 18.83c.
11. Through London.

Ratio and Proportion.—Page 330.

1. \$12000. 2. \$16$\frac{7}{8}$. 3. 8 hr. 4. 16 da. 6. 15 hr.
7. 30 da. 8. 24 da. 9. 10.45 a. m. 10. 1 hr. 33 min.
11. 135 men. 12. 63 gal., 105 gal., 135 gal., 153 gal.
13. 11$\frac{7}{11}$ hr. 14. 19.5. 15. 1000 men. 16. $\frac{754}{1269}$ hr.

Mensuration.—Page 332.

1. 3 ft. 2. 75$\frac{1}{6}$ yd. 3. 270 ft. 4. \$3346.87$\frac{1}{2}$. 5. 16$\frac{1}{2}$ ft.
6. 11 sq. ft. 7. £4 1s. 6$\frac{23}{27}$ da. 8. \$60.75; \$20.42$\frac{1}{2}$. 9. 1s. 1$\frac{8}{9}$d.
10. \$1238.70. 11. \$1.12. 12. 1200 gal. 13. 38 barges.
14. £44 13s. 3d.; $\frac{1}{9}$ ft. 15. 47$\frac{1}{15}$ in. 16. \$81.12; 7 yd.
17. 27 in. 18. 474 pictures. 19. 32 ft.; 16 ft.; 8 ft.
20. 48000. 21. 27 ft.; 18 ft.; 12 ft. 22. $\frac{1}{56448}$ in.
23. 28$\frac{1}{2}$; 427$\frac{1}{2}$ lb. 24. \$12.31. 25. 26 yd. 26. 8s. 2$\frac{1}{3}$d.
27. \$20.95. 28. 2$\frac{1}{2}$ in. 29. 256. 30. 168$\frac{3}{4}$ gal. 31. 11$\frac{1}{9}$ times.
32. 15$\frac{5}{8}$ times. 33. 1$\frac{1}{3}$ in. 34. 216; 64; 27; 23$\frac{17}{21}$ gal.
35. $6\sqrt[3]{3}$; $5\sqrt[3]{3}$; $4\sqrt[3]{3}$. 36. 28$\frac{1}{8}$ ft.; 25 ft. 37. 391 ft.
38. 12637 ft.; 12012 ft. 39. 2.1 ac. 40. 11250 sq. yd.
41. 12 sq. chains. 42. 2016 sq. in. 43. 67$\frac{1}{2}$ in.; 54 in.
44. 13 ft. 45. 5540 sq. ft 46. 264 yd.; 165 yd. 47. 70$\frac{2}{5}$ yd.
48. 104 yd. 49. 23.95 ft. 50. 11$\frac{1}{4}$ ft. 51. 2214 in.; 243602 sq. in.
52. 230.94 sq. ft. 53. 2.25 sq. yd. 54. 225 sq. yd.
55. 1400 sq. yd. 56. 220 yd. 57. 187$\frac{1}{2}$ sq. in.
58. 498.83 sq. in. 59. 4056 sq. ft. 60. 32 in.
61. 2.8873 ft. 62. 7854 sq. yd. 63. 1269.74... sq. ft
64. 113$\frac{1}{3}$ ft. 65. 504 sq. ft.; 88 ft.
66. Circle, 616 sq. in.; square, 484 sq. in.; 372.58 sq. in.
67. 88 in. 68. 924 sq. in. 69. 40 mi. 70. 330 mi.
71. 83.136 rods. 72. $\frac{91}{216}$. 73. 98.521... cu. ft.
74. 15.874 in. from the apex.
75. 7.951 in.; 9.410 in.; 12.192 in.; 24.446 in.